Example of Gaussian Elimination

Consider the system of linear equations:

$$\begin{array}{ccccccc} x & + & y & + & z & = & 12 \\ x & - & y & & & = & 2 \\ x & & & - & z & = & 4 \end{array}$$

Solve using Gaussian elimination:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 4 \end{array}\right] \qquad \begin{array}{rcl} \\ R2 & = & R2 - R1 \\ R3 & = & R3 - R1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 0 & -2 & -1 & -10 \\ 0 & -1 & -2 & -8 \end{array}\right] \qquad \begin{array}{rcl} \\ R2 & = & -\frac{1}{2}R2 \\ R3 & = & -R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 0 & 1 & 1/2 & 5 \\ 0 & 1 & 2 & 8 \end{array}\right] \qquad \begin{array}{rcl} \\ \\ R3 & = & R3 - R2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 0 & 1 & 1/2 & 5 \\ 0 & 0 & 3/2 & 3 \end{array}\right] \qquad \begin{array}{rcl} \\ \\ R3 & = & \frac{2}{3}R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 0 & 1 & 1/2 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Now that the matrix is in row–echelon form, we use back substitution:
Row 3 gives us $z = 2$.
Substitute z into Row 2:

$$\begin{aligned} y + \left(\frac{1}{2}\right) z &= 5 \\ y + \left(\frac{1}{2}\right) 2 &= 5 \\ y + 1 &= 5 \\ y &= 4 \end{aligned}$$

Substitute y and z into Row 1:

$$\begin{aligned} x + y + z &= 12 \\ x + 4 + 2 &= 12 \\ x &= 6 \end{aligned}$$

The solution to the system of linear equations is $x = 6, y = 4, z = 2$.

We check this by substituting the solution back into the original equations:

Equation 1:

$$\begin{aligned} x + y + z &= 6 + 4 + 2 \\ &= 12 \end{aligned}$$

Equation 2:

$$\begin{aligned} x - y &= 6 - 4 \\ &= 2 \end{aligned}$$

Equation 3:

$$\begin{aligned} x - z &= 6 - 2 \\ &= 4 \end{aligned}$$

Since all three equations are satisfied, we can be sure this is the correct answer.