Introduction to Matlab

School of Mathematics and Statistics
University of South Australia

February, 2008.

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1 General Information

1.1 Overview

This booklet (guide) aims to introduce students to a number of the essential concepts of Matlab , Release 14, Version 7 (a **MathWorks** product). It can be used in conjunction with with the **MATLAB Practicals Booklet** provided in the courses **Mathematical Methods for Engineers 1** and **Calculus 1**.

This booklet is designed to assist the learning of Matlab relevant to the syllabuses for the first year mathematics courses in the Applied Science and Engineering degree programs at the Mawson Lakes campus of the University of South Australia. The three course textbooks are **Calculus** (7th edition) by C.H. Edwards and D.E. Penney, **Using MATLAB in Calculus** by G. Jensen, and **Understanding Linear Algebra Using MATLAB** by E. & M. Kleinfeld. It is best to read, attempt and fully comprehend the concepts being introduced in this guide before progressing to similar sections in Jensen. However, it is expected that they will complement each other most beneficially.

During the early learning of this software, for example in the courses *Mathematical Methods for Engineers 1* and *Calculus 1*, it is essential that students bring along this guide and MATLAB Practicals Booklet, and a floppy disk or a flash drive (memory stick) to each computer practical session. Although you can obtain assistance from the Practical Supervisor during class, it is expected that you must spend considerable unsupervised time working through the guide and Matlab Practicals Booklet to be able to master the Matlab software.

1.2 Professional Matlab

A number of courses in the later years of engineering programs (especially those related to Electronics, Computer Systems and Telecommunications) will use Professional Matlab together with a collection of various associated mathematical toolboxes. Many postgraduate students in other engineering and science disciplines use Matlab , as well as technical and research establishments that employ graduate engineers. An increasing number of the world's leading universities (and most Australian universities) are incorporating Matlab into their teaching programs. **All** students using this booklet will be expected to know Matlab well enough so that extended computer code can be written in Matlab up to at least the second year of **all** programs.

In this introductory booklet, only Matlab and the Symbolic Math Toolbox will be mentioned.

1.3 What is Matlab ?

At the simplest level, Matlab is a powerful calculator. At the more advanced level, Matlab is a high-performance language for technical computing. It integrates computation, visualisation, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Matlab is an interactive system whose basic data element is an **array** that does not require dimensioning. This allows you to solve many technical problems related to science and engineering, especially those involving data that can be held in matrices and vectors. Because Matlab has been designed around engineering and scientific applications, it requires only a fraction of the time to write a program compared to scalar non-interactive languages such as C or FORTRAN.

Matlab is structured using a large collection of files, called M-files. These are of two quite different types, namely **script** M-files and **function** M-files. You can use these M-files to do arithmetic calculations, numerically solve equations, create graphics, etc. You can use the M-files supplied in Matlab to build up sophisticated computer programs. But, most importantly, you will need to create and store your own M-files to supplement those already provided by Matlab in order to solve your specific problems.

By using the Symbolic Math Toolbox, your Matlab M-files can also include differential and integral calculus, linear algebra, algebraic simplifications, algebraic solution of equations, variable-precision arithmetic, etc.

1.4 Logging On and Logging Off

Matlab is available in the following Computer Pools at Mawson Lakes:
GP1-01, GP1-02, GP1-06, GK3-19, F1-13, F1-15, F1-17, P1-12, P1-13, P1-15. It is remotely possible that Matlab might be temporarily unavailable on a PC at times of maximum usage by other students.

Log onto a computer. Your user name is your Mail ID. The first time you log on, your password is a combination of your surname and your birthday. It uses the first four letters of your surname, followed by the day and month of your birthday. Anne Bloggs, born on February 25, has the password `blog2502`. If your surname has fewer than four letters, fill the rest with the letter x. Hence, Peter Ho, born on December 7, has the password `hoxx0712`.

A group of icons will then appear. One icon is “Change Password”. It is advisable to change your password the first time. Double click the icon. Choose a password that has from 6 to 8 characters, each of which must be a letter or a digit. Close the window afterwards.

There is an icon displaying the computer pool room number followed by the word “Applications”; for example, “GP1-01 Applications”. Double click this icon. Then double click the “Matlab ” folder, followed by the “R2007b” folder and the “Matlab R2007b” icon (or whatever Release is installed).

The default Matlab **Desktop Layout** which appears is shown at the back of this guide. In the “Command History”, you can view previously used functions, and can copy and execute selected lines. However, the “Command Window” is the most important, and this is where you enter variables and commands, and where you define functions and run M-files. We suggest you opt to expand this window by clicking on “View”, followed by “Desktop Layout” and then choosing “Command Window Only”. You can always bring back the other windows later by choosing “Default” under “Desktop Layout”.

In Command Window, you will see “`>>`”: `>>` is the Matlab prompt. When the Command Window is the active window, a cursor should appear to the right of this prompt, indicating that Matlab is waiting to answer a mathematical query or execute a command. We will return to this prompt in Section 2.

When you have finished your Matlab session, and saved on disk or flash drive everything that you wish to save, you need to exit from Matlab . To do this, enter either the command `quit` or `exit` after the prompt, or else click **File** followed by **Exit** Matlab . Perform one of these to return to the icons. (Later, when you have finished doing everything in a session, log off the computer.)

1.5 Getting general HELP on using the computer in the pools

If you are having trouble using a computer in one of the pools, assistance is available. Return to the Desktop Window containing the icons. Double click on “Utilities and Network Apps”, then click on “Internet Explorer”. Click on “Bookmarks”. You can then acquire HELP by clicking on either “UniSA ITS HelpDesk” or “UniSA Computer Pools Information”.

1.6 Getting rid of windows and retrieving them

Quit often while using MATLAB you will have a number of windows in operation simultaneously. For example, you might have the MATLAB Command Window, a couple of windows containing M-files you have constructed, at least one window containing graphical output, and even a Microsoft Word window where you have copied output in preparation for the final printed report. You can reduce an individual window to an icon on the bar at the bottom of the screen, out of the way, and can retrieve it later on. Each window has some grey buttons at the top right-hand corner with symbols on them.

If you wish to move a window out of the way temporarily, click on the left button with a ’-’. It can be recalled later by clicking on the corresponding icon on the bar at the bottom of the screen. The centre button increases or decreases the size of the window. The right button, with an ’x’, removes the window totally.

Some windows may get in the way of the icons. We assume you know how to use the mouse button to move windows around the screen.

1.7 Saving to and Loading from a Storage Device

NOTE: Even though you are not in a position to be storing files on to a storage device such as a disk or a flash drive at this stage, we have included this in the General Information section. You should read this subsection now, and then return later when you have constructed M-files or output which needs to be saved on to a disk or flash drive.

While you are working in a computer pool, any files that you create which need to be accessed should be saved on to the PC’s hard drive, in this case the Scratch e: drive. However, at the end of a session, when you log off the computer, you will lose all such files that you have saved **unless you save them to disk or flash drive**.

1.7.1 Working on a storage device

Soon you will be constructing M-files. These will be explained in more detail later, but previous information has already signalled that there are two quite different types, namely **script** M-files and **function** M-files. A script M-file will contain a sequence of MATLAB commands designed to solve a particular problem. A function M-file is of a different form, and enables you to extend the list of standard MATLAB functions by constructing, naming and saving your own functions. As long as you are in the MATLAB environment you can access all of the M-files which constitute the professional version no matter on which disk drive or flash drive you are working. However, if you need to access an M-file which has been stored on a floppy disk or flash drive, you must change your working directory to that drive. After the `>>` prompt, type in `cd a:` (or another letter, instead of `a`, for the appropriate drive) which changes the directory to the **a:** drive (the floppy disk or flash drive). Then, to get a list of the files that are stored on the disk or flash drive, you can enter `dir` after the prompt.

1.7.2 Saving M-files to a storage device

Let us assume you have an active window (such as the M-File "Editor/Debugger" window) containing an M-file, and you have placed a disk in the disk drive.

Click on **File**, then on **Save As...**.
In the **Save in:** box, specify the location where you want this M-file saved. If you merely wish to save it during this session, you must specify the **e:** Scratch drive on the computer. If you wish to save it on your floppy disk or flash drive, you need to choose the **a:** or another appropriate drive.

In the **File name:** box, you must enter the name you have chosen for this M-file. All MATLAB M-files must have names that finish with **.m** such as **test1.m** or **project.m** or **myfunctn.m** or **prac3wk8.m**, etc. **NOTE:** every year some students cause all sorts of anguish to themselves by selecting a name for their M-file which happens to already be the name of one of MATLAB 's own M-files. This must not be done! For example, do not name your M-file **sin.m** because this is where the trigonometric function $\sin x$ is stored in MATLAB . Never name your M-file **solve.m** because that is the name of a file in the Symbolic Math Toolbox which solves an equation. As there are hundreds of M-files in MATLAB , you need to be careful when naming your own.
Once you have selected a name, click on **Save**.

Thereafter, if you make any changes to this M-file, and you need to save those changes, you click on **File**, then on **Save** in the "Editor/Debugger" window. Alternatively, you could click on the "Save" icon.

1.7.3 Loading M-files from a storage device

After the MATLAB prompt, change directory to the floppy. Type in `dir` just to check what is on the floppy disk or flash drive. Click on **File**, then **Open** and double click the required M-file. This will open the file inside the "Editor/Debugger" window where additions and corrections can be made, and later saved as described above. A typical example of such a window is shown at the back of this guide.

2 Simple Mathematics: MATLAB as a Calculator

2.1 Basic arithmetic operations

You can use MATLAB as a calculator. Firstly, we will perform simple arithmetic using the operations + (addition), - (subtraction), * (multiplication), / (division), ^ (raise to a power, or exponentiation). We will also include parentheses (brackets).

Enter the MATLAB Command Window as described in Section 1.4. Type in each of the following arithmetic expressions after the `>>` prompt and press the Enter key at the end of each line. **Before doing so, mentally calculate the answer you expect to see in each case**.

```
(7+6)/(10-2)
 7+6/(10-2)
  7+6/10-2

 9+(6*2)/(3*4)
 9+6*2/(3*4)
 9+6*2/3*4

 1.00001^100000
```

You will notice that the last expression is very close to the exponential number *e*. This will be explained later in lectures.

Some students are unsure of the use of parentheses (brackets). They often use many unnecessary brackets, thereby making expressions hard to interpret. But other students omit brackets when they are absolutely needed. The order in which these operations are evaluated in a given expression is given by the usual rules of precedence, which can be summarised as follows:

Expressions are evaluated from left to right, with the power operation having the highest order of precedence, followed by both multiplication and division having equal precedence, followed by both addition and subtraction having equal precedence. Brackets can be used to alter this usual ordering.

Exercise: Each of the following two expressions has the value 12 (do the calculations yourself). Type in the equivalent MATLAB expressions so that you obtain the answer 12 in each case. If you make an error and wish to try again, you can retype the expression or you can use the up-arrow ↑ to recall a previous line and then edit it.

$5+\dfrac{9^{3/2}+1}{2(5-3)}$ (needs 3 or 4 pairs of brackets) $\dfrac{17}{\frac{2}{3}+\frac{3}{4}}$ (needs one pair of brackets)

2.2 Variable names

You would have noticed that MATLAB gives the result of each calculation the default name `ans`. This clearly will become confusing and unacceptable when a sequence of commands is executed, especially in an M-file. Hence, you should always give the result a **variable name**. This variable name can then be used in subsequent calculations in a manner similar to putting a result in the memory of a calculator. The rules for variable names are

- variable names are case sensitive; `crows, Crows, cROWs, CROWS` are all different variables
- variable names can contain up to 31 characters; `prideofsouthaustralia` is acceptable
- variable names must start with a letter, followed by a mixture of letters, digits or underscores. Other symbols and spaces are not allowed; `x, ABC, item3week1`, `profit_1999`, `Pride_of_South_Australia` are all acceptable variable names

Type in the following sequence of commands after the prompts on consecutive lines

```
x=7.231
y1=x+1/x
y2=x-1/x
difference=y1^2-y2^2
```

Exercise: explain why the answer must be 4 for any initial `x`. Then use the up-arrow repeatedly to change `x` to any other non-zero value, find new `y1, y2` and demonstrate that `difference` is always 4.

Note: this continual use of the up-arrow will become unmanageable and very confusing when we have a longer sequence of commands. That is where the use of M-files will become crucial.

2.3 Comments, clear, semicolons and Ctrl-C

2.3.1 comments using % sign

All text after a percent sign (%) is taken as a comment statement and is simply ignored by Matlab . It helps you and readers understand what certain variables represent or what outcomes your commands are trying to achieve.

2.3.2 clear previous variables

If you have been working with Matlab in a long session you may wish to re-use a variable name for a different purpose. This is especially important if, say, you had an array variable `x` and now you want to use `x` for a scalar variable or a shorter array `x`. Then you can delete the previous variable from the Matlab workspace using the command `clear x`. You can also delete all variables no longer required using `clear all`. This is useful if you are starting a new problem .

2.3.3 suppress output using ;

A semicolon (;) after a calculation command tells Matlab to suppress the result. When you have a large amount of data, or certain variables are not needed to be shown in the final output, they need not be displayed. They will still be held in memory ready for future use. For example, suppose you want to know where the straight line passing through the points $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (8, 7)$ intersects the y-axis. Type in the following commands:

```
clear all
x1=3, y1=4
x2=8, y2=7
%let m be the slope of the line y=m*x+b
m=(y2-y1)/(x2-x1);
% y-y1=m*(x-x1) intersects y-axis when x=0
% this gives b=y1-m*x1
y_intercept=y1-m*x1
```

Exercise: add a command to find where the straight line intersects the x-axis.

Exercise: You know that two points on the curve $y = \sin x$ are $x_1 = \frac{\pi}{6}$, $y_1 = \frac{1}{2}$ and $x_2 = \frac{\pi}{3}$, $y_2 = \frac{\sqrt{3}}{2}$. In MATLAB , `pi` represents the number π while `sqrt(3)` represents the number $\sqrt{3}$.
Use the up-arrow to change the commands above to find the intercepts on both axes of the straight line joining these two points.

2.3.4 Ctrl-C

Sometimes you will inadvertently set MATLAB into a never-ending loop or cause masses of numbers to scroll down the screen. You can interrupt MATLAB at any time by pressing **Ctrl-C**.

2.4 Elementary Mathematical Functions

All of the mathematical functions on your calculator are available in MATLAB , as well as many other elementary functions. To view a list of the common functions, click on "Help" at the top of the MATLAB Command Window, then click on "MATLAB Help", and then double click on MATLAB `Functions Listed by Category`.
Then click on `Elementary Math Functions`.
(Note there are also `Specialized Math Functions` beyond the scope of this introduction.)
For brief information about any of the elementary functions, merely double click on that function name. You can use the back-arrow near the top of the window to move back to previous windows. Of special significance, note there are the six trigonometric functions as well as their corresponding inverse functions. For example, the MATLAB expression `sin(x)` represents $\sin x$. **Note the use of brackets around the `x`.** Of course, MATLAB only uses natural radian measure and never degrees. Type in the following (non-comment) commands:

```
theta=3*pi/4
y1=sin(theta), y2=cos(theta), y3=tan(theta)
% note than you can put more than one command on a given line,
% separated by commas. Now find the reciprocal of each:
z1=csc(theta), z2=sec(theta), z3=cot(theta)
% Find alpha, beta such that sin(alpha)=0.721 and tan(beta)=-1.78
alpha=asin(0.721), beta=atan(-1.78)
```

There are also the six hyperbolic functions and their inverse functions, to be discussed later in Chapter 6 of Grossman. Of special importance, note that

- the exponential function e^x is represented in Matlab by `exp(x)`
- the natural logarithm function $\ln x$ is represented in Matlab by `log(x)`
- the square-root function $\sqrt{x}$ is represented in Matlab by `sqrt(x)`
- the absolute value function $|x|$ is represented in Matlab by `abs(x)`

Exercises: Let $x = 1.253$. Show that the variables u, v, w, y, z have the values shown by typing in the corresponding Matlab commands:

mathematical expression	answer	Matlab command
$u = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$	0.1820	`u=1/sqrt(2*pi)*exp(-0.5*x^2)`
$v = x \arctan x - \frac{1}{2}\ln(1+x^2)$	0.6523	`v=x*atan(x)-0.5*log(1+x^2)`
$w = \frac{\sqrt[3]{5+\cos 4x}}{\lvert\sin 3x\rvert}$	3.0107	`w=(5+cos(4*x))^(1/3)/abs(sin(3*x))`
$y = \sin^2(\pi x)$	0.5094	
$z = \frac{e^{\sin x}}{\sqrt{x^2+1}}$	1.6128	

2.4.1 Other useful functions

`round(x)`	rounds `x` to the nearest integer
`floor(x)`	rounds `x` **down** to the nearest integer
`ceil(x)`	rounds `x` **up** to the nearest integer
`rem(y,x)`	the remainder after dividing `y` by `x`
`sign(x)`	$=-1$ if `x`<0 ; $=0$ if `x`$=0$; $=1$ if `x`>0
`rand`	generates a random number between 0 and 1

Exercise: You have \$50. Each item costs \$3.35. How many items can you buy and how much change will you have left?

```
number_of_items=floor(50/3.35)
% change default format to one with two decimal places
format bank
change=rem(50,3.35)
```

Note: to view other possible formats, type in `help format` after the MATLAB prompt.

Exercise: There are 211 students to be arranged into tutorial classes which can hold 24 students per class. Use MATLAB commands to determine the number of tutorial classes needed and how many spare places are left.

2.5 Assignment statements

The preceding MATLAB commands that assign the value of the expression after the '=' sign to the variable before the '=' sign are **assignment statements**. Note that all variables in the expression after the '=' sign must have previously been allocated a value, or else an error occurs. For example, enter the following commands:

```
a=3
B=4
c=sqrt(a^2+b^2)
```

You will see the error message `??? Undefined function or variable 'b'.`
Consider the following:

```
clear all
x=7
x=x^2-12
```

The last line has nothing at all to do with a mathematical equation. It is a MATLAB assignment statement that calculates $x^2 - 12$ at $x = 7$ and stores the result in the variable `x`, thereby over-writing the previous value.

3 Script M-files

As has been mentioned already, for simple problems it is easy to enter your requests after the prompt in the MATLAB Command Window. However, as the number of commands increases or whenever you wish to make major changes, this becomes very tedious. Instead, you should place MATLAB commands in a simple text file, and then tell MATLAB to open the file and evaluate all the commands precisely as it would if you had typed them after the prompt. These files are called **script M-files** and must have filenames ending with the extension '.m'. Examples of possible M-file names are **example1.m, proj1Y2000.m, question5.m, mechdevice.m**, etc.

3.1 Creating, saving, executing and editing script M-files

To create a script M-file, click on "File" in the MATLAB Command Window, then on "New" followed by "M-file". This opens the M-file "Editor/Debugger" window, where you can enter your sequence of MATLAB commands, in simple text format without any prompts. See the picture at the end of this guide. As a first example, reconsider the problem of finding the intercepts on the axes from Section 2.3.3. Type in the following:

```
clear all
x1=3
y1=4
x2=8
y2=7
%let m be the slope of the line y=m*x+b
m=(y2-y1)/(x2-x1);
% y-y1=m*(x-x1) intersects y-axis when x=0
% this gives b=y1-m*x1
y_intercept=y1-m*x1
x_intercept=x1-y1/m
```

Now save this file as described in Section 1.7.2. If you are working on the hard drive, specify **e:** in the **Save in:** box, or else **a:** or another drive if you wish to save on a disk or flash drive. In the **File name:** box type in **example1** and enter. You will note that the header now shows the directory location as well as **example1.m** as the name of this M-file.

Return to the MATLAB Command Window and enter `example1` after the prompt. Assuming you have changed to the same directory, this will execute the sequence of commands in **example1.m**.

Return to the "Editor/Debugger" window. Use the mouse, arrows, backspace and delete facilities to change `x1,y1,x2,y2` to the values specified in the second exercise in Section 2.3.3. Then **Save** and again enter `example1` after the `>>` prompt. Of course this can most simply be accomplished using the up-arrow once.

3.2 MATLAB Search Path

Suppose you type in `crow` after the prompt; in other words `>> crow`
How is this to be interpreted? MATLAB interprets this statement using the following hierarchy:

1. It checks to see if you merely wish to print out the value of a variable named `crow` in the current MATLAB workspace. If not,
2. it checks to see if `crow` is a built-in MATLAB command. If not,
3. it checks to see if you wish to execute an M-file named **crow.m** that you have saved in your current working directory. (There is a further possibility not discussed here.)

3.3 Revision exercises

Exercise 1: Suppose you want to find the two zeros of the quadratic $ax^2 + bx + c$. (As you would expect, MATLAB already has a procedure to do this automatically, but let us solve this independently.) Firstly, to solve $5x^2 - 17x - 12 = 0$, you should create, save and execute an M-file **quadzeros.m** (for example)

```
clear all
a=5, b=-17, c=-12
x1=(-b+sqrt(b^2-4*a*c))/(2*a)
x2=(-b-sqrt(b^2-4*a*c))/(2*a)
```

Then change this to find the zeros of each of the quadratics

$$3x^2 + 5x - 13 \qquad 2x^2 - 7x + 23 \qquad 2.34x^2 + \pi x - \sqrt{6}$$

Exercise 2: Suppose the three sides of a triangle have lengths (in cm) $a = 11.7, b = 6.9, c = 7.1$. Find the three angles A, B, C using the cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

and the sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \; .$$

Convert the angles into degrees.

Construct the following in an M-file **triangle.m**, and then execute it.

```
clear all
a=11.7; b=6.9; c=7.1;
% let cA  represent cos(A),  and sB represent sin(B)
cA=(b^2+c^2-a^2)/(2*b*c);
A=acos(cA);
sB=b*sin(A)/a;
```

```
B=asin(sB);
C=pi-A-B;
% convert angles into degrees for output
A=A*180/pi; B=B*180/pi; C=C*180/pi;
fprintf('angle A is %6.2f degrees \n', A)
fprintf('angle B is %6.2f degrees \n', B)
fprintf('angle C is %6.2f degrees \n', C)
```

You will notice that the `fprintf` command produces neat output. In each case, the variable to be printed is at the end, while `%6.2f` stipulates where the value is to be printed along with how many positions (6) and how many decimal places (2). The `\n` moves to a new line after printing whatever precedes it, in preparation for more output.

Exercise 3: Consider the two simultaneous linear equations

$$\begin{aligned} a_{11}x + a_{12}y &= b_1 \\ a_{21}x + a_{22}y &= b_2 \end{aligned}$$

MATLAB provides various ways to solve this automatically, given values for the coefficients, as we shall see later. But for now, you will write your own M-file. You will need to give values for the four coefficients and the numbers on the right hand side, find $detA = a_{11}a_{22} - a_{12}a_{21}$ so that

$$x = \frac{a_{22}b_1 - a_{12}b_2}{detA} \quad \text{and} \quad y = \frac{a_{11}b_2 - a_{21}b_1}{detA} ,$$

providing $detA \neq 0$. Test your M-file on the following two problems:

$$\begin{aligned} 2x - 3y &= 2 \\ 5x + 6y &= 59 \end{aligned} \qquad\qquad \begin{aligned} 3x - 11y &= 17 \\ -6x + 22y &= 23 \end{aligned}$$

The solution of the left system is $x = 7$, $y = 4$. The right system clearly does not have a solution.

Entering Long Expressions: if an expression does not fit on a single line, use three dots ... to indicate that the statement continues on the next line. Do not leave any unnecessary blank spaces and press **Enter** or **Return** immediately to advance to the next line, and continue entering the statement. Be careful to create this break at a sensible point in the expression, preferably after a comma or mathematical operation.

4 Arrays

All of the computations considered so far involve single (scalar) numbers. One of the powers of MATLAB is its ability to do computations on arrays, where the same operation is performed on all numbers in a data array using a single command. Special types of arrays such as matrices and vectors will be discussed in more detail in Section 9, as will mathematical operations specific to them.

4.1 Constructing arrays

We will illustrate four somewhat different ways of constructing a simple array of numbers. The whole array is represented by a variable name along similar lines to scalar variables. For example, `a, x, Y, A3, Xarray` are suitable names for arrays providing they have not already been used for another purpose. The first method of constructing an array is merely to list the numbers in the array, enclosed inside square brackets and divided either by spaces or commas. Type in the following after the prompt:

```
x=[3, -5.1, 0, 6, 11, -17, pi]
y=[8  2  -12  3/4  1.234567 1 10]
```

This is fine if you have only a few numbers to input into an array (seven in each of the above two examples). However, if there are hundreds, we need another method. This is possible if the numbers in the array are equally spaced. We use the **colon notation**. Type in the following after the prompt:

```
A=[-3:21]
B=[3.33:12]
```

Clearly [a:b] is an array starting at a with increments of one until we reach b but not going beyond b. If a and b are **integers** with $a < b$, the number of elements in the array is $b - a + 1$. Why?

Suppose we want an array to extend over the interval from a to b with n equal subintervals of length h. Then clearly $h = \dfrac{b-a}{n}$ and the number of elements in the array will be $n + 1$. Why? For example, suppose you wanted an array of values from 2.5 to 4.0 with 25 subintervals, which implies that the values need to be incremented by a step-size of $h = 0.06$. Why? The number of values in the array will be 26. Show that the following three expressions are equivalent by entering each expression (MATLAB command only) after the prompt:

- `C=[2.5:0.06:4.0]` — `[a:h:b]` is an array from a to b with step-size h.
- `C=linspace(2.5,4.0,26)` — `linspace(a,b,n+1)` is an array from a to b with n subintervals, $n+1$ points (avoids having to find h).
- `C=2.5 + 1.5*[0:0.04:1]` — divides the interval from 0 to 1 into 25 equal subintervals of length 0.04 and then scales & shifts.

Which of the above do you prefer?

Of course, when you have an array with a large number of elements you should follow statements like those above with a semi-colon ";" to avoid printing lines of probably unwanted numbers in your output.

Example 1:
An array from -2π to 2π using 150 subintervals is

```
array1=linspace(-2*pi,2*pi,151);
```

Example 2:
An array from 3 to 76 with steps of 0.5 is

```
array2=[3:0.5:76];
```

Exercise: Suppose we need an array (in hours) to represent the 24 hours of a day, from midnight to midnight, and we want to have time-steps of one minute. Then a suitable array might be:

```
day=[0 : 1/60 : 24];
```

What equivalent statement uses the `linspace` command?

You can append the elements of one array to the end of another array. Enter the following:

```
clear all
x1=[1 6 -7]
x2=[5 9 0 -1 11]
x=[x1 x2]
```

4.2 Array subscripts

Given an array, you may need to access only one or some of the individual elements contained in the array, rather than the whole array itself. If A is an array with n elements in it, and i is an integer such that $1 \leq i \leq n$, then in Matlab $A(i)$ is the i^{th} element in the array (often denoted by A_i in mathematics). To illustrate this, enter the following into an M-file and then execute it. Firstly you will construct an array containing 9 random numbers between 0 and 1. Then you will use these elements of A to introduce scalar variables or new arrays which are merely sub-arrays of A. In each case the new variable names before the "=" signs have been suggested so that you can read the output conveniently. **Please study carefully the expressions following the "=" signs. They will be used extensively later in this course and in subsequent courses.**

```
A=rand(1,9)
% the next two lines define scalar variables
a1=A(1)
a7=A(7)
% the remaining lines define new arrays
a3456=A(3:6)
a1357=A(1:2:7)
a852=A(8:-3:2)
```

Exercise 1: After executing the above M-file, what would you expect to see if you now entered each of the following after the prompt? Because we have not allocated a variable name for either expression, both of course will be named `ans` by default.

```
a852(3)
a1357(2:3)
```

Exercise 2: The Matlab command

```
Q=linspace(0,100,141);
```

creates an array which has 140 subintervals and 141 points. Print out only

- the 46^{th} element of Q
- the first four elements of Q
- the last three elements of Q.

4.2.1 Special array functions: sum, prod, length, max, min, sort, find, diff

The functions `sum, prod, length, max, min, sort, find` of an array are self explanatory by creating and executing the following in an M-file (do not type in the comment statements). `diff(Z)` is a new array containing the differences between consecutive elements of `Z`.

```
clear all
% create an array with 11 elements,
% each of which is a random number between 0 and 5
Z=5 * rand(1,11)
Zsum=sum(Z)
Zprod=prod(Z)
Zlength=length(Z)
% 'max' finds the maximum value and its subscript (index) number
[Zmax,i] = max(Z)
[Zmin,j] = min(Z)
% 'sort' and 'find' create new arrays
Znew=sort(Z)
Zbig=find(Z>3.0)   % for example, finds subscripts where Z(i) is >3.0
Znewbig=find(Znew>3.0)  % have now moved to end of array
```

4.3 Calculating with arrays

Performing arithmetic calculations on arrays containing large data sets is considerably easier with MATLAB than with other computer languages such as C or Pascal. It is precisely the ease of array manipulation that makes MATLAB suitable for mathematical, scientific and engineering applications.

4.3.1 Basic arithmetic operations on arrays

If X is an array and a, b are scalar values, then $a*X+b$ creates a new array by multiplying all the elements of X by a and then adding b to each position. What do you expect $X/a+b$ to give?
Enter `X=[1:8]` after the prompt. Then execute

```
y=10*X+7
z=(y-7)/10   %why is this the same as X?
v=3*X/2 -1
```

If A, B are two arrays **containing the same number of elements** and c_1, c_2 are scalars, then expressions of the form $c_1A \pm c_2B$ will create new arrays. Enter the following:

```
clear all
A=[1:2:11] , B=[8:-1:3]
C=B-A
D=(3*A-4*B)/2 + 5
```

However, there are some other very important differences. Later you will see that MATLAB includes standard operations with matrices and vectors, which are also examples of arrays. Hence, to distinguish those operations, a dot must be placed immediately before the $*$ / ^ operators in each of the following circumstances, where A, B are arrays of the same length and c is a scalar.

```
 A .* B      A ./ B        c ./ A        A .^ B       c .^ A        A .^ c
```

In other words, a dot is required before the operator whenever two arrays are multiplied together, whenever you divide by an array, and whenever an array occurs in any power expression. These always create a new array of the same length as A (and B). Enter the following and take careful note of the answers:

```
clear all
X=[1 2 3 4] , Y=[-1 4 2 1/2]
K1=X.*Y     %  multiply the corresponding elements
K2=X./Y     %  divide the corresponding elements
K3=5./Y     %  divide 5 by each element of Y
K4=X.^Y     %  raise the numbers in X to the corresponding power in Y
K5=2.^X     %  raise 2 to the power of each number in X
K6=Y.^3     %  cube each element in Y
```

Example 1: Create an array *curo* that contains the cube roots of all integers from 24 to 30.

```
clear all
A=[24:30];
curo=A.^(1/3)          % note the dot and brackets
```

Rather than introducing the array A at all, this could be done immediately using

```
clear all
curo=[24:30].^(1/3)
```

Example 2: What is the value of the sum

$$\sum_{n=1}^{100} \frac{1}{n^2} \quad = \quad \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{100^2}\,?$$

Enter the following:

```
clear all
% create an array n
n=[1:100];     % note the semi-colon (do not print out the 100 values)
b=1./n.^2;     % note the two dots (and the semi-colon)
total = sum(b)    % you should get total = 1.6350
```

Or you could be clever and find this total without actually introducing the array names `n, b`.

```
clear total
total = sum([1:100].^(-2))    % note the dot as [1:100] is an array
```

Exercise: Consider the 40 values $\frac{n^7}{2^n}$ for $n = 1, 2, 3, \ldots, 40$. Use MATLAB 's `max` function to determine the largest of these values and the number n for which it occurs. You should obtain that it is the 10th value equal to 9.7656e+003 which you **must** know is scientific notation for 9765.6.

4.3.2 Using mathematical functions on arrays

Given an array X and a function f, then the MATLAB command `Y=f(X)` produces a new array Y such that the elements of Y are $Y(i) = f(X(i))$ (or in mathematics $Y_i = f(X_i)$). This, coupled with the basic arithmetic operations on arrays discussed in Section 4.3.1, enables powerful but simple data manipulation.

Example 1: Enter the following into an M-file and then execute it.

```
clear all
X=[0:pi/6:2*pi]
Y=sin(X)
Z1=1-2*Y.^2      % note the dot
Z2=cos(2*X)      % why are Z1 & Z2 equal?
```

Example 2: Consider the values $\tan^2(m)$ for $m = 1, 2, \ldots, 200$. How many of these values are greater than 100?

```
clear all
m=[1:200];
b=tan(m).^2;             % note the dot
c=find(b>100)            % finds the positions, not the actual values
number=length(c)         % answer is 13 values
```

Example 3: You should know the classic limit $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$. Demonstrate this by evaluating $\dfrac{\sin x}{x}$ for each of $x = 0.1, 0.01, \ldots, 0.00000001$.

```
clear all
format long           % use 16 characters for each value
x=0.1.^[1:8]          % note the dot because [1:8] is an array
y=sin(x)./x           % note the dot because x is an array
```

Exercise: The output from an engineering system is $y = t^2 e^{-t} \sin t$ for $t \geq 0$. We want to find the maximum value which certainly occurs within $0 \leq t \leq 3$. Although MATLAB has other more efficient ways of doing this (see later), you could merely investigate this function by dividing the interval [0,3] into 10,000 subintervals (10,001 points). Fill in the missing code in the second and third lines in the following M-file, recalling that the exponential function e^x is `exp(x)` in MATLAB . You should obtain `ymax=0.5239, posn=5759, time=1.7274`.

```
clear all
t=linspace( , ,    );
y=               ;
[ymax,posn]=max(y)       % use array max function
time=t(posn)             % finds corresponding value in the t array
```

4.4 Using arrays to plot graphs

In this section we will use MATLAB 's **plot** command to produce graphs. In Sections 6.2 and 8.4 you will see there are two other commands to create graphs, namely **fplot** which uses function M-files and **ezplot** which is inside the Symbolic Math Toolbox.
If $x = \{x(1), x(2), \ldots, x(n)\}$ and $y = \{y(1), y(2), \ldots, y(n)\}$, then the MATLAB command `plot(x,y)` opens a graphics window, called a **Figure** window, scales the axes to fit the

data, plots the points $(x(1), y(1))$, $(x(2), y(2))$, $\ldots$, $(x(n), y(n))$, and then graphs the data by connecting the points with straight lines. If a **Figure** window already exists, another `plot` command clears the current **Figure** window by default (unless instructed not to do so) and draws a new plot. Type in the following as an attempt to graph $y = \sin x$ for $0 \le x \le 2\pi$. [You could replace the last two lines with `plot(x,sin(x))`.]

```
clear all
x=linspace(0,2*pi,11);
y=sin(x);
plot(x,y)
```

Clearly this attempt is unacceptable because we have not used sufficient points!.
Change the 11 to 201 and execute the code again.
In general, most graphs can be satisfactorily plotted using arrays of, say, 101 to 201 points, although erratic or wildly fluctuating functions would require many more points. Please note that you only need two points to plot a single straight line. For example, to plot a straight line from the point (1,7) to the point (3,-5) you need the command
`plot([1 3],[7 -5])`.
Note: Students often get this wrong by forgetting that the first array always contains the x-coordinates, not the two coordinates of the first point. Similarly, the second array contains the y-coordinates.

Exercise 1: Plot the graph of $y = \dfrac{xe^x}{x^2 - \pi^2}$ for $-3 \le x \le 2$ using a step-size of 0.02. You will need three dots in the expression to generate the array `y`.

Exercise 2: Plot the graph of $y = \sin 9x + \sin 10.5x + \sin 12x$ for $-\pi \le x \le \pi$ using 601 points.

4.4.1 Plotting several graphs on the same axes.

Example 1: Suppose you want to plot the oscillations $y1 = \cos t$, $y2 = \cos 3t$ and their sum $y3 = y1 + y2$, for $0 \le t \le 4\pi$, on the same axes. Here t is measured in seconds and $y1, y2, y3$ are measured in cm.

```
clear all
t=linspace(0,4*pi,201);
y1=cos(t); y2=cos(3*t); y3=y1+y2;
plot(t,y1,t,y2,t,y3)
```

You notice that we found the three y-arrays in a single line of code and they were plotted in different colours. But the empty space at the right end of the graphs is annoying, and how can we label the output so that each graph can be identified? There are many facilities provided by MATLAB to assist you in producing attractive, meaningful graphs. Change the above M-file to the following (**this is important!**):

```
clear all
t=linspace(0,4*pi,201);
y1=cos(t); y2=cos(3*t); y3=y1+y2;
axis([0 4*pi -2 2])        % specifies the axes limits
hold on
plot(t,y1,'b--',t,y2,'g:',t,y3,'r')
legend('y1=cos(t)','y2=cos(3*t)','y3=y1+y2')
plot([0 4*pi],[0 0],'k')        % adds the t axis in black
xlabel('time in seconds')
ylabel('displacement in cm')
title('oscillations')
hold off
```

<u>A short diversion</u>
To get assistance with the MATLAB commands featured above and in the next example, you can use the **help** facility. Enter each of the following after the >> prompt and carefully read the information until you understand precisely what each of the lines in the previous (or next) M-file is accomplishing.

- `help axis`
- `help plot`
- `help hold`
- `help legend`
- `help text`
- `help print`   a menu of print options for your Figure

Alternatively, if you do not know the precise name of a MATLAB command for which you need help, you can click on the **Help** button at the top of the MATAB Command Window, then click on "MATLAB Help" followed by "MATLAB Functions Listed by Category" and then on a topic of interest. For example, to get help on many graphics related commands, click on `Plotting and Data Visualization`.

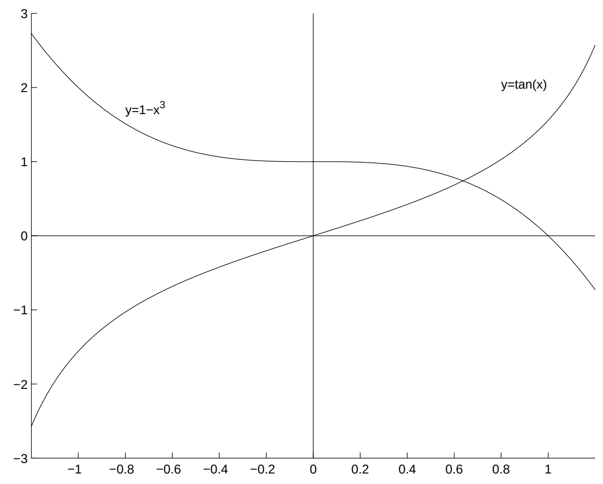
Another help option is the **lookfor** facility. Suppose you were interested in commands involving the use of complex numbers. After the `>>` prompt enter `lookfor complex`.

Example 2: By using `hold on` and `hold off`, you can plot several functions on the same axes using a number of `plot` commands. The functions $f(x) = x^2$ and $f^{-1}(x) = \sqrt{x}$ are inverse functions for $x \geq 0$ and hence their graphs must be reflections through the line $y = x$. Execute the following M-file, carefully noting the results of each command:

```
clear all               % file available in M-files folder as file1.m
axis([0 4 0 4]), axis square
hold on
x1=linspace(0,2,101);
y1=x1.^2;             % note the dot
plot(x1,y1)
x2=linspace(0,4,101);
y2=sqrt(x2);
plot(x2,y2)
plot([0 4],[0 4],'k:')      % dotted line in black from (0,0) to (4,4)
text(1.5,3.5,'y=x^2')
text(3.1,1.7,'y=sqrt(x)')
grid on             % adds grid lines if you want them
title('reflection property of inverse functions')
```

4.4.2 zoom on

MATLAB provides an interactive tool to expand sections of a plot to see more detail. This is particularly useful if you need to obtain accurate information about where two graphs intersect, or to find the coordinates of an extreme point. The command **zoom on**, either in your M-file or after the `>>` prompt turns on the zoom mode. Then go to the Figure window, place the pointer where you want an enlargement and click with the left button. Continue this process and you can zoom in to obtain three or four decimal place accuracy.



Example: Suppose you want to find graphically the point of intersection of $y = \tan x$ and $y = 1 - x^3$. (Note that there are other non-graphical ways of doing this, described in Sections 6.2 and 8.2.) Firstly, plot both of these functions on the same axes for $-1.2 \leq x \leq 1.2$, with at least 800 points to enable zooming. You might also like to add the x and y axes to obtain a figure similar to that on the previous page.
Enter the command `zoom on`. Then zoom in at the point of intersection until you are able to find its coordinates to the accuracy of (0.6376 , 0.7408).

Exercise: Plot $y = 10e^{-t}\sin t$ for $0 \leq t \leq 2\pi$ and then use **zoom on** to find its maximum value.

4.4.3 Creating separate graphs in one M-file

If you want two separate Figures to be created in one M-file, you can use the **figure(n)** command where `n` is a number associated with the window and precedes the set of plotting commands.

Example: At time t seconds, $t \geq 0$, a moving point has coordinates $x = \sin 2t$, $y = \cos 3t$ (metres), and so its speed is given by

$$speed(t) = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{4\cos^2 2t + 9\sin^2 3t}.$$

Plot the path taken by the point over one cycle $0 \leq t \leq 2\pi$, where t is regarded as a parameter, and also plot the speed against time. Use the following M-file. (You can add your own labels, etc.)

```
clear all                 % file available in M-files folder as file2.m
t=linspace(0,2*pi,500);
x=sin(2*t);
y=cos(3*t);
figure(1)
plot(x,y)
axis equal      % same scale (metres) on each axis
speed=sqrt(4*cos(2*t).^2+9*sin(3*t).^2);   % note the dots
figure(2)
axis([0 2*pi 0 4])
hold on
plot(t,speed)
hold off
```

5 Control Loops in an M-file: for, while, if

MATLAB , along with other computer programming languages and programmable calculators, offers features that allow you to control the sequencing and execution of commands by incorporating decision making facilities. The **while** and **if** commands are often used in conjunction with the **relational operators**

Relational Operator	Mathematical Meaning
`<`	less than $<$
`<=`	less than or equal to $\leq$
`>`	greater than $>$
`>=`	greater than or equal to $\geq$
`==`	equal to $=$
`~=`	not equal to $\neq$

5.1 for loops

A `for` loop allows a group of commands to be repeated a fixed predetermined number of times. It is typically of the form

```
for  variable=start:step:finish
      commands .......
      ................
end
```

Note that `variable` is not an array variable, but a scalar variable that runs through all the numbers in the array `start:step:finish` one at a time.

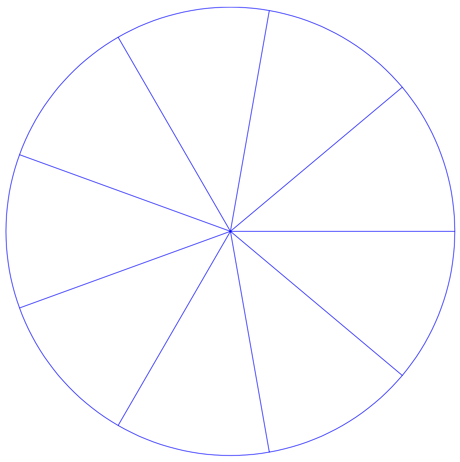
Example 1: Calculate the following using **both** (i) arrays **and** (ii) `for` loops:

$$1 + 1.1 + 1.2 + \ldots + 3 \qquad \text{and} \qquad 1 \times 1.1 \times 1.2 \times \ldots \times 3$$

(i)
```
clear all
x=[1:0.1:3];
total=sum(x)
product=prod(x)
```

(ii)
```
clear all
s=1;
p=1;
for t=1.1:0.1:3
    s=s+t ; p=p*t ;
end
total=s
product=p
```

Example 2: Write a simple M-file to produce the following graphic. Use the fact that $x = \cos(t)$, $y = \sin(t)$, $0 \le t \le 2\pi$, is the (assumed known!) parametric representation of the unit circle centred at the origin.



```
clear all
t=linspace(0,2*pi,181);
% use 181 points to plot circle
x=cos(t); y=sin(t);
plot(x,y)
axis equal, axis off
hold on
for i=1:20:161
    % each line needs 2 x-values and 2 y-values only
    plot([0 x(i)],[0 y(i)])
end
hold off
```

Example 3: Suppose you wish to construct a table of values which converts degrees C into degrees F using the standard formula $F = 1.8C + 32$ for $C = 25, 26, 27, \ldots, 40$.

```
clear all
fprintf(' deg C \t deg F \n')   % \t leaves space  \n goes to new line
for C=25:40
    F=1.8*C+32;    % do not print yet, use fprintf instead to line up
    fprintf('%4.0f  \t  %5.1f  \n',[C F])
end
```

5.2 while loops

A `while` loop allows a group of commands to be repeated an indefinite (unknown) number of times until an expression becomes false (is no longer satisfied). It is typically of the form

```
while  expression
      commands .......
      ................
end
```

The commands between the `while` and `end` statements are executed as long as **all** elements in `expression` are satisfied.

Example: For what value of n does the summation

$$\sum_{k=1}^{n} \frac{1}{\sqrt{k}} = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}}$$

first exceed 30?

```
clear all
s=0;
k=0;
while s<30
    k=k+1;
    s=s+1/sqrt(k);
end
number_of_terms=k
sum=s
```

5.3 if-else-end constructions

The simplest form is

```
if  expression
      commands evaluated if expression is true
end
```

but could be of the more detailed form

```
if  expression1
      commands evaluated if expression1 is true
elseif expression2
      commands evaluated if expression2 is true
elseif expression3
      commands evaluated if expression3 is true
.....................
else
      commands evaluated if no other expression is true
end
```

Example 1: Let us consider a very simple situation firstly. Suppose a shop offers a 5% discount on bills over \$100. Execute the following M-file for both \$78.85 and then \$122.40.

```
clear all
bill=78.85;
if bill>100
    bill=bill*0.95;
end
format bank
final_charge=bill
```

Example 2: The following M-file calculates the current amount of income tax due on a given taxable income. It also introduces MATLAB 's **input** command which enables data to be input from the keyboard when required by printing out a suitable message. Execute the following M-file for a variety of different taxable incomes:

```
clear all              % file available in M-files folder as file3.m
income=input('enter taxable income:>>')
format bank
```

```
if income <= 5400
    tax=0
elseif income <= 20700
    tax=(income-5400)*0.20
elseif income <= 38000
    tax=3060+(income-20700)*0.34
elseif income <= 50000
    tax=8942+(income-38000)*0.43
else
    tax=14102+(income-50000)*0.47
end
```

Example 3: Most `if` constructions occur inside a `for` or `while` loop.
In how many ways can $x = 59,650$ be written as the sum of squares of two integers m and n? In other words, find all integer pairs m, n such that $m^2 + n^2 = x$, which is equivalent to asking whether $n = \sqrt{x - m^2}$ is an integer. Execute the following:

```
clear all
x=59650
for m=1:sqrt(x)          % m cannot exceed sqrt(x) if m^2+n^2=x
    n=sqrt(x-m^2);       % next test if n is an integer
    if n = = round(n)    % no space between the two equal signs
        pair=[m,n]
    end
end
```

6 Function M-files

Almost all of the Matlab commands that you have been using are stored in **function M-files**. Before fully explaining the construction of this type of M-file, it is worth looking more deeply inside Matlab to view some of them.

1. The Matlab `type` command enables you to view the content of any M-file, including the many hundreds stored inside the Matlab software. After the prompt, enter `type('cos')`. The response is merely "`cos is a built-in function`" and so no documentation is available as to how Matlab actually evaluates $cos(x)$.

2. However, now enter `type('gammainc')`. The M-file **gammainc.m**, located in Specialized Math Functions, contains a special function, namely the *Incomplete Gamma Function* $= \gamma(x, a)$. You do not need to be familiar with this function. Examine the

first line

```
function b = gammainc(x,a)
```

The first word `function` is essential and indicates this is a function-type M-file. `gammainc` is the name of the function **and must correspond to the name of the M-file, gammainc.m**.
There are two **input variables**, namely `x` (which could be a scalar or an array) and `a` (which is usually a scalar). There is one output variable `b` which is the same length as `x`.

Now go to the end of the M-file **gammainc.m**. You will notice that the output variable `b` is finally given a value.

To see this function working, after the `>>` prompt enter `gammainc(4,1.5)` (ans=0.9540) and `gammainc([3 3.5 4],2.28)` (ans= 0.7427 0.8189 0.8745). Note that the input and output variables `x,a,b` are dummy variables merely used to define the function.

3. Enter `type('linspace')` after the prompt. This displays the content of **linspace.m**, in Elementary Matrices and Matrix Manipulation, which is the function M-file containing the linspace command. The first line is

   ```
   function y = linspace(d1,d2,n)
   ```

   You are aware that the input dummy variables `d1,d2` are the ends of the interval and `n` is the number of points. Notice that the only executable commands in this M-file are

   ```
   if nargin = = 2
       n = 100;
   end
   y = [d1+(0:n-2)*(d2-d1)/(n-1) d2];
   ```

   The first three lines set `n=100` by default if you forget to input a value for `n`. The last line determines the output array `y` containing the `n` points. **Exercise:** check that this formula is correct.

4. Enter `type('quadL')` to view **quadL.m** in Function Functions - Nonlinear Numerical Methods. This function finds a numerical approximation to $\int_a^b funfcn(x)dx$ where $funfcn(x)$ is any function, either built-in by MATLAB or defined by you in a function M-file funfcn.m as explained in the next Section 6.1. The first line is

   ```
   function [Q,fcnt] = quadl(@funfcn,a,b,tol,trace,varargin)
   ```

   The last three input variables are optional. The ouput variable `Q` contains a value for the integral, while `fcnt` is optional if you require extra information. At the end of this M-file notice that values are calculated for these output variables. (Do not bother looking at all the commands in between.)

Exercise: Enter `quadL(@tan,0,1.5)` to show that an accurate estimate of $\int_0^{1.5} \tan x \, dx$ is 2.6488.

6.1 Constructing Function M-files

Why might you wish to define and construct your own functions and store them in function M-files?

- the mathematical formulation of a function might be quite long and complicated. It might be better to define it separately rather than inside a script M-file.
- the same expression of one or more variables might be required a number of times, either within a single M-file or on different occasions. It is more economical to define the expression as a function in an M-file.
- many MATLAB commands operate on functions which must be held in function M-files (see Section 6.2).

Once a function has been constructed by you in an M-file, it can be used in the same ways as all other MATLAB functions. It can be evaluated, used in any other M-file, or differentiated/integrated in the Symbolic Math Toolbox (see Section 8.5). In fact, your function can later be used in constructing yet another new function.

The first time MATLAB executes a function M-file, it opens the file and *compiles* the commands into an internal representation in memory that speeds their execution for all ensuing function calls.

Suppose you wish to construct a function $myfunc$. Function M-files must follow specific rules.

1. The function name and the M-file name must be identical. Therefore, a function $myfunc$ must be stored in a file named **myfunc.m**.
2. The first executable statement must be of the form
   ```
   function   output_variables = myfunc(input_variables)
   ```
3. Do not include any `clear` commands inside a function M-file.
4. Each executable command most probably will finish with a semicolon to avoid printing out numbers each time the function is accessed. Any required values can be printed in the Command Window or in the calling script M-file.

5. Each output variable should be given a value within the M-file.

6. Function M-files terminate execution and return output values when they reach the end of the M-file, or if they encounter the command **return** beforehand.

Note: You cannot *run* a function M-file by entering the name of the M-file after the `>>` prompt as you do with a script M-file. You must give values for the input variables and evaluate as an arithmetic expression.

Example 1: The number of different combinations of r items from n items is given by the binomial coefficient

$$\mathrm{C}_r^n = \binom{n}{r} = \frac{n(n-1)\dots(n-r+1)}{r!}$$

where $n \geq 1$ and $r \geq 0$ are integers, $r \leq n$. Create a function M-file **comb.m** containing the function `comb(n,r)`:

```
function y=comb(n,r)
if r = = 0          % no space between the two equal signs
   y=1;
   return
end
top=prod([n:-1:n-r+1]);
bottom=prod([1:r]);
y=top/bottom;
```

Now enter after the prompt

```
comb(4,0) ,  comb(3,3) , comb(5,2) , comb(9,5) , comb(25,14)
```

Example 2: Consider the function

$$trigs(x,n) = \sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots + \frac{\sin(2n-1)x}{2n-1}$$

where x is the point (or array of points) where the function is being evaluated and n is the number of terms used in this series of sine functions. For example,

$$trigs(1.27,4) = \sin 1.27 + \frac{\sin 3(1.27)}{3} + \frac{\sin 5(1.27)}{5} + \frac{\sin 7(1.27)}{7}.$$

Construct the function M-file **trigs.m**:

```
function y=trigs(x,n)
% commence the sum for y with the first term
y=sin(x);           % y is an array if x is an array
for k=2:n
    kodd=2*k-1;
    y=y+sin(kodd*x)/kodd;
end
```

Now evaluate $trigs(1.27, 4)$ (=0.8347) and $trigs(1.27, 40)$ (=0.7822).
You can, of course, now plot a graph of $trigs(x, n)$ for a given n. Construct the following script M-file and then run it.

```
clear all
x=linspace(-2*pi,2*pi,801);
y=trigs(x,4);
axis([-2*pi 2*pi -1 1])
hold on
plot(x,y)
hold off
```

Change the `trigs(x,4)` to each of `trigs(x,40)` and `trigs(x,200)` and execute. The graphs you see are part of *Fourier Series* theory which many students will meet next year.

6.2 Functions operating on functions: fplot, fzero, fminbnd, quadL

Suppose a function $y = func(x)$ has been defined in a function M-file **func.m**. Then

- `fplot(@func,[a b])` or `fplot('func(x)',[a b])` plots the function for $a \le x \le b$ without requiring you to set up arrays
- `fzero(@func,[a b])` or `fzero('func(x)',[a b])` finds a root of the equation $func(x) = 0$ inside the interval $[a, b]$ providing $func(a)$ and $func(b)$ have opposite signs
  `fzero(@func,c)` or `fzero('func(x)',c)` finds a root of the equation $func(x) = 0$ by commencing a search at $x = c$.
- `fminbnd(@func,a,b)` or `fminbnd('func(x)',a,b)` finds the coordinates of a minimum point for $a \le x \le b$.
- `quadL(@func,a,b)` or `quadL('func(x)',a,b)` finds an accurate value for $\int_a^b func(x)\,dx$.

Note: `quadL` uses arrays in its calculations. Hence the M-file **func.m** must use the array dot notation in the definition of $func(x)$. This is not required for `fplot, fzero` or `fminbnd`.

Example: Consider the function $f1(x) = \dfrac{x - \cos x}{2 + x^2}$. Define it in the function M-file **f1.m**.

```
function y=f1(x)
% use dots as quadL(@f1,a,b) occurs in calling M-file
y=(x-cos(x))./(2+x.^2);    % ";" to avoid printing inside f1.m
```

Now construct and execute the following M-file. This will graph the function for $-10 \leq x \leq 10$, find its zero (from the graph, between 0 and 1), find the x and y coordinates of the minimum and integrate the function from the x-intercept to $x = 10$.

```
clear all
fplot(@f1,[-10 10])
hold on
plot([-10 10],[0 0],'k')  % adds x-axis
hold off
x_intercept=fzero(@f1,[0 1])
[xmin,ymin]=fminbnd(@f1,-3,3)  % finds both the x-coordinate
          % and y-coordinate of minimum
accurate_integral=quadL(@f1,x_intercept,10)
```

As well as the graph, the following output should result:

```
x_intercept = 0.7391
xmin = -0.4562
ymin = -0.6132
accurate_integral = 1.8910
```

Exercise 1: Consider the output $OP(t) = 1 - 4e^{-t}\sin t$ for $t \geq 0$. Define this function in an M-file and then use `fplot` to graph it for $0 \leq t \leq 5$, find the two zeros, find its minimum value and evaluate $\displaystyle\int_0^5 OP(t)\,dt$.

Exercise 2: Knowing that $\tan x$ has a vertical asymptote at $x = \frac{\pi}{2}$, see what occurs if you enter after the prompt `fplot(@tan,[1 2])`. Remember that $\tan x$ is already stored in **tan.m**. Knowing that $\sqrt{x}$ only exists for $x \geq 0$, see what occurs if you enter `fplot(@sqrt,[-1 4])`.

7 Polynomials

MATLAB has a number of commands to make manipulating and evaluating polynomials very simple. In the Help Window, in the `\mat\polyfun` topic you will see the following commands:

```
roots          - Find polynomial roots
poly           - Convert roots to polynomial
polyval        - Evaluate polynomial
polyvalm       - Evaluate polynomial with matrix argument
residue        - Partial-fraction expansion (residues)
polyfit        - Fit polynomial to data
polyder        - Differentiate polynomial
conv           - Multiply polynomials
deconv         - Divide polynomials
```

In MATLAB (but not in the "Maple" Symbolic Math Toolbox) a polynomial is represented by an array (row vector) of its coefficients in descending order. For example, enter the polynomial $p(x) = 3x^5 - 4x^4 + 7x^2 - 9x + 3$ as `p=[3 -4 0 7 -9 3]`. What polynomial $q(x)$ does `q=[4 5 -6]` represent?
MATLAB does not provide a special function for adding or subtracting polynomials other than using arrays of the same length. Hence you would need to pad out the shorter array with leading zeros in this circumstance. Note that when `poly` forms a polynomial with given roots, it makes the leading coefficient equal to one. A few of these concepts are illustrated by executing the following M-file. Examine carefully the output.

```
clear all              % file available in M-files folder as file4.m
p=[3 -4 0 7 -9 3]
q=[4 5 -6], qpadded=[0 0 0 4 5 -6];
newp=p+2*qpadded
proots=roots(p)
qroots=roots(q)
p1=poly(proots)       % check that p=3*p1, as expected
q1=poly(qroots)       % check that q=4*q1, as expected
y=polyval(p,2.31)     % evaluate p(2.31)
pder=polyder(p)       % check answer is dp/dx
qder=polyder(q)       % check answer is dq/dx=8*x+5
pq=conv(p,q)          % multiply p(x)*q(x)
[Q,R]=deconv(p,q)     % Q=quotient, R=remainder for p/q
```

Exercise: Add the commands `x=linspace(-1.7,2,300); plot(x,polyval(p,x))` to the previous M-file to see a graph of $y = p(x)$ for $-1.7 \le x \le 2$.

8 The ("Maple") Symbolic Math Toolbox

In all previous sections of this introduction MATLAB has been used as a powerful programmable graphics calculator. A variable x must have a numerical value (or array values) before expressions involving x can then be evaluated. For example if you merely enter the single command `sin(x)` after the prompt you receive the error message
`??? Undefined function or variable 'x'.`

However, commands in the Symbolic Math Toolbox enable you to enter formulae such as `sin(2*x)`, `x*exp(x)`, `x^2+3*atan(x)`, `cos(x)^2+sin(x)^2`, etc, where `x` is a **symbolic variable**. Each expression can be given a variable name (also symbolic) thereby allowing algebraic, trigonometric and other functional manipulations and simplifications as well as permitting differential and integral calculus. You will be solving and computing with mathematical symbols rather than numbers.

The tools in the Symbolic Math Toolbox are built upon the core of the powerful software program called **Maple**® which is used in many universities in Australia and overseas. For convenience only in this guide we will refer to the smaller Symbolic Math Toolbox attached to MATLAB as "Maple". The interface between MATLAB and "Maple" is smooth and it is easy to move in and out of this Symbolic Math Toolbox.

The simplest way to enter "Maple" is to make one or more variables into symbolic symbol(s) using the `syms` command. Enter the following and examine the output:

```
syms x
y=x^2-5/x          % y is automatically another symbolic variable
z=y*x-8            % z is automatically another symbolic variable
```

You notice that no attempt has yet been made to simplify z. Add the command
`z=simplify(z)`.

Note that in earlier versions of MATLAB the first two lines above were equivalent to the single command `y='x^2-5/x'`. This notation for constructing a symbolic expression using single quotation marks is still valid in many parts of "Maple" although it is being phased out because MATLAB already uses single quotation marks in other situations. Often in the "Maple" help menu you will still see the single quotation formulation but it is recommended you ignore it wherever possible to avoid confusion.
Numbers can be held in various possible formats in "Maple". The default format is an integer or rational fraction. If this is not possible, a real number will be approximated

by a fraction of large integers. The **sym** function converts the argument into a symbolic expression in "Maple". Conversely, the **double** function converts a symbolic number back into a double-precision value in MATLAB . In each of the following cases the symbolic expression in the middle column results from the corresponding `sym` command, while the last column results if you then apply the `double(...)` command to each symbolic value in the middle column:

```
sym(11/13)          11/13                            0.8462
sym(3.85)           77/20                            3.8500
sym(pi)             pi                               3.1416
sym((4/5)^2)        16/25                            0.6400
sym(sqrt(49))       7                                7
sym(sqrt(51))       sqrt(51)                         7.1414
sym(exp(1))         6121026514868073*2^(-51)         2.7183
sym(sin(1))         7579296827247854*2^(-53)         0.8415
```

Alternative formats can be viewed by entering `help sym`.

Exercise: What do you **expect** to result if you enter the following?

```
syms x
y=x+1.4*sqrt(x+3)+5.49
pretty(y)     % output to look like type-set mathematics
```

8.1 Algebraic operations

A selection of commands in "Maple":

```
simplify         performs algebraic and other functional simplifications
simple           tries a number of simplification techniques including
                 trigonometric identities; may need using twice
collect          collects like terms
factor           attempts to factor the expression
expand           expands all terms
poly2sym         converts array of polynomial coefficients in \mat into
                 a symbolic expression in ``Maple" with x as the variable
sym2poly         converts symbolic polynomial in ``Maple" into coefficient
                 array in MATLAB
```

At the simplest level, enter `factor(28809)`.

Example 1: Suppose $p(x) = x^2(x+4)^2 - 8x(x+1)^2 + 13x - 6$. Expand and factor in "Maple", convert into an equivalent MATLAB array, find the roots numerically, re-assemble the polynomial coefficients from the roots and then change back into a symbolic form.

```
clear all                % file available in M-files folder as file5.m
syms x
p=x^2*(x+4)^2-8*x*(x+1)^2+13*x-6;
p=expand(p)
p=factor(p)
P=sym2poly(p)           % change into MATLAB array P
polyroots=roots(P)      % numerically find roots in \mat
Q=poly(polyroots)       % re-assemble polynomial from its roots
q=poly2sym(Q)           % change back to Maple expression
```

You will notice that the coefficients of x^3 and x^2 in $q(x)$ are not exactly zero but are infinitesimally small due to tiny round-off errors introduced when numerically solving for the roots in MATLAB .

Example 2: Using trigonometric identities you should be able to show that

$$\frac{\sin x}{\cos x - \sin x} + \frac{\sin x}{\cos x + \sin x} \quad \text{is equivalent to } \tan 2x.$$

Use `simple` in "Maple" to check this.

```
clear all                % file available in M-files folder as file6.m
syms x
y=sin(x)/(cos(x)-sin(x))+sin(x)/(cos(x)+sin(x));
pretty(y)
y=simple(y)
```

8.2 Solving symbolic equations

You have seen that `fzero` numerically finds where a function is zero in MATLAB .
"Maple" uses the `solve` facility which can solve n simultaneous algebraic or transcendental equations for n unknowns. It firstly attempts to find an exact analytic solution. If this is not possible, it then attempts to find a numeric solution in *variable precision* format.

The command is of the form `[a1,a2,...,an]=solve(f1,f2,...,fn,v1,v2,...,vn)`.

Here, `f1,f2,...,fn` are symbolic expressions that are to be made zero,
`v1,v2,...,vn` are the variables **in alphabetical order** to be solved for,
and `a1,a2,...,an` are the corresponding answers.

8.2.1 Solving a single equation

To solve a single equation $f(x) = 0$, this reduces to `a=solve(f,x)`.

Of course there might be more than one solution for `a`.

What happens if you do not specify the variable for which the equation is to be solved? If there is only one symbolic variable in the expression, it will solve for it by default. Otherwise `x` is the default variable. If there are two or more symbolic variables, none of which is `x`, and you forget to specify the one for which the solution is required, it appears to choose the last alphabetical one.

Example 1: Solve the equation $\dfrac{10}{x^2+1} = 4 - x$. This is equivalent to solving $\dfrac{10}{x^2+1} - 4 + x = 0$. There are various ways:

```
    clear all
    syms x
    f=10/(x^2+1)-4+x;     % f is now a symbolic expression
    soln=solve(f,x)       % note the three solutions
```

or, knowing that `x` is the default variable,

```
    clear all
    syms x
    soln=solve(10/(x^2+1)-4+x)          % no need to use f or specify x
```

In earlier versions of Matlab , you could use

```
    clear all
    soln=solve('10/(x^2+1)=4-x')
```

but this is being phased out.

Exercise: Change to `10/(x^2+1)+2+x=0` and solve again.

Example 2: Solve the general quadratic equation $ax^2 + bx + c = 0$ for x.

```
clear all
syms a b c x
soln=solve(a*x^2+b*x+c)
```

giving the output
```
soln=[1/2/a*(-b+(b^2-4*a*c)^(1/2))]
     [1/2/a*(-b-(b^2-4*a*c)^(1/2))]
```

However, if for some reason you wanted to solve the same equation for b, you would need

```
clear all
syms a b c x
soln=solve(a*x^2+b*x+c,b)
```

giving the ouput `soln=-(a*x^2+c)/x`.

Example 3: We know $\sin\theta = 0.5$ has an infinite number of solutions. Enter

```
clear all
syms theta
angle=solve(sin(theta)-0.5)
```

giving the output `angle=1/6*pi`.

Example 4: Unlike Example 3, the equation $\sin\theta = 0.5 - \cos\theta$, $-\pi \leq \theta \leq \pi$, does not have obvious solutions. But "Maple" can find two solutions exactly. The answers are hardly in a form that you would use and so it is better to convert them into numeric values.

```
clear all
syms theta
angle=solve(sin(theta)-0.5+cos(theta))     % hardly usable
angle=double(angle)                        % looks better in MATLAB
```

giving the output
```
angle=[atan((1/4-1/4*7^(1/2))/(1/4+1/4*7^(1/2)))]
      [atan((1/4+1/4*7^(1/2))/(1/4-1/4*7^(1/2)))+pi]
angle=-0.4240
       1.9948
```

Example 5: By means of a simple sketch you can see that $e^x = 4 - x^2$ has two solutions. However, there is no analytic way to find them. Enter

```
clear all
syms x
soln=solve(exp(x)-4+x^2)
```

giving the output `soln=1.0580064010906363086213874461232`. This indicates that a numerical solution process was needed in "Maple", and the answer is in variable precision format. If you wish, you can follow with `soln=double(soln)`. Note that only the solution nearest to zero was found.

8.2.2 Solving simultaneous equations

Find the points of intersection of the two circles $2x^2 - x + 2y^2 - 8y = 0$ and $x^2 + 2x + y^2 - 6y + 1 = 0$.

```
clear all                % file available in M-files folder as file7.m
syms x y
eqn1=2*x^2-x+2*y^2-8*y;
eqn2=x^2+2*x+y^2-6*y+1;
[X,Y]=solve(eqn1,eqn2)
```

giving the output
```
X = [ 2]          Y = [ 3]
    [ -14/41]         [ 3/41]
```

which implies the two points $(x, y) = (2, 3)$ and $(x, y) = (-\frac{14}{41}, \frac{3}{41})$.

8.3 Variable substitution and expression evaluation: subs, eval

Suppose you have a symbolic expression `f` which includes the symbol `x` and you wish to substitute for `x` another symbol `c` or a numerical value $x0$. Then you can use the general **subs** command `g=subs(f,old,new)` which in our cases would be `g=subs(f,x,c)` or `g=subs(f,x,x0)`. Here `old, new` can be arrays. The result `g` is still a symbolic variable or symbolic constant in "Maple".

Example 1: Consider a function of the two Cartesian coordinates $f(x, y) = \dfrac{2xy}{(x^2 + y^2)^2}$. Change to polar coordinates using $x = r\cos\theta$, $y = r\sin\theta$ and then determine the value of f at an arbitrary point on the unit circle $r = 1$.

```
clear all
syms x y r theta
f=2*x*y/(x^2+y^2)^2;
F=subs(f,[x y],[r*cos(theta) r*sin(theta)]);
F=simple(F)        % previous answer is messy
f_on_unit_circle=subs(F,r,1)
```

which gives the output

```
F=sin(2*theta)/r^2
f_on_unit_circle=sin(2*theta)
```

An alternative is to use the **eval** command. It is of the form `ans=eval(S)` where `S` is a symbolic expression for which at least one of its symbolic variables has just been given a value. If **all** variables are given numerical values, the answer is a number in MATLAB , not "Maple".

Example 2: Let us compare simple MATLAB and "Maple" codes which both evaluate the expression $y = (x^3 + 2)\sec x$ at $x = 0.123$.

MATLAB **code**	**Maple code using subs**	**Maple code using eval**
`clear all`	`clear all`	`clear all`
`x=0.123;`	`syms x`	`syms x`
`y=(x^3+2)*sec(x)`	`S=(x^3+2)*sec(x);`	`S=(x^3+2)*sec(x);`
	`y=subs(S,x,0.123)`	`x=0.123;`
	`y=double(y)`	`y=eval(S)`

Example 3: Reconsider Example 1 at the top of the page. Change the last line to the corresponding two lines in the following M-file:

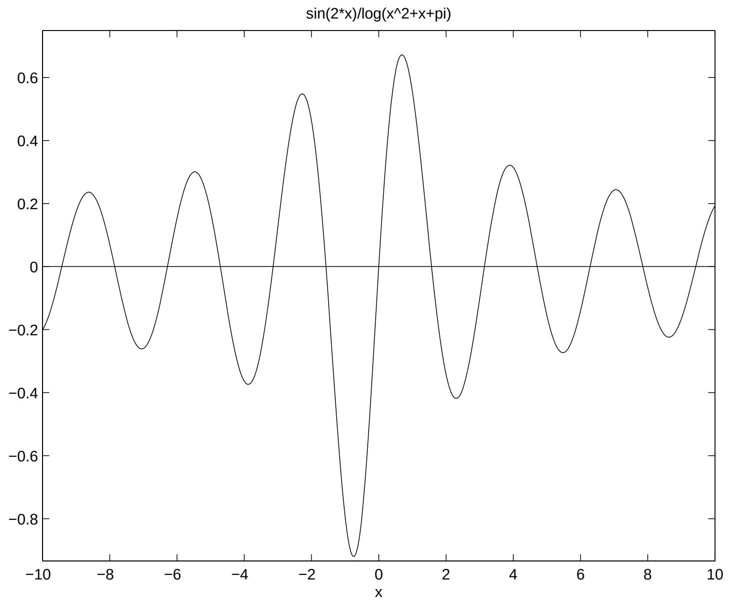
```
clear all
syms x y r theta
f=2*x*y/(x^2+y^2)^2;
F=subs(f,[x y],[r*cos(theta) r*sin(theta)]);
F=simple(F)
r=1;
f_on_unit_circle=eval(F)
```

8.4 ezplot

If `S` is s symbolic expression, then `ezplot(S,[a b])` graphs the function $y = S(x)$ (or $y = S(t)$) over the domain $a \leq x \leq b$. If `[a b]` is omitted, the default domain is $-2\pi \leq x \leq 2\pi$. Also, if no `title` command is used, the default title is the expression `S`. Of course you are not using arrays and so the dot notation is not applicable.

Example: In "Maple", plot the function $y = \dfrac{\sin 2x}{\ln(x^2+x+\pi)}$ for $-10 \leq x \leq 10$.

```
clear all                % file available in M-files folder as file8.m
syms x
y=sin(2*x)/log(x^2+x+pi);
ezplot(y,[-10 10])          % notice title
hold on
plot([-10 10],[0 0],'k')    % plot x-axis
hold off
```



8.5 Differential and integral calculus

Although there are many other commands that will be needed in later semesters of your degree program, we will restrict the discussion to `diff` and `int` in this introduction. The default variable is `x`; otherwise it is `t`; but if you wish to differentiate or integrate with respect to some other symbol, say `v`, it must be specified.

Suppose `S` is a symbolic expression involving the symbolic variable `v` (and perhaps other symbols). Then

- `diff(S,v)` represents $\dfrac{dS}{dv}$
- `diff(S,v,n)` represents $\dfrac{d^nS}{dv^n}$
- `int(S,v)` represents the indefinite integral $\int S(v)\, dv$ (without the "+c")
- `int(S,v,a,b)` represents the definite integral $\int_a^b S(v)\, dv$

Example 1: Let $G(x) = e^{-x}\cos x$. Find $G'(x)$, $G''(x)$ and $\int G(x)\, dx$, giving answers in factored form.

```
clear all                % file available in M-files folder as file9.m
syms x                   % use (first) default variable
G=exp(-x)*cos(x)
dG=diff(G);              % diff(G,x) is the same
dG=factor(dG)
d2G=diff(G,2);           % diff(dG) gives same result
d2G=factor(d2G)
intG=int(G);             % int(G,x) is the same
intG=factor(intG)
```

giving the results
```
G = exp(-x)*cos(x)
dG = -exp(-x)*(cos(x)+sin(x))
d2G = 2*exp(-x)*sin(x)
intG = -1/2*exp(-x)*(cos(x)-sin(x))
```

Example 2: Evaluate $\displaystyle\int_{-a}^{a} \frac{1}{t^2+1}\, dt$ and $\displaystyle\int_{-\infty}^{\infty} \frac{1}{t^2+1}\, dt$.

Note that the MATLAB /"Maple" equivalent of ∞ is `inf`.

```
clear all
syms t a                  % use (second) default variable t
ans1=int(1/(t^2+1),-a,a)          % gives ans1 = 2*atan(a)
ans2=int(1/(t^2+1),-inf,inf)      % gives ans2 = pi
```

Example 3: There is no explicit expression for $\int \frac{\sin v}{v + e^v}\, dv$ because this antiderivative does not exist. Nevertheless the definite integral $\int_0^{\pi} \frac{\sin v}{v + e^v}\, dv$ can be evaluated numerically by introducing the `double` command as described previously.

```
clear all                 % file available in M-files folder as file10.m
syms v                           % not a default variable
S=sin(v)/(v+exp(v));
intS=int(S,v)                    % Note: no antiderivative exists!
defint=double(int(S,v,0,pi))
```

which produces the following output:

```
Warning: Explicit integral could not be found.
> In c:\\mat\toolbox\symbolic\@sym\int.m at line 58
  In c:\\mat\bin\file10.m at line 4

 intS = int(sin(v)/(v+exp(v)),v)

Warning: Explicit integral could not be found.
> In c:\\mat\toolbox\symbolic\@sym\int.m at line 58
  In c:\\mat\bin\file10.m at line 5

defint = 0.3993
```

9 Matrices and Vectors

Each array that was discussed in Section 4 was, in effect, a **row vector** or **row matrix**. But you are aware that a rectangular array represents a **matrix** and a single array column represents a **column vector**.

To construct a matrix with m rows and n columns (called an "m by n matrix", written $m \times n$ matrix), each row in the array ends with a semicolon. For example, run the following M-file **mat.m**:

```
A=[3 -5 1;-2 0 -4;6 7 9]
B=rand(3,3)
C=[1 -3;-1 5;2 8]
```

The element `A(i,j)` is in the `i`th row and `j`th column. Hence, after the prompt, enter `a23=A(2,3)`, `b31=B(3,1)`, `c12=C(1,2)`. The comma separates the row number(s) from the column number(s).

What is `A(3,[2 3])`, `B([1:2:3],[1:2:3])`?

A single colon ":" before the comma means "take all rows", whereas a single colon after the comma means "take all columns". Hence `A(:,2)` is column number 2 in the matrix `A` while `B(1,:)` is the first row of `B`.

Providing matrices have the same shape they can be added or subtracted. Providing they have compatible shapes they can be multiplied using the established rules for matrix multiplication. Hence calculate after the prompt `D=2*A-B`, `F=A*B`, `G=A*C`, `Asq=A^2`. However, `B+C` and `C*A` produce error messages.

Note: `A.^2` does not square the matrix but squares each element in the matrix. Similarly, `A.*B` is not matrix multiplication but merely multiplies the corresponding positions in the two matrices.

`det(A)` is the **determinant** of `A`, written |`A`|.

`A'` is the **transpose** of `A` and is written in mathematics as $\mathtt{A}^{\mathtt{T}}$. It is formed by interchanging the rows and columns. Hence, if you need to input the column vector

$$v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$$

you could enter `v=[v1;v2;...;vn]` or `v=[v1 v2 ...vn]'`.

The **magnitude** or **Euclidean norm** of the vector v, given by

$$||v|| = \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}\ ,$$

is represented in MATLAB by `norm(v)`.

If `A` is a square matrix with $|\mathtt{A}| \neq 0$, then `inv(A)` represents the inverse of `A`, denoted in mathematics by $\mathtt{A}^{-1}$.

The $n \times n$ **identity matrix** I is represented in MATLAB by `eye(n)`. For the matrix `A` at the beginning of this section, verify that `A*inv(A)=inv(A)*A=eye(3)`.

10 Other Useful Commands and Examples

Previous students have found other commands useful and have suggested they be included in this introduction. Also included are examples that further illustrate efficient usages of commands already discussed.

10.1 global

Often a variable (which could be a scalar variable, or an array, or a symbolic variable representing a mathematical expression) has been determined in one M-file and might also be needed in another M-file, for example a function M-file. However, it might not be practical or possible to pass it across as an input variable to the function. Two M-files can share a common MATLAB workspace for a variable `var` if the command

```
global var
```

occurs in both M-files before it is defined or used in either.

10.2 Finding limits in "Maple"

Suppose you need to find

$$\lim_{x\to 1} \frac{2^x - 2}{2x - 2}$$

for which both the numerator and denominator tend to zero. This is an example of an *indeterminate form* of which many others will be investigated in later mathematics using *L'Hôpital's Rule*. One method is to find the limit graphically. Enter:

```
clear all
syms x
f=(2^x-2)/(2*x-2);
ezplot(f,0.95,1.05)
zoom on
```

You might notice a small hole in the graph at $x = 1$ because the expression is undefined there. However, by zooming in, you see that the function is tending towards a value 0.6931 as $x \to 1$. Do you recognise this value? To obtain an exact answer, use the "Maple"s **limit** command. Add to the above M-file the extra command `lim=limit(f,x,1)` to obtain the answer `lim = log(2)` (which is $\ln 2$).

Exercise 1: Change the previous M-file to demonstrate that $\lim_{x\to 0} \frac{\sin x}{x} = 1$, both graphically and using `limit`.

Exercise 2: Change the previous M-file to find

$$\lim_{x\to 0.25} (\tan \pi x)^{\tan 2\pi x} \qquad \text{(clearly undefined at } x = 0.25\text{)}$$

to obtain 0.3679 graphically and $1/e$ using `limit`.

10.3 Function calculator: funtool

The command `funtool` opens an interactive graphical function calculator that uses mouse clicks to perform operations on symbolic expressions. It can simultaneously manipulate and graph (in two separate windows) two functions $f(x)$ and $g(x)$ and allows constant horizontal shifts. A third window controls the calculator, and contains the symbolic functions `f` and `g`, the chosen domain of these functions, and a constant value `a` . Below this are four rows of buttons that enable you to combine and operate on `f` and `g` in a variety of ways as well as controls for the calculator itself.
Try `funtool` when you have some time to help visualise how functions behave and interact.

10.4 diff commands in MATLAB and "Maple"

You should have discovered that the `diff` command has a different meaning in MATLAB compared to "Maple". In MATLAB `diff(vec)` finds the differences between elements in the array `vec`. In "Maple", if `S` is a symbolic expression, `diff(S)` finds its derivative.

However, you know that

$$\frac{df}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} .$$

Hence, if the differences are divided by a tiny step-size they should be crude approximations to the derivative.

10.5 Finding the maximum value of a function

You are aware that the MATLAB commands `max, min` find the maximum and minimum elements in an array (and their locations) while `fminbnd` helps to find the minimum value of a function defined in a function M-file. Seeing there is **no such command as `fmaxbnd`**, how can we print out the maximum value of a function $f(x)$, for $a \le x \le b$, without using the `zoom on` facility? For example, find the x and y coordinates of the maximum turning point of $y = xe^{-x}\cos x$, $0 \le x \le \pi$.

Using MATLAB

The value of x where $f(x)$ has a maximum is the same as the value where $-f(x)$ is a minimum. Hence, set up the function M-file **negf.m**

```
function y=negf(x)
y= - x*exp(-x)*cos(x);
```

and run the following M-file

```
clear all
[temp1,temp2]=fminbnd(@negf,0,pi);
xmax=temp1, ymax= -temp2     % gives xmax=0.5959, ymax=0.2718
```

Using "Maple"

Solve for where the derivative is zero and then substitute back into $y = f(x)$. However, be aware that there might be other (real or complex) solutions to the equation $\dfrac{df}{dx} = 0$.

```
clear all
syms x
f=x*exp(-x)*cos(x);
xmax=double(solve(diff(f)))    % gives xmax=0.5959
x=xmax;
ymax=eval(f)                   % gives ymax=0.2718
```

11 Trouble Shooting

In previous years students have experienced various problems. We have collected some "frequently asked questions" with answers in the hope it will save you some time.

<u>**Question:**</u> I tried to execute my M-file **proj.m** using `>> proj` but received

```
??? Undefined function or variable 'proj'.
```

or

I tried to use my function $proj(x)$ held in **proj.m** but received

```
??? Can not find function 'proj'.
Error in ==> c:\\mat\toolbox\\mat\specgraph\fplot.m
On line 85 ==> x = xmin; y = feval(fun,x);
```

Reason: You are not working in the same directory where the M-file **proj.m** is held.
Answer: Change directory or save the M-file into the correct directory (see Page 8).
****

<u>**Question:**</u> I attempted to evaluate an elementary MATLAB function such as `mod(101,7)` (or another MATLAB function) but received the following message

```
??? ??? Error using ==> mod
Too many input arguments.
```

I then asked for help using `help mod` and received

```
No help comments found in mod.m.
```

Reason: Both responses indicate that you have inadvertently named one of your own M-files **mod.m** previously. It has taken precedence over MATLAB 's inbuilt function.
Answer: Change the name of your own M-file to one that does not exist in MATLAB .
****

<u>**Question:**</u> Why do I receive one of the following error messages, or similar?

```
??? Error using ==> .*
Matrix dimensions must agree.
```

or

```
??? Error using ==> plot
Vectors must be the same lengths.
```

Reason: When doing a calculation involving two arrays, or graphing using the `plot` command, the arrays must be of the same size.
Answer: Check the size of each array involved.
****

<u>**Question:**</u> Why did I not get any numbers printed on the screen?
Reason: Semicolons at the end of a MATLAB assignment statement prevent answers being displayed.
Answer: Leave off semicolons where you want numbers to be shown in the output.

Question: Why did I get thousands of numbers printed on the screen?
Reason: Semicolons at the end of a MATLAB assignment statement prevent answers being displayed.
Answer: Place semicolons where you do not want numbers to be shown in the output.
****

Question: Why do I continually get the following error message?

```
??? Error using ==> ^
Matrix must be square.
```

or

```
??? Error using ==> *
Inner matrix dimensions must agree.
```

Reason: You have asked for something like `x^2` or `f(x)*g(x)` where `x` is currently an array. It is only possible to square an array `x` or multiply `f(x)*g(x)` if `x` is a square matrix.
Answer: If you *think* `x` is a scalar variable, it is possible that you were also using the same symbol to represent an array and forgot to clear it. But more likely `x` is an array and you should be using the dot notation `x.^2` or `f(x).*g(x)`. The location might be in a script M-file or, more obscurely, in a function M-file where you did not allow for later array evaluations.
****

Question: Why are parts of my graphical figures missing?
Reason: You have probably lost graphs or graphical components.
Answer: Make sure each figure is numbered. Within a figure, use the `hold on` ... `hold off` facility to avoid losing command outcomes.
****

Question: When I access a function M-file, either by doing an evaluation after the `>>` prompt or else calling from another M-file, why do I get no answer back or else receive the empty answer: [ ] ?
Reason: Although the first line in your function M-file was of the form

```
function y=fred(x)
```

you have not assigned any value to `y` during the file.
Answer: Make sure output variables are given values in function M-files.
****

Question: When using **solve** on a transcendental equation in "Maple", why do I get complex solutions, or else a real solution which is not the one I want?
Reason: Consider, for example, the equation $xe^x + x^3 - 2 = 0$. Using `syms x` , `ans=solve(x*exp(x)+x^3-2)` yields a complex solution only.
Answer: Use `fzero` in MATLAB instead. If the function has been defined in **func.m**, then `ans=fzero(@func,[0 1])` yields `ans=0.7482` which is probably the solution you were seeking within the specified interval.

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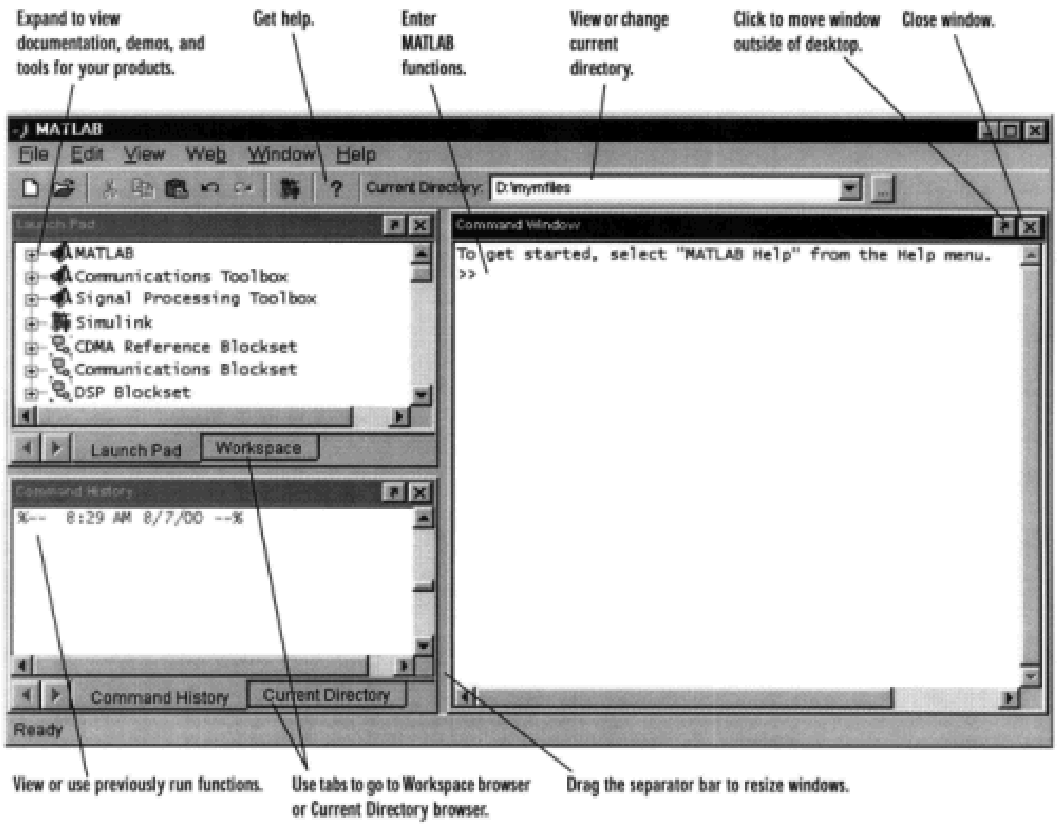
MATLAB Desktop

When you start MATLAB, the MATLAB desktop appears, containing tools (graphical user interfaces) for managing files, variables, and applications associated with MATLAB.

The first time MATLAB starts, the desktop appears as shown in the following illustration, although your Launch Pad may contain different entries.

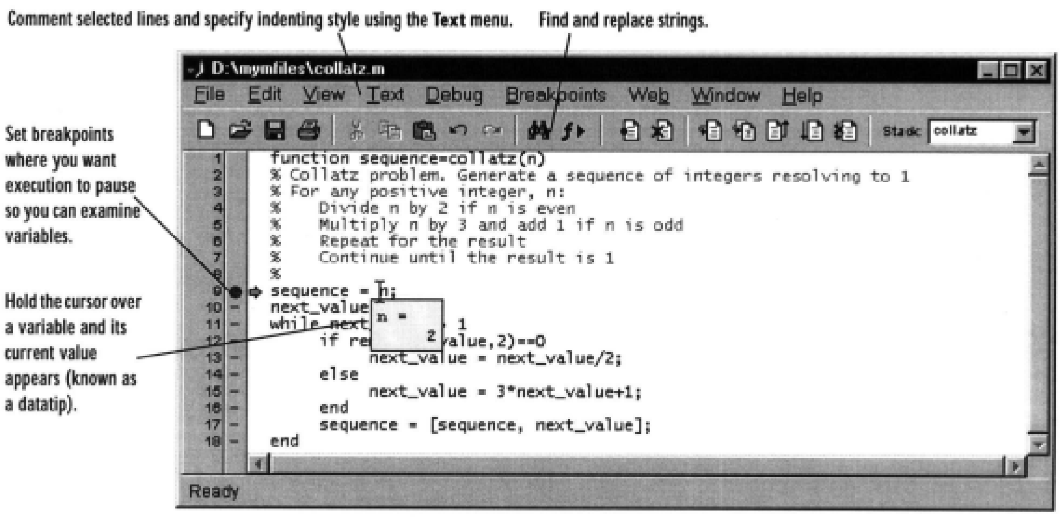

Expand to view documentation, demos, and tools for your products.
Get help.
Enter MATLAB functions.
View or change current directory.
Click to move window outside of desktop.
Close window.
MATLAB
File Edit View Web Window Help
Current Directory: D:\mymfiles
Launch Pad
MATLAB
Communications Toolbox
Signal Processing Toolbox
Simulink
CDMA Reference Blockset
Communications Blockset
DSP Blockset
Launch Pad
Workspace
Command History
Current Directory
Command Window
Ready
View or use previously run functions.
Use tabs to go to Workspace browser or Current Directory browser.
Drag the separator bar to resize windows.

```
To get started, select "MATLAB Help" from the Help menu.
>>
```

```
%-- 8:29 AM 8/7/00 --%
```

Editor/Debugger

Use the Editor/Debugger to create and debug M-files, which are programs you write to run MATLAB functions. The Editor/Debugger provides a graphical user interface for basic text editing, as well as for M-file debugging.


Comment selected lines and specify indenting style using the **Text** menu.
Find and replace strings.
Set breakpoints where you want execution to pause so you can examine variables.
Hold the cursor over a variable and its current value appears (known as a datatip).
D:\mymfiles\collatz.m
File Edit View Text Debug Breakpoints Web Window Help
Stack: collatz
n =
2
Ready

```
function sequence=collatz(n)
% Collatz problem. Generate a sequence of integers resolving to 1
% For any positive integer, n:
%    Divide n by 2 if n is even
%    Multiply n by 3 and add 1 if n is odd
%    Repeat for the result
%    Continue until the result is 1
%
sequence = n;
next_value
while next_          1
    if re          value,2)==0
        next_value = next_value/2;
    else
        next_value = 3*next_value+1;
    end
    sequence = [sequence, next_value];
end
```

You can use any text editor to create M-files, such as Emacs, and can use preferences (accessible from the desktop **File** menu) to specify that editor as the default. If you use another editor, you can still use the MATLAB Editor/Debugger for debugging, or you can use debugging functions, such as dbstop, which sets a breakpoint.

If you just need to view the contents of an M-file, you can display it in the Command Window by using the type function.