

Comparisons of proportions

3.1. One-sample Test: Normal distribution approximation method

Properties of the sample proportion: Let Y be a binomial random variable with parameter n and π . The sample proportion p is a point estimate of π . Its mean equals π and its variance is given by $\pi(1-\pi)/n$, which is estimated by $p(1-p)/n$. From the central limit theorem:

$$p \approx N(\pi, \pi(1-\pi)/n) \text{ and } Z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \approx N(0,1),$$

which is a good approximation when $n\pi(1-\pi) \geq 5$.

One-sample z test: both one-tail and two-tails tests		
Hypotheses	p-value	Rejection region
$H_0: \pi = \pi_0$ vs $H_1: \pi \neq \pi_0$	$P(Z < - z_0 \text{ or } Z > z_0)$	$ z_0 > z_{\alpha/2}$
$H_0: \pi = \pi_0$ vs $H_1: \pi > \pi_0$	$P(Z > z_0)$	$z_0 > z_{\alpha}$
$H_0: \pi = \pi_0$ vs $H_1: \pi < \pi_0$	$P(Z < z_0)$	$z_0 < -z_{\alpha}$

(i) Translate the scientific hypothesis into a hypothesis on the parameters.

Example: Is the proportion of live newborns more than 85%?

$H_0: \pi = 0.85$ versus $H_1: \pi > 0.85$ ($\pi_0 = 0.85$)

(ii) Decide type I error: $\alpha = 5\%$; $z_{\alpha} = z_{0.05} = 1.645$

(iii) Choose and compute the test statistic: $z_0 = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$,

where p is the sample proportion and n the sample size. For a sample size $n = 100$, if we observe $p = 90\%$, then $z_0 = (0.90 - 0.85) / [(0.85 * 0.15/100)^{1/2}] = 1.40$

(iv) Write down a conclusion based on either the rejection region or the p-value.

$z_0 = 1.40 < 1.654$, then we do not reject the null hypothesis.

3.2. Chi-square Test of independence

Basic of the Chi-Square test statistics

Rationale: Although Z and T test statistics are widely used, they both require data to be quantitative, that is, variables are measured on a numeric scale. They are not applicable to categorical measurement scales. For a given phenomenon, the Chi-square test Statistic is used to compare observed frequencies, denoted by O to expected frequency, denoted by E , and helps decide if the deviations are statistically significant. Expected frequencies are obtained under the null hypothesis that there is no relationship between both variables (variables are independent).

Chi-Square test Statistics: This test statistic is defined by:

$$X^2 = \frac{(O-E)^2}{E} \sim \chi_1^2 : \text{Chi-square distribution with one } df \text{ (} df: \text{degrees of freedom).}$$

For a r by c contingency table (r : number of rows and c : number of columns):

$$X^2 = \sum \frac{(O-E)^2}{E} \sim \chi_v^2 : \text{Chi-square distribution with } v \text{ } df, \text{ where } v = (r-1) * (c-1),$$

where the expected frequency in each cell is given by:

$$E = \frac{(\text{Total of the column}) * (\text{Total of the row})}{n : \text{total sample size}}.$$

Note: (1) This test statistic should be used only if $E \geq 5$. (2) χ : Greek small letter Chi and X : Greek capital letter Chi. Sometimes, the term “Khi-square” is used instead.

Example of the aspirin efficacy in preventing thrombosis among patients under dialysis.

	Development of thrombosis: observed and expected frequencies				Total
	Yes		No		
	O	E	O	E	
Placebo Group	18	13.95	7	11.05	25
Aspirin Group	6	10.04	12	7.96	18
Total	24		19		43

e.g.: $E = 13.95 = (24) * (25) / 43$; $E = 11.05 = (19) * (25) / 43$.

The observed value of the Chi-square statistic is equal to:

$$\chi^2 = \frac{(18-13.95)^2}{13.95} + \frac{(6-10.04)^2}{10.04} + \frac{(7-11.05)^2}{11.05} + \frac{(12-7.96)^2}{7.96} = 6.34$$

Test of independence between two variables: r by c contingency table

Rationale: The Chi-square statistic is used for various hypothesis tests. The first we consider here is the test of independence (or association) between two variables. For given two variable defining a r by c contingency table, the Chi-square test Statistic will compare observed cell frequencies, O , to expected cell frequencies, E . Small deviations will confirm the null hypothesis of no association while the large deviations will invalidate it.

Example: A researcher is interested in studying the relationship between blood groups and the presence and severity of symptoms of a certain disease. In a sample of 1500 patients, recorded data are as follows below.

Presence of symptoms and severity of them	Blood Group				Total
	A	B	AB	O	
None	543	211	90	476	1320
Less severe	44	22	8	31	105
Severe	28	9	7	31	75
Total	615	242	105	538	1500

Test of independence of two variables: r by c contingency table

- (i) H_0 : No association between the 2 variables, that is, variables are independent;
 H_1 : There is an association or relationship between them
(ii) Test statistic: Chi-Square given by:

$$X^2 = \sum \frac{(O-E)^2}{E} \quad \square \quad \chi^2_{(r-1)(c-1)}$$

- (iii) p -value = $P(X^2 > \chi^2)$, where χ^2 is the observed sample value;

Rejection region: we reject H_0 if $\chi^2 > \chi^2_{(r-1)(c-1), \alpha}$, where the critical value is such that
 $P(X^2 > \chi^2_{(r-1)(c-1), \alpha}) = \alpha$ (Tables or use StaTable to compute critical values or p -values).

$$\begin{aligned} \text{Response: } \chi^2 &= (543 - 541.20)^2 / 541.20 + (211 - 212.96)^2 / 212.96 + (90 - 92.40)^2 / 92.40 \\ &+ (476 - 473.44)^2 / 473.44 + (44 - 43.05)^2 / 43.05 + (22 - 16.94)^2 / 16.94 + \\ &(8 - 7.35)^2 / 7.35 + (31 - 37.66)^2 / 37.66 + (28 - 30.75)^2 / 30.75 + \\ &(9 - 12.10)^2 / 12.10 + (7 - 5.25)^2 / 5.25 + (31 - 26.90)^2 / 26.90 \\ &= 0.01 + 0.02 + 0.06 + 0.01 + 0.02 + 1.51 + 0.06 + 1.18 + 0.25 + 0.79 + \\ &0.58 + 0.62 = 5.12 \end{aligned}$$

$$p\text{-value} = P(X^2 > 5.12) = 0.528517; \text{ Critical value: for } \alpha = 5\%, \chi^2_{6,0.05} = 12.5916$$

3.3. Comparison of proportions: Independent samples

Rationale: The study variable (or dependent variable) is dichotomous and measured within two groups, defined by a grouping factor (or independent variable). In this case, data could be summarized in 2-by-2 contingency table as follows.

Characteristic of interest			or	Characteristic of interest			
Yes	No	Total		Yes	No	Total	
Group 1	a	b		Group 1	y_1	$n_1 - y_1$	n_1
Group 2	c	d		Group 2	y_2	$n_2 - y_2$	n_2
Total	$a + c$	$b + d$		Total	$y_1 + y_2$	$n - (y_1 + y_2)$	n

Sample proportions of success, $p_1 = y_1/n_1$ and $p_2 = y_2/n_2$, are estimates of population proportions of success π_1 and π_2 .

Confidence intervals for the difference of two proportion:

The 100% $(1 - \alpha)$ confidence interval limits are based on the approximation by the $N(0, 1)$ distribution (Central Limit Theorem). These limits are estimated by:

$$(p_1 - p_2) \pm z_{\alpha/2} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}},$$

which is a good approximation when $n_1 p_1 \geq 5$ and $n_2 p_2 \geq 5$.

Example: A research study aimed at evaluating the efficacy of aspirin in the prevention of thrombosis among patients under dialysis gave the following results shown below.

	Development of thrombosis		Total
	Yes	No	
Placebo Group	18	7	25
Aspirin Group	6	12	18
Total	24	19	43

Response: $p_1 = y_1/n_1 = 18/25 = 0.72$, $p_2 = y_2/n_2 = 6/18 = 0.33$ and the 95% CI limits are:

$$(0.72 - 0.33) \pm 1.96 \sqrt{\frac{0.72(1-0.72)}{25} + \frac{0.33(1-0.33)}{18}} = 0.39 \pm 0.28$$

Two independent groups: normal distribution

Two-samples z test for two proportions		
Hypotheses	p-value	Rejection region
$H_0: \pi_1 = \pi_2$ vs $H_1: \pi_1 \neq \pi_2$	$P(Z < - z_0 \text{ or } Z > z_0)$	$ z_0 > z_{\alpha/2}$
$H_0: \pi_1 = \pi_2$ vs $H_1: \pi_1 > \pi_2$	$P(Z > z_0)$	$z_0 > z_{\alpha}$
$H_0: \pi_1 = \pi_2$ vs $H_1: \pi_1 < \pi_2$	$P(Z < z_0)$	$z_0 < -z_{\alpha}$

where $z_0 = \frac{p_1 - p_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$, where $p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$: common proportion of success.

Example: A research study aimed at evaluating the efficacy of aspirin in the prevention of thrombosis among patients under dialysis. Based on the results shown above, is aspirin efficient in preventing thrombosis? Use $\alpha = 5\%$.

Response: (i) $H_0: \pi_1 = \pi_2$ vs $H_1: \pi_1 > \pi_2$ (π_1 : placebo group, π_2 : aspirin group).

(ii) $\alpha = 5\%$, then $z_{\alpha} = 1.645$;

(ii) $p_1 = y_1/n_1 = 0.72$, $p_2 = y_2/n_2 = 0.33$, $p = 24/43 = 0.55814$, then:

$$z_0 = \frac{0.72 - 0.33}{\sqrt{0.55814(1 - 0.55814)\left(\frac{1}{25} + \frac{1}{18}\right)}} = 2.52$$

(iv) $p\text{-value} = P(Z > 2.52) = 0.0059$; (v) Use $p\text{-value}$ or rejection region to reject H_0 .

Sample size determination: normal approximation

One-Sample Test ($H_0: \pi = \pi_0$): Given α , $1 - \beta$, π_0 and π_1 , if the alternative is two-sided, then the sample size is given by:

$$n = \left[z_{\alpha/2} \sqrt{\pi_0(1-\pi_0)} + z_{\beta} \sqrt{\pi_1(1-\pi_1)} \right]^2 / (\pi_0 - \pi_1)^2$$

For one-sided alternatives, change $\alpha/2$ to α into the formula.

Two-Samples Test ($H_0: \pi_1 = \pi_2$): Given α , $1 - \beta$, π_1 and π_2 , if the alternative is two-sided, then the sample size per group (*equal sample size case*) is given by the following formula:

$$n = \left[z_{\alpha/2} \sqrt{2\pi(1-\pi)} + z_{\beta} \sqrt{\pi_1(1-\pi_1) + \pi_2(1-\pi_2)} \right]^2 / (\pi_1 - \pi_2)^2, \pi = (\pi_1 + \pi_2)/2$$

where π is the common proportion. For one-sided alternatives, change $\alpha/2$ to α into the formula.

Comparisons of two proportions: Test of independence between two variables**Test of independence of two variables: an alternative to the Z test for a 2-by-2 table**

H_0 : No association between the 2 variables versus H_1 : There is an association between them. If the first variable (or dependent variable) is dichotomous and the second variable (or independent variable) is a dichotomous grouping factor, then “No association between the 2 variables” means that $H_0: \pi_1 = \pi_2$ (proportions of success in both groups).

(i) $H_0: \pi_1 = \pi_2$ vs $H_1: \pi_1 \neq \pi_2$ (two-sided alternative only)

(ii) Test statistic: Chi-Square. An equivalent formula for a 2-by-2 table is given by:

$$X^2 = \sum \frac{(O-E)^2}{E} = \frac{n(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)} \quad \square \quad \chi_1^2$$

(iii) p -value = $P(X^2 > \chi^2)$, where χ^2 is the observed sample value;

Rejection region: we reject H_0 if $\chi^2 > \chi_{1,\alpha}^2$, where the critical value is such that

$P(X^2 > \chi_{1,\alpha}^2) = \alpha$ (see Tables or use StaTable to compute critical values or p -values).

Note: The square of the z score observed in the Z test is approximately equal to the χ^2 observed sample value. This is due to the fact that, for Z follows a $N(0,1)$ distribution, then Z^2 follows a Chi-Square distribution with one degree of freedom.

Example of the aspirin efficacy in preventing thrombosis among patients under dialysis.

(i) $H_0: \pi_1 = \pi_2$ vs $H_1: \pi_1 \neq \pi_2$ (two-sided alternative only)

(ii) The observed value of the Chi-square statistic is equal to:

$$\chi^2 = \frac{(18-13.95)^2}{13.95} + \frac{(6-10.04)^2}{10.04} + \frac{(7-11.05)^2}{11.05} + \frac{(12-7.96)^2}{7.96} = 6.34$$

(iii) p -value = $P(X^2 > 6.34) = 0.011804$; Critical value: for $\alpha = 5\%$, $\chi_{1,0.05}^2 = 3.8415$; then we can conclude using either p -value or rejection region. We reject H_0 .

More than two independent groups

Rationale: It is often important to determine whether the distribution of a characteristic of interest is similar for various groups (homogeneity). The study variable (characteristic of interest or dependent variable) is dichotomous and it measured within more than two groups (or samples of binary data) defined by a grouping factor of interest (or independent variable). Suppose that we have m ($m > 2$) groups. In this case, data could be summarized in 2-by- m contingency table:

Study variable	Group 1	Group 2	Group 3	...	Group m	Total
Yes: success	y_1	y_2	y_3	...	y_m	$\sum_{i=1}^m y_i$
No: failure	$n_1 - y_1$	$n_2 - y_2$	$n_3 - y_3$...	$n_m - y_m$	$n - \sum_{i=1}^m y_i$
Total	n_1	n_2	n_3	...	n_m	$n = \sum_{i=1}^m n_i$

The sample proportions of success, $p_1 = y_1/n_1$, $p_2 = y_2/n_2$, $p_3 = y_3/n_3$, ..., $p_m = y_m/n_m$, are estimates of population proportions of success π_1 , π_2 , π_3 , ..., π_m .

Test of homogeneity:

H_0 : No association between the 2 variables vs H_1 : There is an association between them. This null hypothesis is equivalent to $H_0: \pi_1 = \pi_2 = \pi_3 = \dots = \pi_m$ (proportions of success in all groups are equal: homogeneity of groups). The alternative hypothesis is that "At least one proportion is different from the others".

- (i) $H_0: \pi_1 = \pi_2 = \pi_3 = \dots = \pi_m$ vs H_1 : At least one proportion is different from the others
(ii) Test statistic: Chi-Square. An equivalent formula for a 2-by- m table is given by:

$$X^2 = \sum \frac{(O-E)^2}{E} = \sum_{i=1}^m \frac{n_i (p_i - p)^2}{p(1-p)} \sim \chi_{m-1}^2, \text{ where } p = \frac{\sum_{i=1}^m y_i}{n} : \text{ global proportion of success.}$$

- (iii) $p\text{-value} = P(X^2 > \chi^2)$, where χ^2 is the observed sample value;

Rejection region: reject the null hypothesis H_0 if $\chi^2 > \chi_{m-1, \alpha}^2$, where the critical value is such that $P(X^2 > \chi_{m-1, \alpha}^2) = \alpha$ (see Tables or use StaTable to compute critical values or p -values).

Example of a test of homogeneity:

A given plant manufactures a certain product potentially harmful and believed to be associated to respiratory disease. A sample of 500 workers from that plant are interviewed and tested to determine their levels of exposition to the potential toxic agent and the presence of symptoms of respiratory disease. Given the data below, is there any evidence of harmful effect of the agent? Use $\alpha = 1\%$.

Presence of symptoms	Level of exposition			Total
	High (1)	Limited (2)	None (3)	
Yes: success	185	33	17	235
No: failure	120	73	72	265
Total	305	106	89	500

Response:

- (i) $H_0: \pi_1 = \pi_2 = \pi_3$ vs H_1 : At least one proportion is different from the others
(ii) Test statistic: Chi-Square.

Presence of symptoms	Level of exposition			Total
	High (1)	Limited (2)	None (3)	
Yes: success	185	33	17	235
No: failure	120	73	72	265
Total	305	106	89	500
p_i	$p_1 = 0.60656$	$p_2 = 0.31132$	$p_3 = 0.19101$	$p = 0.47$
$n_i * (p_i - p)^2$	5.68783	2.66901	6.92735	
$n_i (p_i - p)^2 / p (1 - p)$	22.83352	10.71461	27.80951	

- (iii) $\chi^2 = \sum n_i (p_i - p)^2 / p (1 - p) = 22.83352 + 10.71461 + 27.80951 = 61.35764$;
Critical value for $\alpha = 1\%$ and a Chi-square distribution with 2 *df* is equal to 9.2103;
since $\chi^2 > 9.2103$, we reject the null hypothesis.
(iv) There is a relationship between the levels of exposition and the presence of symptoms of respiratory disease.
Indeed, observed proportions: $p_1 = 0.60656 > p_2 = 0.31132 > p_3 = 0.19101$, are not homogeneous.

Using SPSS: Pearson Chi-Square test and Fisher Exact Test

Example: The table below gives results of the study related to the medical controversy over the efficacy of Vitamin C in preventing the common cold within children. Does the number of children developing cold differ between the two groups? Use $\alpha = 5\%$.

Status	Vitamin C group	Placebo group	Total
Yes	21	11	32
No	36	35	71
Total	57	46	103

- You can enter the data in the short way as presented in the table above. You have to define two binary variables (One for the outcome variable of interest: cold development status: yes/no, the other for the group membership: Vitamin C/Placebo), then define a third variable, say count, that contains the frequencies or cell sizes.
- Open the data file in SPSS, then choose “Data”, then “Weight cases”; Click on “weight cases by” and enter the variable that contains the cell sizes (e.g., “count”), then “OK”.
- Go to analysis “Analyze”, then “Descriptive Statistics”; then “Crosstabs”.
- The “Crosstabs” window will appear with, at the upper left side, the list of variables in your data set, and at the right side, two boxes where to enter variables “Row(s)” and “Column(s)”. For the crosstabs, you have to choose row variables and column variables. Click on the variables list (left side) to select row and columns variables, then click on the arrows to send the selections at the right side of the window.
- Select the type of the type of test statistics: in the lower right side of the window appear three choices: “Statistics, Cells and Format”. Click on “Statistics”. A “Crosstabs: Statistics” will appear with many possible choices.
- Both variables being nominal, choose “Chi-square” then click “Continue”.
- Back to the “Crosstabs” window, you can click on “Cells” to choose the “Cell Display”. Choose, within “counts”, “observed” and “expected” then click “Continue”.
- Back to the “Crosstabs” window, click on “OK”.

Example: Efficacy of Vitamin C

Groups * coldStatus Crosstabulation

			Cold Status		Total
			No	Yes	
Groups	Vitamin C	Count	21	36	57
		Expected Count	17.7	39.3	57.0
	Placebo	Count	11	35	46
		Expected Count	14.3	31.7	46.0
Total		Count	32	71	103
		Expected Count	32.0	71.0	103.0

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	1.987(b)	1	.159	.200	.116
Continuity Correction(a)	1.429	1	.232		
Likelihood Ratio	2.015	1	.156		
Fisher's Exact Test					
Linear-by-Linear Association	1.968	1	.161		
N of Valid Cases	103				

(a) Computed only for a 2x2 table

(b) 0 cells (.0%) have expected count less than 5. The minimum expected count is 14.29.

Pearson Chi-Square: This is the Chi-square statistics (no correction is done); $\chi^2 = 1.987$; p -value = 0.159 which is > 5%, leading to non-rejection of the null hypothesis.

Fisher's Exact Test: For a 2-by-2 contingency table, there exists an exact test, called "Fisher's exact test". Both two-sided and one-sided p -values are computed. p -value (two-sided) = 0.200, which > 5%, leading to non-rejection of the null hypothesis.

3.4. Two-samples Test for correlated proportions: paired samples

Rationale: As seen previously, there are two scenarios in which one will end up with correlated data: One group data in which each individual is measured twice or two groups data in which each individual from one group is matched to one individual in the second group.

Example: Suppose we want to compare two different treatments for a rare form of cancer. Since relatively few cases of this disease are seen, the two treatment groups should be as comparable as possible. To accomplish this goal, a matched study is set up such that a random member of each matched pair gets treatment *A* (*Chemotherapy*), whereas the other member gets treatment *B* (*Surgery*). Patients are assigned to pairs matched on age (in general within 5 years), gender, and clinical conditions. Patients are followed for 5 years, with survival as the outcome variable. Based on data (to be shown below), is there any difference between treatments *A* and *B*? Use $\alpha = 5\%$.

Wrong Data Presentation: The outcome variable is dichotomous (e.g. Survival: Yes/No) and it is measured within two groups (Treatment *A* group and Treatment *B* group). In this case, one could think data be summarized in two-by-two contingency table as follows.

	Characteristic of interest		Total	e.g.	Survival for 5 years		Total
	Yes	No			Yes	No	
Group 1	a	b	a + b	Treatment A	106	515	621
Group 2	c	d	c + d	Treatment B	95	526	621
Total	a + c	b + d	n	Total	201	1041	1242

Warning: Such presentation assumes that: (1) the patient is the basic unit (sample size equals 621 per group), (2) both groups are independent. In this case, the temptation will be to apply immediately the Chi-Square test to compare both groups, which is **WRONG!!!** Why? Because **groups are not independent** (Matched samples).

Not to do: $\chi^2 = [n (a d - b c)^2] / [(a + c) (b + d) (a + b) (c + d)]$
 $= [1242 (106 \cdot 526 - 515 \cdot 95)^2] / [(201) (1041) (621) (621)] = 0.71822$
 $p\text{-value} = 0.3967 > 5\%$, there is no association between survival and treatments.

Note: We will see later that this conclusion is erroneous!

Matched-Pair Data, the Right Data Presentation: The outcome variable is dichotomous (e.g. Survival: Yes/No) and measured within two matched groups (Treatment A group and Treatment B group). In this case, one should summarize data in two-by-two contingency table as follows.

Outcome variable Group 1	Outcome variable Group 2		Total	Outcome variable Group A	Outcome variable Group B		Total
	Yes	No			Survive	Die	
Yes	a	b	a + b	Survive	90	16	106
No	c	d	c + d	Die	5	510	515
Total	a + c	b + d	n	Total	95	526	621

Very important: (1) The matched pair is the basic unit and pairs are classified according to whether or not each treatment worked for the members of that pair; (2) The sample size n is the number of pairs not the number of individuals; (3) The table can be read as follows:

Outcome variable: Survival			Number of pairs
Group 1: Treatment A	Group 2: Treatment B		
Yes	Yes	Yes	a = 90
Yes	Yes	No	b = 16
No	Yes	Yes	c = 5
No	No	No	d = 510

Test of independence of two correlated variables:

H_0 : No association between variables; H_1 : There is an association between them. "No association between both variables" means that $H_0: \pi_1 = \pi_2$. The test statistic is based on discordant pairs.

(i) $H_0: \pi_1 = \pi_2$ vs $H_1: \pi_1 \neq \pi_2$ (two-sided alternative only)

(ii) Test statistic: The Chi-Square test statistic, "McNemar Chi-Square test statistic", is given by:

$$X^2 = \sum \frac{(O - E)^2}{E} = \frac{(b - c)^2}{(b + c)} \chi_1^2 \quad \text{or} \quad X^2 = \frac{(|b - c| - 1)^2}{(b + c)} \chi_1^2 \text{ (software packages)}$$

(iii) $p\text{-value} = P(X^2 > \chi^2)$, where χ^2 is the observed sample value; Rejection region: reject H_0 if $\chi^2 > \chi_{1,\alpha}^2$, where the critical value is such that $P(X^2 > \chi_{1,\alpha}^2) = \alpha$.

Example on efficacy of Chemotherapy versus Surgery treatments:

(i) $H_0: \pi_1 = \pi_2$ vs $H_1: \pi_1 \neq \pi_2$ (two-sided alternative only)

(ii) Observed value of the Chi-square statistic is equal to: $\chi^2 = \frac{(16-5)^2}{21} = 121/21 = 5.7619$

(iii) $p\text{-value} = P(\chi^2 > 5.76) = 0.016377$; Critical value: for $\alpha = 5\%$, $\chi_{1,0.05}^2 = 3.8415$; then we conclude, using either $p\text{-value}$ or rejection region, that survival rates are not the same for both Chemotherapy and Surgery treatments. We reject H_0 .

Using SPSS: McNemar and Wilcoxon Signed Ranks Tests

Example: The table below gives results from a cohort study on myocardial infarction involving 200 smokers and 200 non-smokers matched on age, gender, arterial blood pressure and total cholesterol levels. Test if the cigarette consumption does have an effect on the development of myocardial infarction. Use $\alpha = 5\%$.

Outcome variable: Myocardial Infarction (MI)		Number of pairs
Group 1: Smokers	Group 2: Non smokers	
Yes	Yes	a = 7
Yes	No	b = 29
No	Yes	c = 14
No	No	d = 150

- You can enter the data in the short way as presented in the table above. You have to define two binary variables (One for the outcome variable of interest: two binary variables (MISmokers: yes=1/no=0 and MINSmokers: yes=1/no=0), then define a third variable, say count, that contains the frequencies or cell sizes.
- Open the data file in SPSS, then choose “Analyze”, then “Non parametric Tests” and “Legacy Dialogs”; then “2 related samples”.
- The “Two-Related-Samples Tests” window will appear with, at the upper left side, the list of variables in your data set, and in the lower left side “Current selections: Variable1 & Variable 2”. Click on the variables of interest (variables 1 and 2), then click on the arrow to send the selection at the right side of the window (it will appear as a difference variable).
- Select the type of the type of test. Indeed, in the lower right side of the window appear three tests “Wilcoxon, Sign and McNemar”. Select “McNemar” or “Wicoxon” or both, then click “OK”.

Example:

McNemar Test		Wilcoxon Signed Ranks Test									
Crosstabs: MISmoke & MINSmoke		Ranks									
	MISmoke	MINSmoke									
		0	1								
	0	150	14								
	1	29	7								
Test Statistics: McNemar Test											
		MISmoke & MINSmoke									
	N	200									
	Chi-Square(a)	4.558									
	Asymp. Sig.	.033									
(a) Continuity Corrected: Use of this form											
		$X^2 = \frac{(b-c -1)^2}{(b+c)} \chi_1^2 \quad (X^2 = 4.558)$									
		$p\text{-value} = 0.033 \text{ (two-tailed)}$									
		<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%;"></th> <th style="width: 50%; text-align: center;">MINSmoke - MISmoke</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">Z</td> <td style="text-align: center;">-2.287(a)</td> </tr> <tr> <td style="text-align: center;">Asymp. Sig. (2-tailed)</td> <td style="text-align: center;">.022</td> </tr> </tbody> </table>					MINSmoke - MISmoke	Z	-2.287(a)	Asymp. Sig. (2-tailed)	.022
	MINSmoke - MISmoke										
Z	-2.287(a)										
Asymp. Sig. (2-tailed)	.022										
		<p>(a) MINSmoke < MISmoke, (b) MINSmoke > MISmoke (c) MINSmoke = MISmoke</p>									
		Test Statistics: Wilcoxon Signed Ranks Test									
		<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%;"></th> <th style="width: 50%; text-align: center;">MINSmoke - MISmoke</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">Z</td> <td style="text-align: center;">-2.287(a)</td> </tr> <tr> <td style="text-align: center;">Asymp. Sig. (2-tailed)</td> <td style="text-align: center;">.022</td> </tr> </tbody> </table>					MINSmoke - MISmoke	Z	-2.287(a)	Asymp. Sig. (2-tailed)	.022
	MINSmoke - MISmoke										
Z	-2.287(a)										
Asymp. Sig. (2-tailed)	.022										
		<p>(a) Based on positive ranks. The equivalent to the form</p> $X^2 = \frac{(b-c)^2}{(b+c)} \chi_1^2 \text{ is given by: } X^2 = Z^2 = 5.23$ <p>$p\text{-value} = 0.022 \text{ (two-tailed)}$</p>									