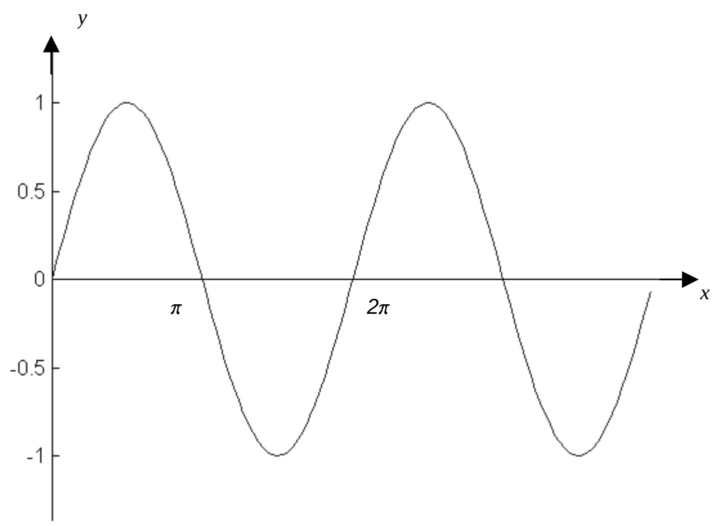
The Basics of Trigonometry

The word **trigonometry** comes from the Greek word *trigonom* meaning 'triangle' and *metron* meaning 'measurement'. Thus, broadly speaking, trigonometry is the study of the properties of triangles and trigonometric functions.

Simple trigonometry is concerned with the calculation of angles and the side lengths of triangles, but of course, trigonometry gets abstract very fast, and we soon forget about simple triangles.

There are two main trigonometric functions: **sine** and **cosine**. These functions are **periodic**, that is, they repeat themselves over and over in the horizontal direction. The length of one of the repetitions or cycles is called the **period**.

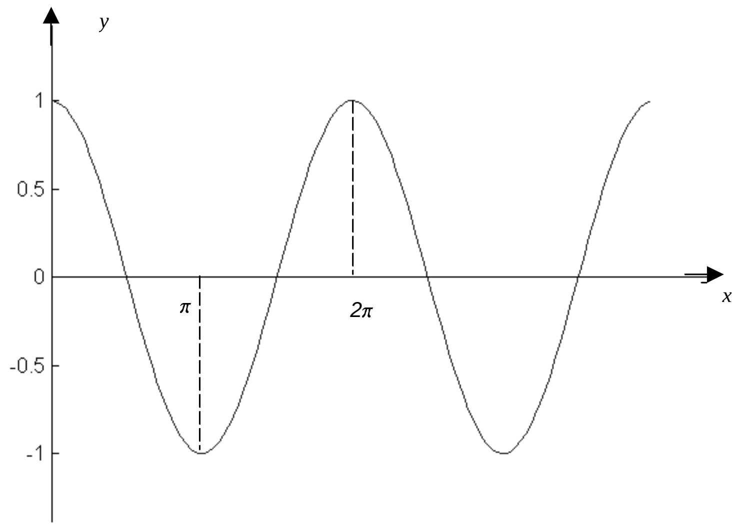
The Sine Function



The domain of $\sin x$ is $x \in \mathbb{R}$.
The range of $\sin x$ is $-1 \leq y \leq 1, y \in \mathbb{R}$.

The period of $\sin x$ is 2π.

The Cosine Function



The domain of $\cos x$ is $x \in \mathbb{R}$.
The range of $\cos x$ is $-1 \leq y \leq 1, y \in \mathbb{R}$.

The period of $\cos x$ is 2π.

Other Functions

The other trigonometric functions are given as follows:

$$\tan x = \frac{\sin x}{\cos x}$$ **tangent**

$$\operatorname{co}\sec x = \frac{1}{\sin x}$$ **cosecant**

$$\sec x = \frac{1}{\cos x}$$ **secant**

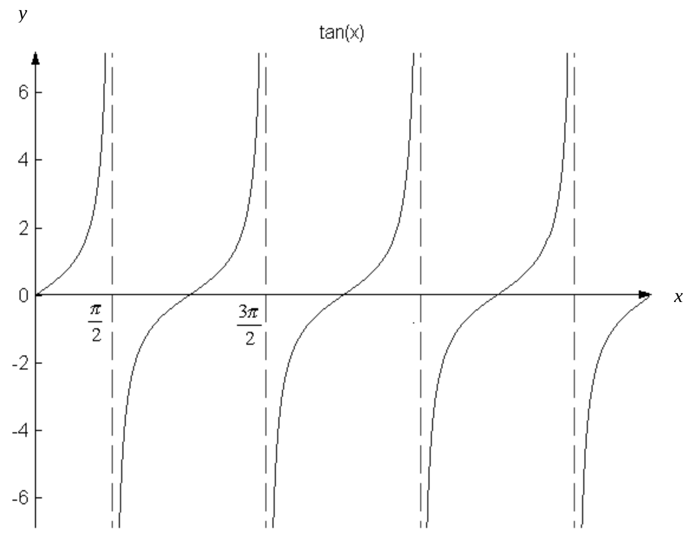
$$\cot x = \frac{1}{\tan x}$$ **cotangent**

The Tangent Function

Once we determine the domain of $y = \tan x = \dfrac{\sin x}{\cos x}$ we can sketch it.

Domain: values for which $\cos x \neq 0$. i.e. $x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$. This implies there are vertical asymptotes at these values of x.

$y = \tan x$ cuts the x axis when $y = 0$, so when $\sin x = 0$. These values are $x = 0, \pi, 2\pi, \ldots$.



Calculating the values of trigonometric functions

You can do this without your calculator. Or, if you are uncomfortable about this, you must be able to recognise the decimal result as a surd.

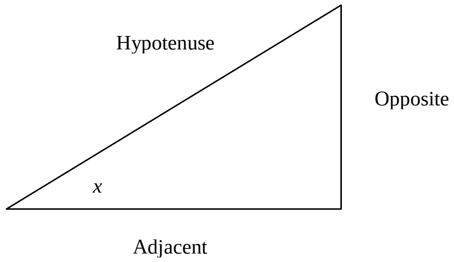
The table of $\sin x$ and $\cos x$ has a pattern to it.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\sin x$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$	0
$\cos x$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$	-1

This can be simplified as follows:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1

Right Angled Triangles



The ratios are as follows:

$$\sin x = \frac{\text{opp}}{\text{hyp}} \qquad \cos x = \frac{\text{adj}}{\text{hyp}} \qquad \tan x = \frac{\text{opp}}{\text{adj}}$$

So, alternatively, the angles can be calculated using these triangles: