Solving Simultaneous Equations

Adapted from Introductory Mathematics, C. Coady and J. Gosling

There are two main methods for solving simultaneous equations – the **elimination** method and the **substitution** method.

These methods are usually only used when solving two equations in two unknowns. For larger problems, matrix algebra (Gaussian Elimination or similar) is used.

Elimination Method

This method involves:

1. *Eliminating* one of the variables, allowing the value of the remaining variable to be determined.
2. *Substituting* this value back into either of the original equations in order to calculate the value of the 'eliminated' variable.
3. *Checking* your answer satisfies both equations.

Example 1

Solve using the *elimination method*:

$$2x + y = 8 \qquad (1)$$
$$3x - y = 7 \qquad (2)$$

Step 1: Add together the two equations to eliminate y. Then solve for x.

$$\begin{aligned} 2x + y &= 8 \\ 3x - y &= 7 \\ 5x &= 15 \\ \therefore x &= 3 \end{aligned}$$

Step 2: Substitute $x = 3$ back into equation (1) and solve for y.

$$\begin{aligned} 2 \times 3 + y &= 8 \\ \therefore y &= 8 - 6 \\ y &= 2 \end{aligned}$$

Step 3: Check that the solution $x = 3, y = 2$ satisfies both equations.
Equation (1): LHS: $2 \times 3 + 2 = 8 =$RHS
Equation (2): LHS: $3 \times 3 - 2 = 7 =$RHS

Therefore the required solution is $x = 3$, $y = 2$.

Note that in order to eliminate a variable it must have the same coefficient in both equations. This may involve having to make some adjustments to one or both equations.

Example 2

Solve using the *elimination method*:

$$2x + 3y = 8 \quad (3)$$
$$3x - 2y = -7 \quad (4)$$

Step 1: To eliminate y, multiply equation (3) by 2 and equation (4) by 3 to give the 'common' coefficient 6, then add the two equations together.

$$\begin{aligned} 4x + 6y &= 16 \\ 9x - 6y &= -21 \\ 13x &= -5 \\ \therefore x &= \frac{-5}{13} \end{aligned}$$

Step 2: Substitute $x = \frac{-5}{13}$ back into equation (3) and solve for y.

$$\begin{aligned} 2 \times \frac{-5}{13} + 3y &= 8 \\ 3y &= 8 + \frac{10}{13} \\ 3y &= \frac{114}{13} \\ \therefore y &= \frac{38}{13} \end{aligned}$$

Step 3: Check that the solution $x = \frac{-5}{13}, y = \frac{38}{13}$ satisfies both equations.
Equation (3): LHS: $2 \times \frac{-5}{13} + 3 \times \frac{38}{13} = \frac{-10+114}{13} = 8 =$RHS
Equation (4): LHS: $3 \times \frac{-5}{13} - 2 \times \frac{38}{13} = \frac{-91}{13} = -7$RHS

Therefore the required solution is $x = \frac{-5}{13}$, $y = \frac{38}{13}$.

Substitution Method

This method involves taking one of the two equations and making one of the variables the subject. The resulting expression is then substituted into the second equation in place of the chosen variable, allowing a solution to be found for the remaining variable. The value of the second variable is easily obtained by substituting back into one of the original equations.

When choosing which variable to make the subject, look for the easiest rearrangement.

Example 3

Solve using the *substitution method*:

$$2x + y = 8 \quad (5)$$
$$3x - y = 7 \quad (6)$$

Step 1: Rearrange equation (5) in terms of y.

$$y = 8 - 2x$$

Step 2: Now substitute $8 - 2x$ in for y in equation (6).

$$\begin{aligned} 3x - (8 - 2x) &= 7 \\ \therefore 3x - 8 + 2x &= 7 \\ \therefore 5x &= 15 \\ \therefore x &= 3 \end{aligned}$$

Step 3: Now substitute $x = 3$ into equation (5)

$$\begin{aligned} 2 \times 3 + y &= 8 \\ \therefore y &= 8 - 6 \\ \therefore y &= 2 \end{aligned}$$

Step 3: Check that the solution $x = 3, y = 2$ satisfies both equations.
Equation (5): LHS: $2 \times 3 + 2 = 8 =$RHS
Equation (6): LHS: $3 \times 3 - 2 = 7 =$RHS

Therefore the required solution is $x = 3$, $y = 2$ (as before).

Note

The best thing about trying to get better at these problems is that you can make up any questions, because you can always check whether your answer is right or wrong!