Year 12 Revision Questions

Normal Distributions Other Than The Standard Normal Distribution

Every normal distribution has roughly the same shape, ie a bell curve, with its own mean and standard deviation. However by a simple transformation, we can relate each one to the standard normal distribution with mean 0 and standard deviation 1. The transformation is $z = \frac{x-\mu}{\sigma}$, where x is a random number from a non-standard normal distribution, μ is the mean and σ is the standard deviation. Then z is a random number from the standard normal distribution.

Question 9, pages 6, 7

A manufacturer produces muesli bars with mass distributed normally with mean 80 gms. It is known that 7% of bars are rejected for being underweight at 75 gms. What is the standard deviation?

The invnormal function of a graphics calculator will return the value a with $P(x \le a)$ equal to some given number. In this case, we seek a with $P(x \le a)=0.07$. However we cannot find this, because we need to know the mean and the standard deviation. But we can use the invnormal function on the standard normal distribution, because we know the mean and standard deviation already.

Using the invnormal function, we find that if $P(z \le a)=0.07$ then a=-1.476, and further, this corresponds to a weight of 75 gms. Using the transformation above, we find the following.

$$-1.476 = \frac{75-80}{\sigma}$$
$$-1.476\sigma = -5$$
$$\sigma = \frac{-5}{-1.476} = 3.39$$

Hence the standard deviation is 3.39.

Normal Approximation To The Binomial Distribution

If a binomial random variable has n trials with probability p of success, then it has mean np and standard deviation $\sqrt{npq} = \sqrt{np(1-p)}$. If the number of trials is large enough, then this is very close to a normal distribution with the same mean and standard deviation. The conditions we need are $n \geq 30$, $np \geq 5$ and $nq = n(1-p) \geq 5$.

The binomial distribution is discrete and the normal distribution is continuous. Hence we use the following approximations.

Binomial	*Normal Approximation*
P(X=a)	P(a-0.5≤X≤a+0.5)
P(a<X<b)	P(a+0.5≤X≤b-0.5)
P(a≤X≤b)	P(a-0.5≤X≤b+0.5)

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35% of all disks are faulty. Let X be the number of faulty discs in a batch of 100. We can use a normal approximation to this Binomial situations, since (1) $n \geq 30$ (2) $np = 100*0.35 = 35 \geq 5$ and (3) $np(1-p) = 100*0.35*0.65 = 22.75 \geq 5$.

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Hence we use a normal approximation with mean $\mu = 100*0.35 = 35$ and standard deviation $\sigma = \sqrt{100*0.35*0.65} = 4.77$.

(1) $P(X < 50) \approx P(X < 49.5)$ under the approximation. Hence

$$\begin{aligned} P(X \leq 49.5) &= P(Z \leq \frac{49.5-35}{4.77}) \\ &= P(Z \leq 3.04) \\ &= 0.9988 \end{aligned}$$