Continuous outcome variable and categorical predictors

Comparison of means: Two-Samples T-Tests

Rationale: There are two possibilities in which one would like to compare two group means: (1) One group in which each individual is measured twice (pre- & post- treatment: each individual is his/her own control) or two groups in which each individual from one group is matched to one individual in the second group (one-to-one matched groups); (2) Two independent groups (experimental and control groups).

The **t-test** is one of the most often tests used in research. It is often used to determine if a change in outcome occurred pre-post test, to measure if differences in an outcome were based on assigned group, and to determine changes that occurred as a result of two different experimental conditions with the same group of participants. The student t-test is appropriate to use only when you are looking for a difference in the mean value of an outcome variable (dependent variable) that is at the interval or ratio level of measurement.

There are two different types of t-tests and the one you choose to use will depend on whether the independent variable was manipulated.

- **Independent t-test** - used when there are two experimental conditions and different participants were assigned to each condition (also referred to as independent-measures or independent-samples t-test).
- **Dependent t-test** - used when there are two experimental conditions and the *same* participants took part in both conditions of the experiment (sometimes may be referred to as matched-pairs or paired-samples t-test).

1. Paired samples: one group measured twice or matched groups

We need to know the following are all satisfied:
1. A simple random sample is obtained;
2. The populations from which both samples are drawn are normally distributed; and
3. The two samples are matched-pairs.

Example: A given dietary supplement available on the market is suspected to affect individual arterial blood pressure. To assess this relationship between blood pressure and the dietary supplement consumption, an experiment is undergone involving 30 patients (15 male and 15 female). Their systolic blood pressure levels are measured before and after the dietary supplement consumption.

Response: Define the difference variable $D = X_1 - X_2$, (X_1: SBP pre; X_2: SBP post), then:

Confidence intervals for the mean difference ($\mu_d = \mu_1 - \mu_2$):
When σ_d is known, a 100 $(1 - \alpha)$% confidence interval for μ_d is defined by:

$$\left(\bar{d} - z_{\alpha/2} * \frac{\sigma_d}{\sqrt{n}}, \bar{d} + z_{\alpha/2} * \frac{\sigma_d}{\sqrt{n}} \right)$$

When σ_d is unknown, a 100 $(1 - \alpha)$% confidence interval (CI) for μ_d is defined by:

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$$\left(\overline{d} - t_{n-1,\alpha/2} * \frac{s_d}{\sqrt{n}}, \overline{d} + t_{n-1,\alpha/2} * \frac{s_d}{\sqrt{n}} \right)$$

where σ_d and s_d are standard deviations of the difference variable (population and sample). We assume that groups are normally distributed.

From the example above: $\overline{d} = -0.5, s_d = 3.06, s_d/\sqrt{n} = 0.559, t_{n-1,\alpha/2} = 2.045$, then:

95% CI is given by: (-0.5 − 2.045*0.559, -0.5 + 2.045*0.559) = (-1.643, 0.643)

Define the difference variable $D = X_1 - X_2$, (X_1: SBP pre; X_2: SBP post), then:

H_0: $\mu_1 = \mu_2$ vs H_1: $\mu_1 \neq \mu_2$ is equivalent to H_0: $\mu_d = 0$ vs H_1: $\mu_d \neq 0$
H_0: $\mu_1 = \mu_2$ vs H_1: $\mu_1 > \mu_2$ is equivalent to H_0: $\mu_d = 0$ vs H_1: $\mu_d > 0$
H_0: $\mu_1 = \mu_2$ vs H_1: $\mu_1 < \mu_2$ is equivalent to H_0: $\mu_d = 0$ vs H_1: $\mu_d < 0$

Response (Effect of dietary supplement on SBP):
- H_0: $\mu_1 = \mu_2$ versus H_1: $\mu_1 \neq \mu_2$ (H_0: $\mu_d = 0$ versus H_1: $\mu_d \neq 0$), two-tails test; $\alpha = 5\%$.
- Paired-sample *t* test since σ_d is unknown; $t_0 = -0.895$
- For a Student *t* distribution with 29 *df*,
  $t_{29, 0.025} = 2.045$ and $-t_{29, 0.025} = -2.045$ are values delimiting the rejection region.
  p-value = P(T < -|t_0| or T > |t_0|) = P(T < -0.895) + P(T > 0.895) = 0.3782
- We can conclude using either the rejection region or the *p-value*.
  $t_0 = 0.895 < 2.045$ or *p-value* = 0.37 > 5%. In both, we accept the null hypothesis.

When data are normally distributed or the sample size is large (n ≥ 30)

Paired-samples T Test with SPSS:
- Open the data file in SPSS, then choose "Analyze";
- From "Analyze", choose "Compare Means", then "Paired-samples T Test";
- The "Paired-Samples T Tests" window will appear with, at the upper left side, the list of variables in your data set, and in the lower left side "Current selections: Variable1 & Variable 2". Click on the variables of interest (variable 1: pre-treatment or group 1, and variable 2: post-treatment or matching group), then click on the arrow to send the selection at the right side of the window (it will appear as a difference variable).
- Before clicking "OK", one can choose, within "Options", "Confidence intervals" for the difference of means.

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2. Two independent groups (experimental and control groups)

Example: Using the blood pressure experimental data, let us test the hypothesis that the pre-experiment SBP levels in males are higher than that in females.

Group 1	Group 2	
X_{11} X_{12} . X_{1n_1}	X_{21} X_{22} . X_{2n_2}	Observations in both groups
n_1 $\overline{X}_1$ S_1	n_2 $\overline{X}_2$ S_2	Sample values
N_1 μ_1 σ_1	N_2 μ_2 σ_2	Population parameters

Confidence intervals for the mean difference *($\mu_d = \mu_1 - \mu_2$)* ***when data are normal***:
When σ_1 and σ_2 are known, a $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$ is given by:

$$\left(\overline{X}_1 - \overline{X}_2\right) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

For σ_1 & σ_2 unknown but supposed equal, a $100(1-\alpha)\%$ CI for μ_d is defined by:

$$\left(\overline{X}_1 - \overline{X}_2\right) \pm t_{n_1+n_2-2,\alpha/2} \sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}, \quad S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

where s_p is the pooled standard deviations, an estimate of the common standard deviation.

Example: Hypertensive and Normotensive patients

Normotensive Hypertensive	$n_1 = 15$ $n_1 = 12$	$\overline{x}_1 = 144$ $\overline{x}_2 = 160$	$S_1 = 6.2$ $S_2 = 3.9$
$S_p^2 = \dfrac{14*(6.2)^2 + 11*(3.9)^2}{15+12-2} = 28.22; t_{25,0.05} = 1.7081; 90\% CI : (144-160) \pm 1.7081 * \sqrt{28.22*\left(\dfrac{1}{15} + \dfrac{1}{12}\right)} = (-16 \pm 3.5)$			

In order to form a $(1-\alpha)\,100\%$ confidence interval for, and test a hypothesis regarding, $\mu_1-\mu_2$, we need to know the following are all satisfied:

1. A simple random sample is obtained.
2. The populations from which both samples are drawn are normally distributed OR both sample sizes are large ($n_1 \geq 30$ and $n_2 \geq 30$).
3. The two samples are independent.

There is actually no exact solution to the situation where σ_1 and σ_2 are unknown and $\sigma_1 \neq \sigma_2$.

When data are normally distributed or the sample size is large (n ≥ 30)

Independent-samples T Test with SPSS:
- Open the data file in SPSS, then choose "Analyze";
- From "Analyze", choose "Compare Means", then "Independent-samples T Test";
- The "Independent-Samples T Tests" window will appear with, at the left side, the list of variables in your data set. Select the variable of interest then click on the arrow to send the selection at the right side of the window. Choose the grouping variable (group means to be compared) then define groups (specific values defining the two groups).
- Before clicking "OK", one can choose, within "Options", "Confidence intervals" for the difference of means.

Example: Given the SBP experimental data, let us test the hypothesis that the pre-experimental SBP in males in higher than that in females.
  (i) H_0: $\mu_1 = \mu_2$ versus H_1: $\mu_1 > \mu_2$ (1: Males, 2: Females); (ii) $\alpha = 5\%$
  (iii) Since variances are unknown but assumed equal (test for equality: later):
       use the two-samples t test statistics and compute t_0 (see table below).
  (iv) The critical region is delimited by $t_{28, 0.05} = 1.701$;
       p-value = P(T > 3.19) = 0.00175
  (v) Since p-value < 0.05 or t_0 =3.19 > 1.701, we reject the null hypothesis.

Group Statistics	n	Mean	Variance	Standard deviation	Standard error
Male	15	120.2	102.029	10.101	2.608
Female	15	108.2	109.886	10.483	2.707

$$S_p^2 = \frac{14*102.029 + 14*109.886}{15+15-2} = 105.9575; \quad t_{28,0.05} = 1.701; \quad t_0 = \frac{(102.2 - 108.2) - 0}{\sqrt{105.9575(1/15 + 1/15)}} = 3.19$$

SPSS OUTPUT: **Independent Samples Test**

		t-test for Equality of Means						
							95% Confidence Interval of the Difference	
		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
SBP PRE	Equal variances assumed	3.193	28	.003	12.000	3.759	4.301	19.699

p-value (one-tailed test) = 0.0015 < 5%, we reject the null hypothesis.

Test of equality of variances

Fisher distribution: Define $F = \dfrac{S_1^2}{S_2^2}$ (the large variance on the numerator), then F follows the Fisher distribution with ($n_1 - 1$) and ($n_2 - 1$) degrees of freedom, that is, $F \sim F_{n_1-1, n_2-1}$ ($n_1 - 1$ and $n_2 - 1$ are numerator and denominator *df*).

In conducting a test of the hypotheses $H_0 : \sigma_1^2 = \sigma_2^2$ *versus* $H_1 : \sigma_1^2 \neq \sigma_2^2$ (Two-tailed test) with significance level α, we reject the null hypothesis if:
$$F_0 = s_1^2 / s_2^2 > F_{n_1-1, n_2-1, \alpha/2} \ or \ F_0 < F_{n_1-1, n_2-1, 1-\alpha/2}$$
where $P\left(F > F_{n_1-1, n_2-1, \alpha/2}\right) = \alpha/2 \ and \ P\left(F > F_{n_1-1, n_2-1, 1-\alpha/2}\right) = 1-\alpha/2$. There are other statistics used to test the equality of variances, such as Levene's test statistics which is also based on F distribution.

Example: Given the SBP experimental data, let us test the hypothesis that the pre-experiment SBP levels in males are higher than that in females.
  (i) H_0: $\sigma_1 = \sigma_2$ versus H_1: $\sigma_1 \neq \sigma_2$ (1: Males, 2: Females); (ii) α = 5%
  (iii) *F* test statistics: F_0 = 109.886/102.029 = 1.07701. Use the two-samples *t* test statistics and compute t_0 (see table below).
  (iv) The critical region is delimited by $F_{14, 14, 0.975}$ = 0.34 and $F_{14, 14, 0.025}$ = 2.98
  (v) Since 0.34 < F_0 = 1.07701 < 2.98, we do not reject H_0. We were right to assume equal group variances.

Back to SPSS OUTPUT: Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means				
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
SBP PRE	Equal variances assumed	.010	.921	3.193	28	.003	12.000	3.759
	Equal variances not assumed			3.193	27.962	.003	12.000	3.759

Remark: If variances were not assumed equal (if we rejected H_0: equality of variances), we would have used results of the second line in the table (*Equal variances not assumed*).

One-way ANOVA: Analysis of variance with one classification factor

Rationale: Comparison of two or more mean groups ($k \geq 2$). The Student t-test for comparing two means is a special case of one-way ANOVA. The total variability in the outcome variable can be split into two components, between-group variability and within-group variability. If there are significant differences between the groups, the between-group variability will be very large compared to the within-group variability.

Definition:

One-way ANOVA examines the relationship between a **quantitative dependent variable** and a **categorical independent variable**. The one-way ANOVA model is written as follows, for each observation Y_{ij} *(i = 1,..., k; j = 1,..., n_i)*: $Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, where μ is the grand or general mean and α_i is the effect of being in the ith group or the difference between the mean of the ith group and the grand mean, ε_{ij} is a random error.

How can looking at the variances yield any information about the population means of the individual treatments of data? Note that any particular observation (data point) can be decomposed as follows:

 Observation = grand mean + (treatment mean - grand mean) + (observation - treatment mean)
This leads to the decomposition of the total variable into:
        Observed variance = "between treatment variance" + "within treatment variance"

Variance decomposition

Variance components: **SST = SSB + SSW**

Total variation or Total sum of squares:

$$SST = \sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(Y_{ij} - \overline{Y}\right)^2 : \text{variability of all observations with respect to the grand mean}$$

Between group variation or Between group sum of squares:

$$SSB = \sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(\overline{Y}_i - \overline{Y}\right)^2 = \sum_{i=1}^{k} n_i\left(\overline{Y}_i - \overline{Y}\right)^2 : \text{variation of group means with respect to the grand mean}$$

Within group variation or Within group sum of squares:

$$SSW = \sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(Y_{ij} - \overline{Y}_i\right)^2 : \text{variability of observations in each group with respect to the group mean}$$

ANOVA Table

Source of variations	Sum of Squares	df	MS	F	p-value
Between-group	SSB	k-1	MSB	MSB/MSW	
Within-group	SSW	n-k	MSW		
Total	SST	n-1			

Assumptions:
- Random samples of independent observations have been selected from each of the *k* populations or groups and the dependent variable has been recorded for each subject.
- The dependent variable has *NORMAL DISTRIBUTION* in each group.
- The variance of the dependent variable is the same in each group: *common variance*.

F test for Overall Comparison of Group Means:

The hypotheses are: $H_0 : \mu_1 = \mu_2 = ... = \mu_k \; versus \; H_1 : Not \; all \; \mu_k \; are \; equal$

To test these hypotheses, we use the statistic:

$$F = \frac{SSB/(k-1)}{SSW/(n-k)} = \frac{MSB}{MSW} \square \; Fisher \; F \; distribution \;,$$

with *k-1* and *(n-k)* degrees of freedom, *MSB* and *MSW* : Mean Squares Between-group and Within-group respectively. Intuitively, if $F_0 = 1$ or close to 1, H_0 is true, otherwise, $F_0 > 1$.

Example 1: Consider the self-perceived health score (*Y*), known as the Cornell Medical Index (CMI) for female heads of household within four neighborhood. In this study, the focus is on the effects of neighborhood characteristics on health. According the ANOVA table below, we fail to reject the null hypothesis of equality of group means.

ANOVA Table

Source of variations	Sum of Squares	df	MS	F	p-value
Between-groups	894.51	3	298.17	1.18	0.3223
Within-groups	24301.60	96	253.14		
Total	25196.11	99			

Example 2: Consider the rated prestige (*Y*), within three types of profession: professional, white collar and blue collar. According the ANOVA table below, we fail to reject the null hypothesis of equality of group means.

ANOVA Table

Source of variations	Sum of Squares	df	MS	F	p-value
Between-groups	33090	2	16545	65.57	< 0.0001
Within-groups	10598	42	252.33		
Total	43688	44			

One-way ANOVA using SPSS

- Open the data file in SPSS, then choose "Analyze";
- From "Analyze", choose "Compare Means", then "One-Way ANOVA";
- The "One-Way ANOVA" window will appear with, at the left side, the list of variables in your data set. Select the variable of interest then click on the arrow to send the selection at the right side of the window in the box "Dependent list". Choose the grouping variable (group means to be compared) then click on the arrow to send the selection at the right side of the window in the box "Factor".
- There are three choices in the bottom of the right side: "Contrasts", "Post Hoc" and "Options".
    - Click first on "Options". The "One-Way ANOVA: Options" window appears with different options to choose. We can choose (select) "Descriptive", "Homogeneity of variance test" and "Brown-Forsythe" options. The "Brown-Forsythe" option is an alternative of F test for ANOVA when variances are unequal. Click "Continue" to come back to the ANOVA window.
    - Click then on "Post Hoc". The "One-Way ANOVA: Post Hoc Multiple Comparisons" window appears with different options to choose (for equal variances and unequal variances). Let us choose (select) "LSD" for the case of Homogeneity of variance and "Tamhane's T2" for the case of unequal variances. We have in this window the choice of the significance level, the default one being 5%. Click "Continue" to come back to the ANOVA window.
- Back to the "One-Way ANOVA" window, click "OK". We will not consider the "Contrast" choice for now.

Example: We would like to compare the efficacy of three treatments to reduce levels of stress. Twenty one (21) patients known as stressed have been randomly assigned to one of the three therapies. Their levels of stress have been measured after two months of with treatment.

Descriptive statistics:

	N	Mean	Std. Deviation	Std. Error	95% CI for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Treatment A	7	4.2429	1.01630	.38412	3.3029	5.1828	2.90	5.70
Treatment B	7	6.1429	.87723	.33156	5.3316	6.9542	4.80	7.40
Treatment C	7	7.4143	1.38375	.52301	6.1345	8.6940	5.30	9.40
Total	21	5.9333	1.70245	.37150	5.1584	6.7083	2.90	9.40

SPSS OUTPUT: ONEWAY for Levels of Stress

Test of Homogeneity of Variances

Two test statistics, **Bartlett's test statistic** and **Levene test statistic** are frequently used to test homogeneity of variances (common variance assumption) :

$$H_0 : \sigma_1 = \sigma_2 = ... = \sigma_k \; versus \; H_1 : Not \; all \; \sigma_k \; are \; equal$$

Levels of Stress

Levene Statistic	df1	df2	Sig.
1.280	2	18	.302

ANOVA

Levels of Stress

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	35.664	2	17.832	14.392	.000
Within Groups	22.303	18	1.239		
Total	57.967	20			

Robust Tests of Equality of Means

Levels of Stress

	Statistic(a)	df1	df2	Sig.
Brown-Forsythe	14.392	2	15.568	.000

(a) Asymptotically F distributed.

Notes:
- **Test of Homogeneity of Variances:** *p-value* > 5%, we do not reject the null hypothesis. Then we can assume variances equal.
- Bartlett's test is unduly sensitive to departures from the normality of data, and a significant result may therefore indicate non-normality rather than non-equality of variances. In this case, Levene test is used.
- **ANOVA Table:** *p-value* for the F statistic is < 5%, we reject the null hypothesis of equality of means. The three treatments are not equally efficient in reducing stress. If the variances were not equal, we should have used the "**Robust Tests of Equality of Means**" proposed by Brown-Forsythe.

Multiple comparison procedures

Rationale: A significant *F* tells us that there are differences between at least 2 means. A post hoc
   analysis is intended to find out which pairs of means do differ significantly. These are
   unplanned comparisons.

T test for the comparison of pairs of group means in One-way ANOVA:

To test the hypotheses, *H$_0$: $\mu_i = \mu_j$ vs H$_1$: $\mu_i \neq \mu_j$ (i $\neq$ j = 1,2,..., k)*, we use the statistic

$$T = \frac{\overline{Y}_i - \overline{Y}_j}{\sqrt{MSW\left(\dfrac{1}{n_i} + \dfrac{1}{n_j}\right)}} \square\, t_{n-k} : Student\ t\ with\ (n-k)\,df$$

Experimentwise error rate:

The comparisons performed are not independent and lead to an overall error rate greater than the
level of significance α. Bonferronni proposed a method that consists in using a significance level,
for each test computed so that the overall will be equal to α. If *k* tests are performed, each test has
to use α/k as the significance level. We reject H$_0$ when $|t_0| > t_{n-k,\ \alpha/2k}$ or *p-value* $< \alpha/k$.

Post Hoc procedures include Least Significant Difference (LSD) method as well as Tukey HSD
(honestly significant difference) test, Sheffe, Bonferroni and Dunnett (when comparing all means
to a control group mean).

Two-Way Analysis of Variance (Two-way ANOVA)

1. Two-way ANOVA data pattern

Rationale: The dependent variable is classified by *two factors* leading to comparing more mean groups ($k \geq 4$). From the one-way ANOVA, a second factor is included to allow us to model and test partial effects of each of the factors as well as their interactions.

Example 1: Consider two factors with 2 levels each (Row factor: R_1, R_2; Column factor: C_1, C_2), then you would have the following two-way tables of means to compare:

	C_1	C_2	Row marginal
R_1	μ_{11}	μ_{12}	$\mu_{1.}$
R_2	μ_{21}	μ_{22}	$\mu_{2.}$
Column marginal	$\mu_{.1}$	$\mu_{.2}$	$\mu_{..}$

At the sample level, you would get:

	C_1	C_2	Row marginal
R_1	$n_{11}; \overline{Y}_{11}$	$n_{12}; \overline{Y}_{12}$	$n_{1.}; \overline{Y}_{1.}$
R_2	$n_{21}; \overline{Y}_{21}$	$n_{22}; \overline{Y}_{22}$	$n_{2.}; \overline{Y}_{2.}$
Column marginal	$n_{.1}; \overline{Y}_{.1}$	$n_{.2}; \overline{Y}_{.2}$	$n_{..}; \overline{Y}_{..}$

Notes:
- If all cells have equal numbers of observations, you have a *balanced* design. Otherwise, you have an "*unbalanced*" design.
- If all cells have at least one observation, you have a *complete* design. Otherwise, you have an "*incomplete*" design.

Definition:
For each observation Y_{ijk} ($i = 1,..., r; j = 1,..., c; k = 1,..., n_{ij}$), the two-way ANOVA model is written as follows: $Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$, where μ is the grand or general mean, α_i is the effect of the ith row or the difference between the mean of the ith row and the grand mean, β_j is the effect of the jth column or the difference between the mean of the jth column, γ_{ij} is the interaction effect of the ith row and the jth column, and the grand mean, ε_{ijk} is a random error.

Example 2: A study of the length of time (in minutes) spent on individual home visits by public health nurses by nurse's age group (4 levels) and the type of patient's illness (4 levels). Does the mean length differ among age groups of nurses? Does the type of illness affect the length of home visit? Is there interaction between nurse's age and type of patient's illness?

2. Two-Way ANOVA with equal cell numbers

The two-way ANOVA with r levels for the row factor, c levels for column factor and equal cell sample size ($n_{ij} = n$) is the simplest balanced design.

ANOVA Model: $Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$

Variance components: **SST = SSR + SSC + SSRC + SSE**

ANOVA Table

Source of variations	Sum of Squares	df	MS	F	p-value
Row effect	SSR	r-1	SSR/r-1	MSR/MSE	
Column effect	SSC	c-1	SSC/c-1	MSC/MSE	
Interaction	SSRC	(r-1)(c-1)	SSRC/(r-1)(c-1)	MSRC/MSE	
Error	SSE	r c(n-1)	MSE/ r c(n-1)		
Total	SST	r c n-1			

Assumptions:
- Random samples of independent observations have been selected from each of the r c populations or groups and the dependent variable has been recorded for each subject.
- The dependent variable has NORMAL DISTRIBUTION in each group.
- The variance of the dependent variable is the same in each group: common variance.
- Analysis of variance is robust to departures from normality, although the data should be symmetric. To check assumptions, you can use homogeneity of variances tests.

The hypotheses are:
- *$H_o(R)$: There is no row factor (main) effect*
- *$H_o(C)$: There is no column factor (main) effect*
- *$H_o(RC)$: There is no interaction effect between rows and columns*

To test these hypotheses, we use the statistic: F statistics with the corresponding degrees of freedom. Intuitively, If $F_0 = 1$ or close to 1, H_0 is true, otherwise, $F_0 > 1$.

<u>*Example 2*</u>: ***ANOVA Table***

Source of variations	Sum of Squares	df	MS	F	p-value
Type of patient	2992.4	3	997.483	67.94	< 0.0001
Nurse's age group	1201.1	3	400.35	27.27	< 0.0001
Interaction	608.5	9	67.606	4.60	< 0.0001
Error	939.6	64	14.681		
Total	5741.5	79			

Two--way fixed effects ANOVA with equal cell numbers using SPSS

- Open the data file in SPSS, then choose "Analyze";
- From "Analyze", choose "General Linear Model", then "Univariate";
- The "Univariate ANOVA" window will appear with, at the left side, the list of variables in your data set. Select the variable of interest then click on the arrow to send the selection at the right side of the window in the box "Dependent variable".
- Choose the grouping variables (or factors) then click on the arrow to send the selection at the right side of the window in the box "Fixed Factors".
- Choose the Model (Full factorial/Custom: by default, the full factorial. It will give you main effects and all interactions. Then choose the type of sum of squares to use. By default, use type III SS (for balanced designs); for unbalanced with missing values, use type IV SS. Check the box "include intercept in the model". This captures the grand mean and test the null hypothesis of grand mean equals zero.
- There are other choices available: "Contrasts", "Plots", "Post Hoc", "Save", and "Options".
  - Click first on "Post Hoc": Post Hoc Multiple Comparisons" window appears with different options to choose (for equal variances and unequal variances). Let us choose (select) "LSD" for the case of Homogeneity of variance and "Tamhane's T2" for the case of unequal variances. Click "Continue" to come back to the Univariate window.
  - Click then on "Options": Options" window appears with different options to choose.
    - Estimated marginal means: choose the factors to display means for; compare main effects; Confidence intervals adjustment method.
    - Display: you can choose (select) "Descriptive", "Homogeneity of variance test". Click "Continue" to come back to the Univariate window.
- Back to the "Univariate ANOVA" window, click "OK". We will not consider the "Contrast" choice for now.

3. Patterns of interactions in a two-way ANOVA: profiles

Normally we would now have to do a post-hoc analysis to determine how the means differ and what exactly is the interaction effect. We can compare the sample means and the corresponding confidence intervals to determine how the means differ but how do we determine the interaction effect? To help determine the interaction effect, we can carry out a profile analysis.
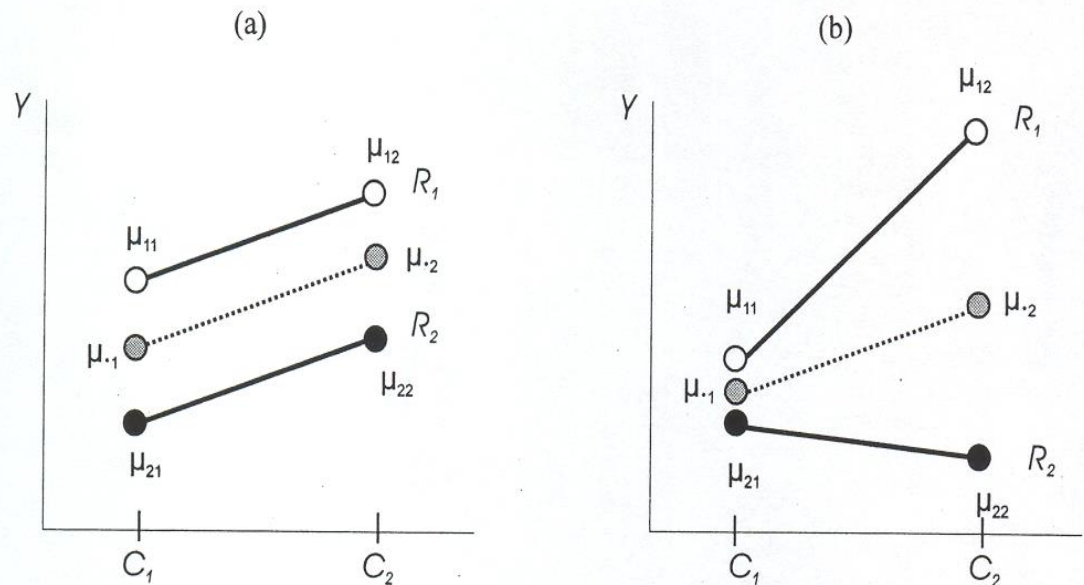
Interactions in two-way Anova

Figure 8.2. Interaction in the two-way classification. In (*a*), the parallel profiles of means (given by the white and black dots connected by solid lines) indicate that R and C do not interact in affecting Y. The R-effect—that is, the difference between the two profiles—is the same at both C_1 and C_2. Likewise, the C-effect—that is, the rise in the line from C_1 to C_2—is the same for both profiles. In (*b*), the R-effect differs at the two categories of C, and the C-effect differs at the two categories of R: That is, R and C interact in affecting Y. In both graphs, the column marginal means $\mu_{\cdot 1}$ and $\mu_{\cdot 2}$ are shown as averages of the cell means in each column (represented by the gray dots connected by broken lines).
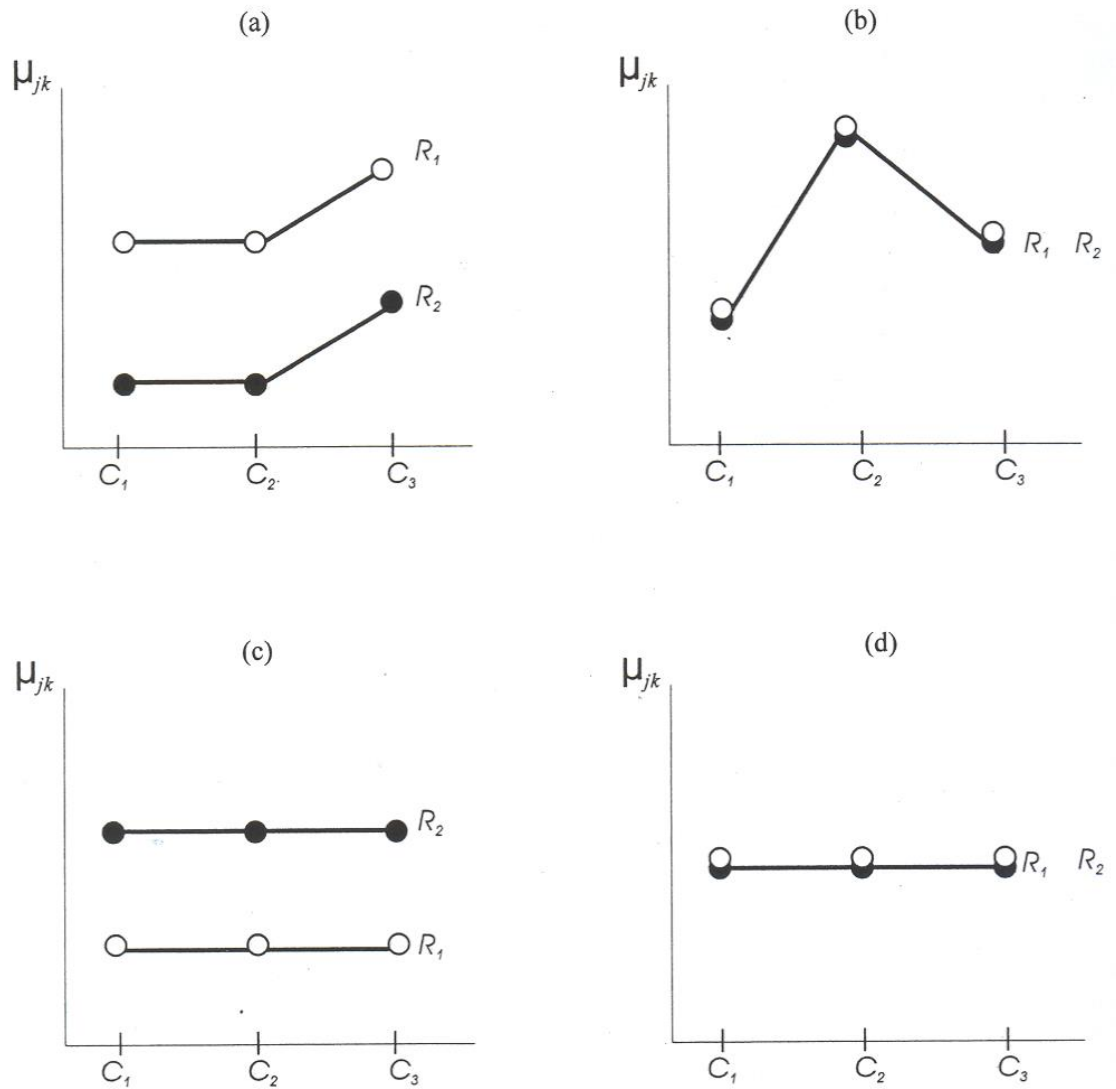
Patterns of relationships in two-way Anova

Figure 8.3. Several patterns of relationship in the two-way classification. In all of these cases, R and C do not interact. (a) Both R and C main effects. (b) C main effects (R main effects nil). (c) R main effects (C main effects nil). (d) No effects (both R and C main effects nil).