Test Your Understanding Week 2

1. Decide whether the following are true or false. Give reasons.

(a) $\forall x(x > 0)$

(b) $\exists x(x \geq 0)$

(c) $\forall x(x^2 \geq 0)$

2. Show using direct proof that:

(a) an even integer times an even integer is even

(b) an odd integer times an odd integer is odd.

3. Use a proof by contradiction to show that for all integers *n*, 5*n*+2 is not divisible by 5.

4. For the following induction problems, the first two true for *n*≥1, prove that the basis step holds. Then state the inductive assumption and the statement to be proved.

(a) That *n* straight lines divide the plane into $\dfrac{n^2 + n + 2}{2}$ regions. (No lines parallel, no three lines having a common point.)

(b) $(1 + x)^n \geq 1 + nx$, for *x*≥1.

(c) That postage of 12 cents or more can be made up using only 4c and 5c stamps.