Example of Sketching a Curve using Derivatives

Consider the curve \( y = 2x^3 + 4x^2 + 2x \).

(a) Find the \( x \)-intercepts.
   \( x \) intercepts when \( y = 0 \).

\[
2x^3 + 4x^2 + 2x = 0 \\
2x(x^2 + 2x + 1) = 0 \\
2x(x + 1)^2 = 0
\]

So, \( x \)-intercepts are \((0,0)\) and \((-1,0)\) (repeated root - graph will touch the \( x \) axis here).

(b) Find the \( y \)-intercept.
   \( y \) intercept when \( x = 0 \).

\[
y(0) = 0 + 0 + 0 \\
     = 0
\]

So, \( y \)-intercept is \((0,0)\).

(c) Find the first derivative, set it to zero, and solve for \( x \).
   \( y' = 6x^2 + 8x + 2 \)

\[
6x^2 + 8x + 2 = 0 \\
(x + 1)(3x + 1) = 0
\]

So, \( x = -1 \) and \( x = -1/3 \).
\( y(-1) = -2(-1 + 1)^2 = 0 \).
\( y(-1/3) = -2(1/3)(-1/3 + 1)^2 = -(2/3)(4/9) = -8/27 \).

There is a maximum at \((-1, 0)\), and a minimum at \((-1/3, -8/27)\).
(d) Find the second derivative, set it to zero, and solve for \( x \).

\[
y'' = 12x + 8
\]

\[
12x + 8 = 0
\]

\[
12x = -8
\]

\[
x = -\frac{2}{3}
\]

So, \( x = -\frac{2}{3} \). \( y(-\frac{2}{3}) = -2(\frac{2}{3})(-\frac{2}{3} + 1)^2 = -\frac{4}{3}(\frac{1}{9}) = -\frac{4}{27} \).

There is a point of inflection at \((-\frac{2}{3}, -\frac{4}{27})\).