Integration Practice

The examples below relate to integration using either the power rule or the chain rule. Break the function up into its component parts $u(x)$ and $u'(x)$ ready for integration.

Power Rule: $\int (u(x))^n u'(x)dx = \frac{1}{n+1}(u(x))^{n+1} + c$

Chain Rule: $\int f(u(x)).u'(x)dx = \int f(u)du$ (Also known as integration by chain rule derivative)

1. $\int \left(x^2+2\right)^3 .2xdx$

2. $\int \left(x^3+3x^2-1\right)^{-3}(3x^2+6x)dx$

3. $\int x\sqrt{x^2+4}dx$

4. $\int (x^2+1)\sin(x^3+3x)dx$

5. $\int \sin(x)\left(\cos(x)+1\right)^4 dx$

Answers

1. $u(x) = x^2+2;\ u'(x) = 2x$

$$\int \left(x^2+2\right)^3 .2xdx = \int u^3.u'(x)dx = \int u^3du = \frac{1}{4}u^4 + c = \frac{1}{4}\left(x^2+2\right)^4 + c$$

2. $u(x) = x^3+3x^2-1;\ u'(x) = 3x^2+6x$

$$\int \left(x^3+3x^2-1\right)^{-3}(3x^2+6x)dx = \int u^{-3}.u'(x)dx = \int u^{-3}du = \frac{1}{-2}u^{-2} + c = \frac{-1}{2}\left(x^3+3x^2-1\right)^{-2} + c$$

3. $u(x) = x^2+4;\ u'(x) = 2x$

$$\int x\sqrt{x^2+4}dx = \frac{1}{2}\int \sqrt{x^2+4}(2x)dx = \frac{1}{2}\int u^{\frac{1}{2}}.u'(x)dx = \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{2}*\frac{2}{3}u^{\frac{3}{2}} + c = \frac{1}{3}\left(x^2+4\right)^{\frac{3}{2}} + c$$

4. $u(x) = x^3+3x;\ u'(x) = 3x^2+3$

$$\int (x^2+1)\sin(x^3+3x)dx = \frac{1}{3}\int \sin(x^3+3).(3x^2+3)dx = \frac{1}{3}\int \sin(u).u'(x)dx = \frac{1}{3}\int \sin(u)du$$

$$= \frac{1}{3}(-\cos(u) + c = \frac{-1}{3}\cos(x^3+3x) + c$$

5. $u(x) = \cos(x) + 1; u'(x) = -\sin(x)$

$$\int \sin(x)\left(\cos(x)+1\right)^4 dx = -\int \left(\cos(x)+1\right)^4 (-\sin(x)dx = -\int \left(u(x)\right)^4 .u'(x)dx = -\int \left(u(x)\right)^4 du$$

$$= -\frac{1}{5}\left(u\right)^5 + c = -\frac{1}{5}\left(\cos(x)+1\right)^5 + c$$