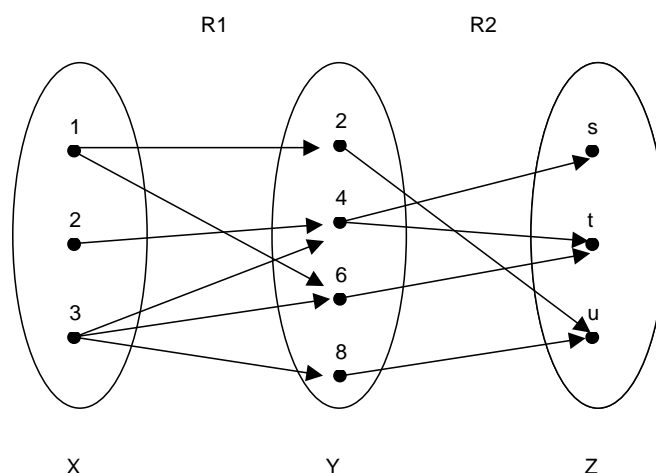## Example of Composition of Relations

Example 2.4.25 from Johnsonbaugh, Discrete Mathematics, Fifth Edition.

Example $X = \{1, 2, 3\}, Y = \{2, 4, 6, 8\}, Z = \{s, t, u\}$

$R_1 = \{(1, 2), (1, 6), (2, 4), (3, 4), (3, 6), (3, 8)\}$

and

$R_2 = \{(2, u), (4, s), (4, t), (6, t), (8, u)\}$

then

$R_2 \circ R_1 = \{(1, u), (1, t), (2, s), (2, t), (3, s), (3, t), (3, u)\}.$

So, in this example, we draw the arrow diagrams of R_1 and R_2.

The composition of relations is defined (in Definition 2.4.24) as:

$$R_2 \circ R_1 = \{(x, z) | (x, y) \in R_1 \text{ and } (y, z) \in R_2 \text{ for some } y \in Y\}$$

This means that we write down each pair (x, z) where x comes from the first set, then we go *through* the middle set Y, and finish on a z. You should be able to see that for 1 in the first set, we go through 2 in the middle set to u, so we write down $(1, u)$.

Also, for 2 in the first set, we go through 4 in the middle set to both s and t, so we write down $(2, s)$ and $(2, t)$.

If an ordered pair appears more than once in the composition of relations, we write it down just once.

Also, it is important to realise that if we draw them in this order (R_1 then R_2) the composition is written back–the–front i.e. $R_2 \circ R_1$.