Tutorial 1 Solutions

<u>1.1</u>

Question	Proposition?	Negation	
1	Yes	2+5≠19	
2	No		
3	Yes	There is no positive integer n with 19340= n .17.	
4	Yes	Audrey Meadows was not the original "Alice"	
		in "The Honeymooners".	
5	No,		
	instruction.		
6	Yes	The line "Play it again, Sam" does not occur in	
		the movie "Casablanca".	
7	Yes	Not every even integer >4 is the sum of two	
		primes.	
8	No, garbage.		

$$p = F, q = T, r = F$$
17.
$$\neg (p \lor q) \land (\neg p \lor r)$$

$$\equiv \neg (F \lor T) \land (T \lor F)$$

$$\equiv (\neg T) \land T$$

$$\equiv F \land T$$

$$\equiv F$$

18.
$$(p \lor \neg r) \land \neg ((q \lor r) \lor \neg (r \lor p))$$

 $\equiv (F \lor T) \land \neg ((T \lor F) \lor \neg (F \lor F))$
 $\equiv T \land \neg (T \lor \neg F)$
 $\equiv T \land F$
 $\equiv F$

21.

p	q	p∨q	<i>¬</i> р	$(p \lor q) \land \neg p$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

It is not the case that (5<9 and 9<7)

$$\neg (p \land q) \equiv \neg (T \land F) \equiv T$$

38. p: Today is Monday, q: It is raining, r: It is hot $\neg (p \lor q) \land r \equiv \text{It}$ is not the case that (today is Monday or it is raining) and it is hot $\equiv \neg (T \lor F) \land T \equiv F \land T \equiv F$

1.2

2. (q if p form) The q part is that follows from the other part, ie Rosa may graduate. Hence:

If Rosa has 160 ¼ hours of credit then she may graduate.

- 3. (q is necessary for p form) The necessary part, obtaining \$2K, is q. Hence: If Fernando is to buy a new computer then he must obtain \$2K.
- 7. (p only if q) The only if part, q, is being well structured. Hence: If the program is readable then it is well structured.
- 33. p: Today is Monday, q: It is raining, r: It is hot $\neg q \rightarrow (r \land p) \equiv \underline{If}$ it is not raining then (today is hot and it is Monday)

$$\underline{\text{if}} \text{ -3<1<3} \underline{\text{then}} \text{ } |1|<3 \qquad \text{ (conditional)}$$

$$p \to q \equiv T \to T \equiv T$$

$$\underline{\text{if}} |1| < 3 \underline{\text{then}} - 3 < 1 < 3 \qquad \text{(converse)}$$

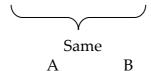
$$q \to p \equiv T \to T \equiv T$$

if (1≤-3 or 1≥3) then |1|≥3

$$\neg q \rightarrow \neg p \equiv T \rightarrow T \equiv T$$

44.

p	q	$p \rightarrow q$	$\neg p$	$\neg p \lor q$
T	Т	T	F	T
T	F	F	F	F
F	T	T	Т	T
F	F	T	Т	T



49.

17.							
p	q	r	$p \rightarrow q$	$q \rightarrow r$	$A \wedge B$	$p \rightarrow r$	
T	T	Т	T	T	T	T	
T	T	F	Т	F	F	F	
T	F	Т	F	Т	F	T	False
T	F	F	F	T			
F	T	Т	T	T			
F	T	F	T	F			
F	F	Т	T	Т			
F	F	F	Т	T			

52.

p	q	p imp1 q	q imp1 p	
T	T	T	T	(=T imp1 T=T)
T	F	F	F	(=F imp1 T=F)
F	T	F	F	(=T imp1 F=F)
F	F	T	T	(=F imp1 F=T)

<u>1.3</u>

$$37. \ \forall x \left(x > 1 \rightarrow \frac{x}{x^2 + 1} < \frac{1}{3} \right)$$

Now $\frac{x}{x^2+1} < \frac{1}{3}$ can only be true if

$$3x < x^2 + 1$$

$$0 < x^2 - 3x + 1$$

And at x = 1, $x^2 - 3x + 1 = -1$ and so at x = 1, $x^2 - 3x + 1 = -1.09 < 0$.

Hence, at
$$x = 1.1$$
, $\frac{x}{x^2 + 1} > \frac{1}{3}$

Here I went straight for the boundary, x=1, since that seemed where the claim was most likely to break down.