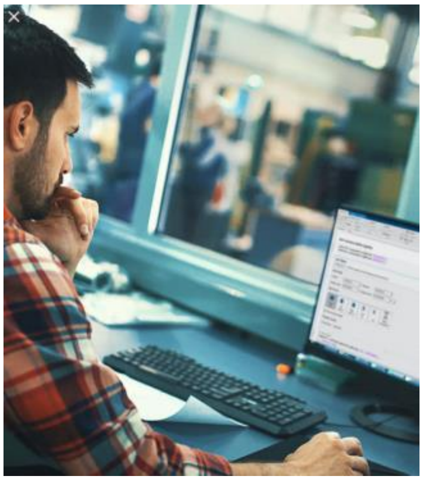
Lecture 8: AC-DC single-phase conversion

Monday, 10 May, 2021 8:27 PM



Right from the outset, it is crucial, crucial to recognize your approach to this lecture should be understanding the problem-solving techniques that are grounded on

- **the physics of devices at the operational level;**
- **the concepts/definitions of average values, RMS values and power factor;**
- **the mathematical theory of differential equations, Fourier series;**
- **symbolic computing and simulation.**

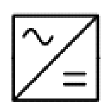
You should not approach this lecture by treating the voltage/current equations as "formulas" that you will just look up when you are given a circuit.

So far in the course, we covered devices (diode, power MOSFET, IGBT) and DC-DC converters (buck, boost, buck-boost), circuit modelling and controller design.

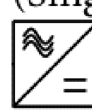
Time to cover other converters, starting with AC-DC converters, i.e., rectifiers.

Back in Lecture 1:

AC-DC conversion, also called *rectification*, refers to the conversion of an AC voltage to a stable DC output voltage. An AC-DC converter is also called a *rectifier*. A rectifier can be *uncontrolled* (where the transferred power depends only on source and load parameters) or *controlled* (where the transferred power depends not only on source and load parameters but also switching parameters) [Har11, Sect. 3.9]. An example of an uncontrolled rectifier is a mercury arc rectifier, the invention of which at the dawn of the twentieth century gave birth to power electronics [Trz16, p. 2].



(Single-phase AC)

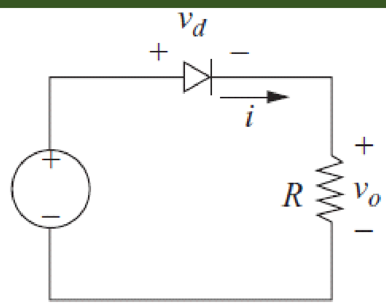


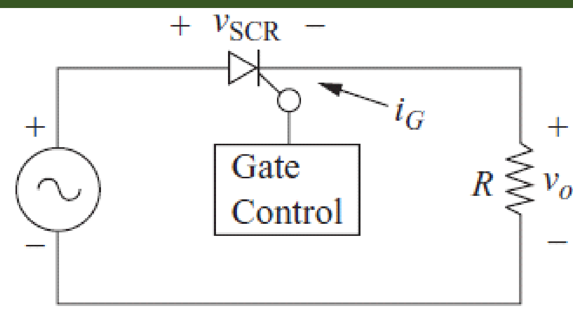
(Three-phase AC)

Rectifiers can be classified as:

Uncontrolled | Controlled

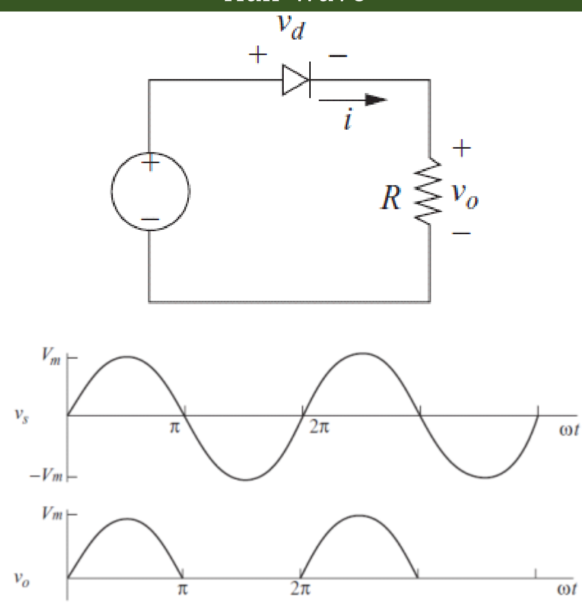
Example

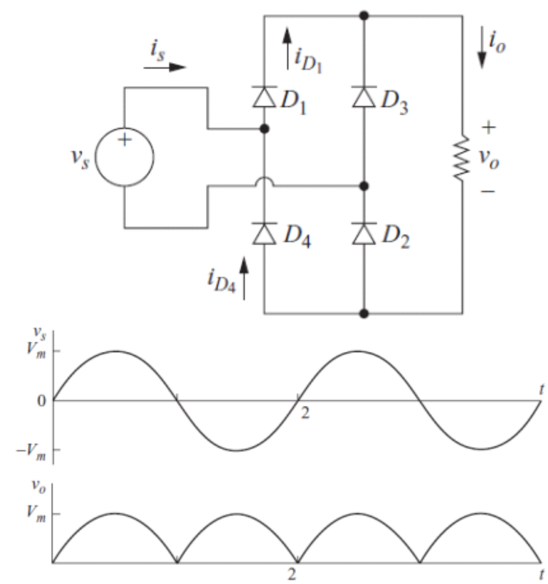




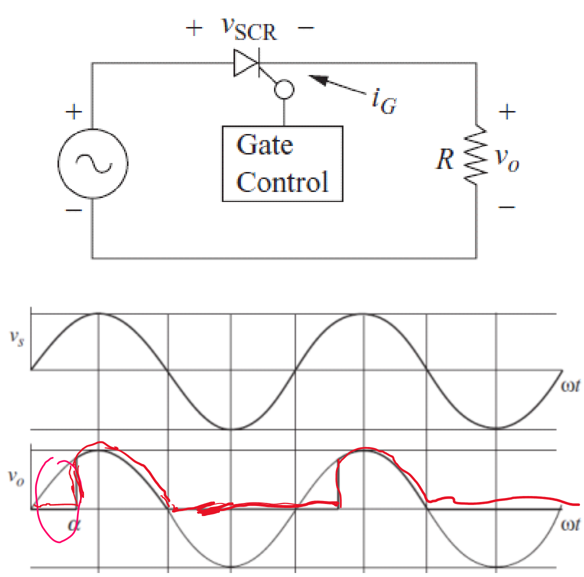
Examples | Half-wave | Full-wave

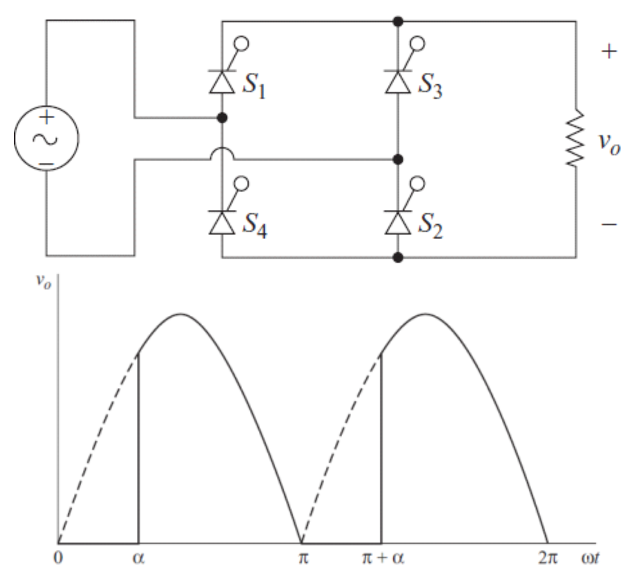
Uncontrolled





Controlled



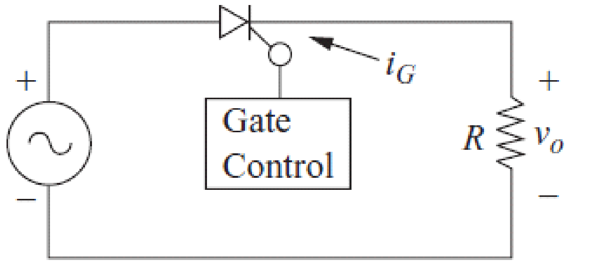


Example | Single-phase | Three-phase

Controlled

+ v_{SCR} −

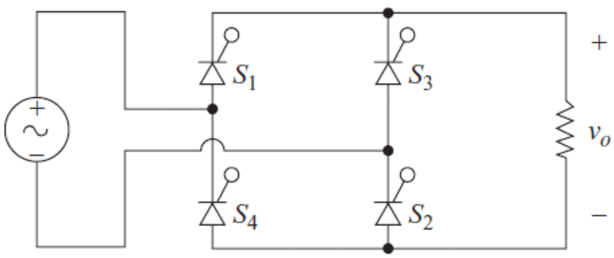
half-wave

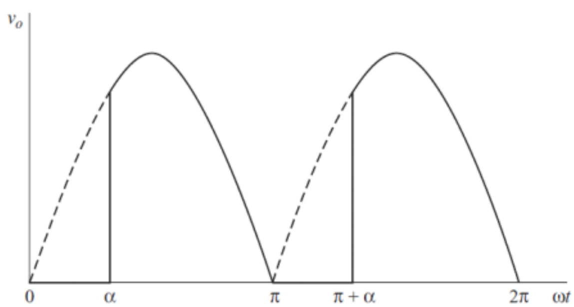


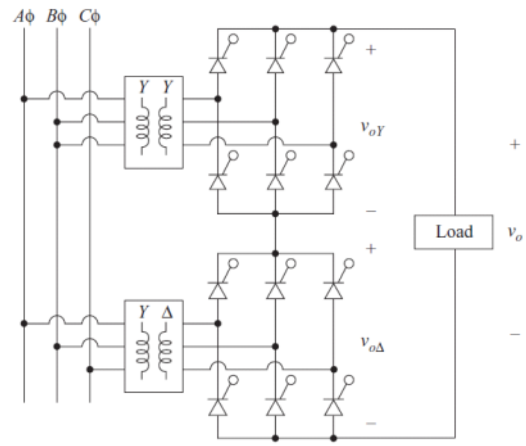
None

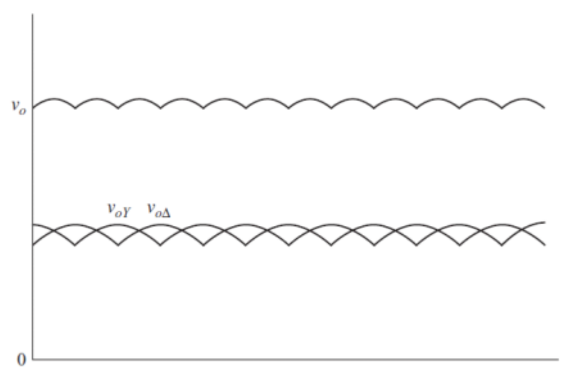
If it is controlled, it had better be full-wave

Controlled full-wave









1. Half-wave rectification

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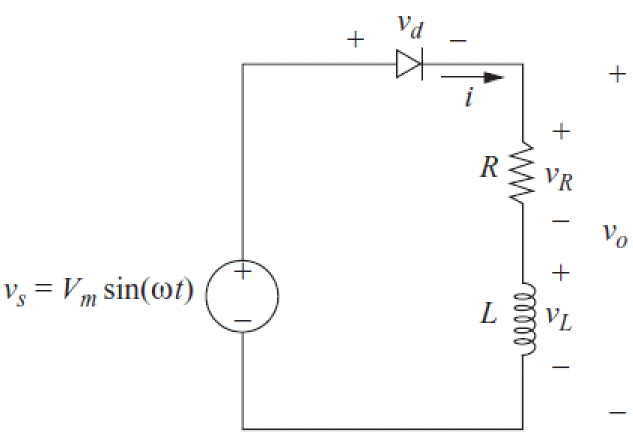
This section discusses uncontrolled and controlled half-wave rectification.

1.1 Uncontrolled half-wave rectification

Our circuit of interest is the half-wave rectifier with a resistive-inductive load.

Why resistive-inductive load? Industrial loads typically contain inductance as well as resistance.

The source is AC, and the objective is to create a load voltage that has a nonzero DC component.



To analyze this circuit, consider two cases separately:

1. When the diode is forward biased and conducting, what is i?
2. When the diode is reverse biased and blocking, $i = 0$.

When the diode is conducting, neglect the diode's forward voltage drop, and apply KVL, we get a first-order differential equation:

$$v_s = v_R + v_L = Ri + L\frac{di}{dt}$$
$$\Rightarrow \frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v_s.$$

Standard time-domain solution (see knowledge base entry) to the first-order differential equation:

$$\dot{x}(t) = ax(t) + f(t)$$

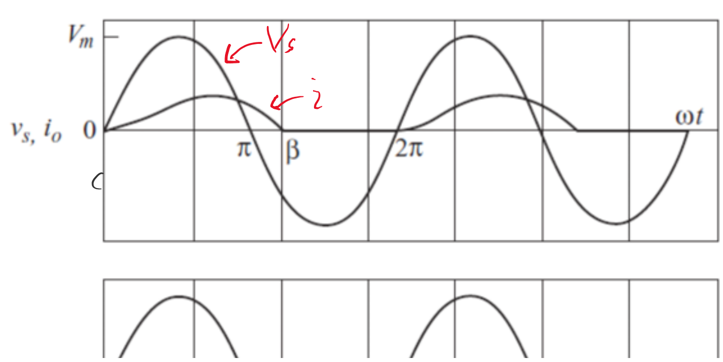
is

$$x(t) = e^{a(t-t_0)}x(t_0) + \int_0^t e^{a(t-u)}f(u)du.$$

Therefore, if $i(0) = 0$,

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v_s$$
$$\Rightarrow i(t) = \frac{1}{L}\int_o^t e^{-\frac{R}{L}(t-u)}v_s(u)\,du.$$

Define time constant $\tau = L/R$. Substituting $v_s = V_m \sin(\omega t)$ into the above, we get



Define time constant $\tau = L/R$. Substituting $v_s = V_m \sin(\omega t)$ into the above, we get

$$i(t) = \frac{V_m}{L}\int_o^t e^{-\frac{(t-u)}{\tau}} \sin(\omega u) du$$

$$\Rightarrow i(t) =$$

$$\frac{L\, V_m\, \omega\, e^{-\frac{R\, t}{L}}}{L^2 \omega^2 + R^2} - \frac{V_m \left(\omega \cos(\omega t) - \frac{R \sin(\omega t)}{L}\right)}{L\left(\frac{R^2}{L^2} + \omega^2\right)}$$

$$\Rightarrow i(t) = \frac{V_m}{Z}\left\{e^{-\frac{t}{\tau}} \sin(\theta) + \sin(\omega t - \theta)\right\},$$

(1.1) $$i(t) = \frac{V_m}{Z}\left\{\sin(\omega t - \theta) + e^{-\frac{t}{\tau}} \sin(\theta)\right\},$$

where $Z = \sqrt{R^2 + \omega^2 L^2}$, $\theta = \text{atan2}(\omega L, R)$.

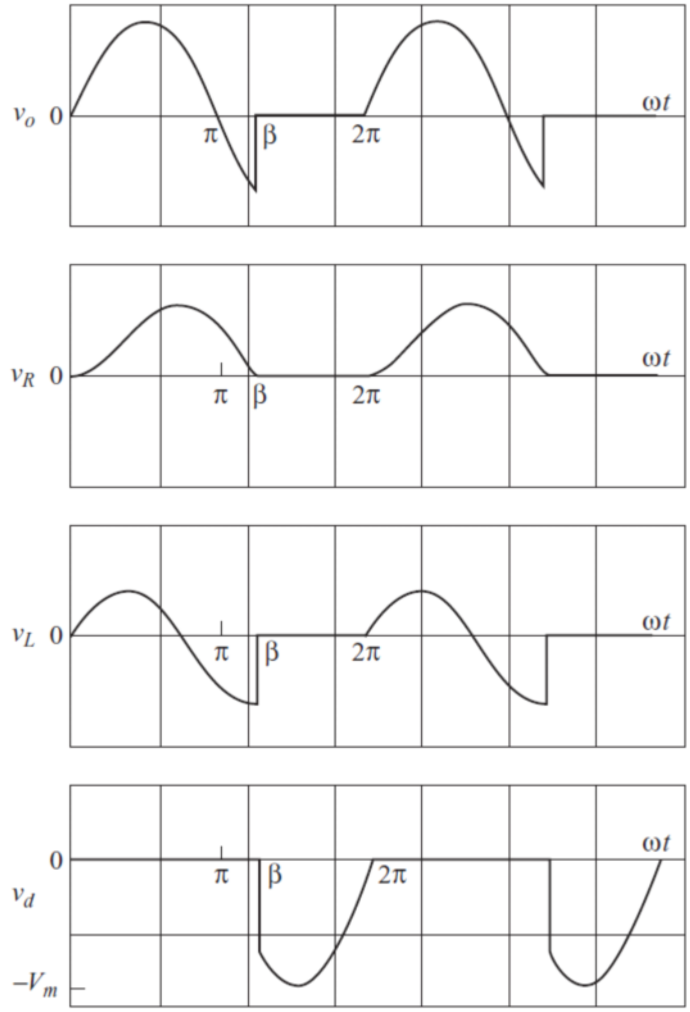
$\omega t_\beta = \beta \Rightarrow t_\beta = \frac{\beta}{\omega}$

The point when the current reaches zero occurs when the diode turns off. The first positive value of ωt that results in zero current is called the ***extinction angle***, denoted β. In other words, Eq. (1.1) is defined for $0 \le t \le \beta/\omega$.

By definition,

$$i\left(\frac{\beta}{\omega}\right) = \frac{V_m}{Z}\left\{\sin(\beta - \theta) + e^{-\frac{\beta}{\omega\tau}} \sin(\theta)\right\} = 0$$

(1.2) $$e^{-\frac{\beta}{\omega\tau}} \sin(\theta) + \sin(\beta - \theta) = 0.$$



Example 1.1

This example was adapted from [Har11, EXAMPLE 3-2]. For the uncontrolled half-wave rectifier, $R = 100\ \Omega$, $L = 0.1$ H, $\omega = 377$ rad/s, and $V_m = 100$ V. Determine (a) an expression for the current in this circuit, (b) the average current, (c) the RMS current, (d) the average power absorbed by the RL load, and (e) the power factor.

Solution: Due to the computational nature of this problem, detailed solution exists in the accompanying Live Script. The following is an outline of the solution. Based on the given data, we can straightaway write down $V_m = 100$, $Z = \sqrt{R^2 + \omega^2 L^2}$, $\tau = L/R$, $\theta = \text{atan2}(\omega L, R)$.

(a) $$i(t) = \begin{cases} \frac{V_m}{Z}\left\{e^{-\frac{t}{\tau}} sin(\theta) + sin(\omega t - \theta)\right\}, & \text{for } 0 \le t \le \frac{\beta}{\omega}; \\ 0, & \text{for } \frac{\beta}{\omega} \le t \le \frac{2\pi}{\omega}; \end{cases}$$

where the extinction angle is the solution to $e^{-\frac{\beta}{\omega\tau}} \sin(\theta) + \sin(\beta - \theta) = 0$, i.e., $\beta = 3.5$ rad, which can be found using MATLAB function `fzero`.

(b) Average current $I_{av} = \frac{1}{T}\int_0^T i(t)dt$, where $i(t)$ is from (a), and $T = \frac{2\pi}{\omega}$. Using the MATLAB function `int`, with integration limits from 0 to $\frac{\beta}{\omega}$, we can work out I_{av} to be 0.3081 A.

(c) RMS current $I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = 0.4736$ A.

(d) Output power $P = \frac{1}{T}\int_0^T v_s(t)i(t)dt = \frac{1}{T}\int_0^T \sin(\omega t)i(t)dt = 22.428$ W.

(e) Power factor $pf = \frac{P}{V_{rms}I_{rms}}$, where $V_{rms} = \frac{V_m}{\sqrt{2}}$. So, $pf = 0.6697$.

1.2 Controlled half-wave rectification

The fundamental limitation of the uncontrolled rectifier is once the source and load parameters are established, the DC level of the output and the power transferred to the load are fixed quantities.

A way to control the output of a half-wave rectifier is to use an SCR instead of a diode.

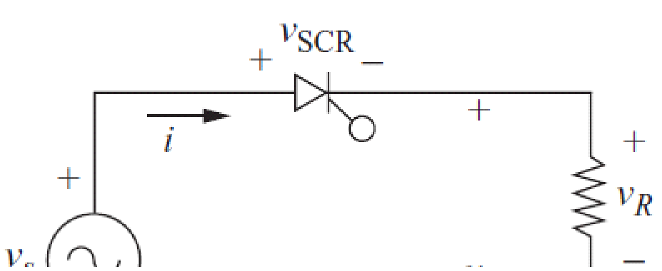
Two conditions must be met before the SCR can conduct:
1. The SCR must be forward-biased.
2. A current must be applied to the gate of the SCR.

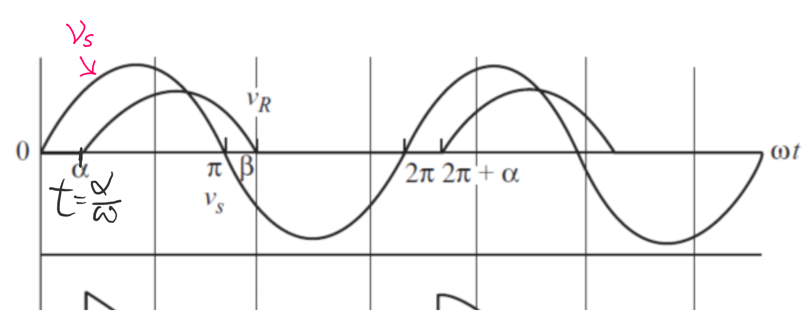
Unlike the diode, the SCR will not begin to conduct as soon as the source becomes positive. Conduction is delayed until a gate current is applied, which is the basis for using the SCR as a means of control.

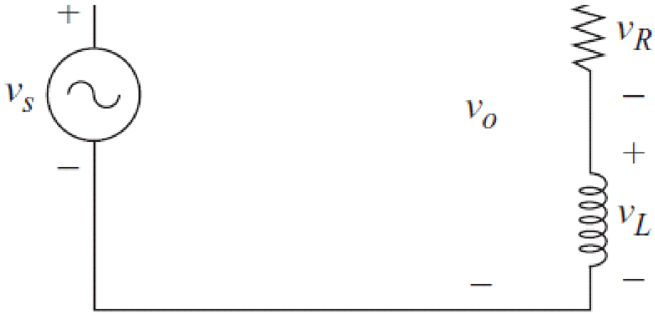
Once the SCR is conducting, the gate current can be removed and the SCR remains on until the (anode) current goes below the holding current level, which is by conventional treated as zero in circuit analyses.

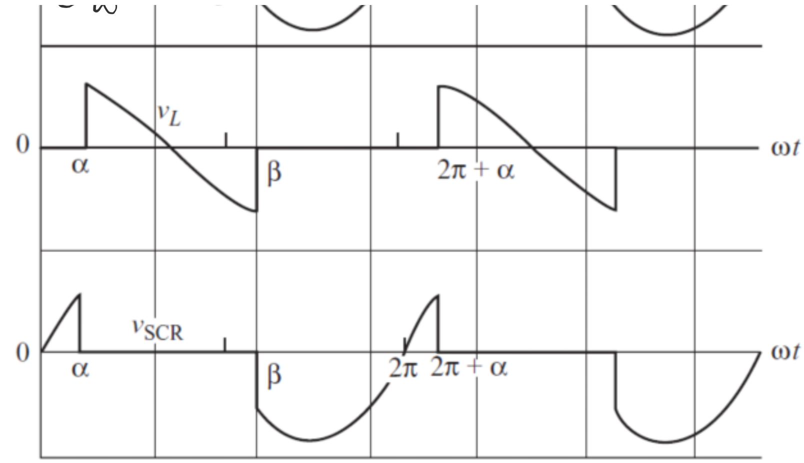
Three types of phase-controlled rectifiers [Ras14, Sect. 10.1]:
- ***Semiconverter***: One-quadrant converter with unipolar v_o ($= v_R + v_L$ in figure below) and output current. Not of interest.
- ***Full converter***: Two-quadrant converter with bipolar v_o but unipolar output current. Covered in this lecture and the next.
  ⚠ v_o can be bipolar, but its DC component remains positive.
- ***Dual converter***: Four-quadrant converter with bipolar v_o and output current. Covered in the next lecture.
  ⚠ Bipolar ≠ AC. Output current flows in one direction for motoring, another direction for regeneration.

Consider the full-converting phase-controlled rectifier below:









Suppose the SCR is fired at time $t = \alpha/\omega$, where α is called the ***delay angle*** [Har11] or ***firing angle*** [Trz16].

Rectifiers like this where the control variable is the (firing) phase angle are called ***phase-controlled rectifiers***.

Using the technique as before, except instead of using the initial condition $i(0) = 0$, which no longer applies, we use the boundary condition $i(\alpha/\omega) = 0$, it is straightforward (see accompanying Live Script) to get

(1.3) $$i(t) = \frac{V_m}{Z}\left\{\sin(\omega t - \theta) - e^{\frac{\alpha - \omega t}{\omega\tau}} \sin(\alpha - \theta)\right\}, \qquad \alpha \le \omega t \le \beta,$$

Compared to (1.1), only the exponent has changed.

The extinction angle β in (1.3) satisfies

(1.4) $$\sin(\beta - \theta) - e^{\frac{\alpha - \beta}{\omega\tau}} \sin(\alpha - \theta) = 0.$$

Compared to (1.2), only the exponent has changed.

How long the conduction lasts is measured by the ***conduction angle***, defined as $\beta - \alpha$.

For an example, see the full-wave case in Example 3.1.

2. Full-wave uncontrolled rectification

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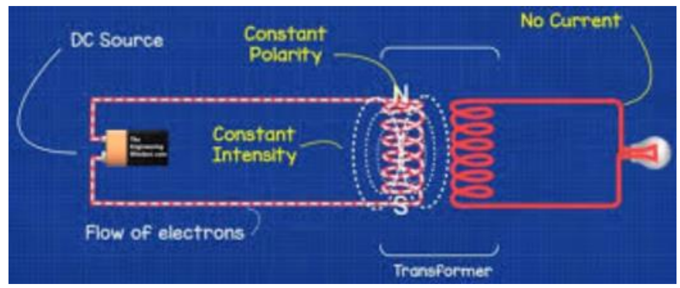
The objective of a full-wave rectifier is to produce a voltage or current that is purely DC or has some specified DC component.

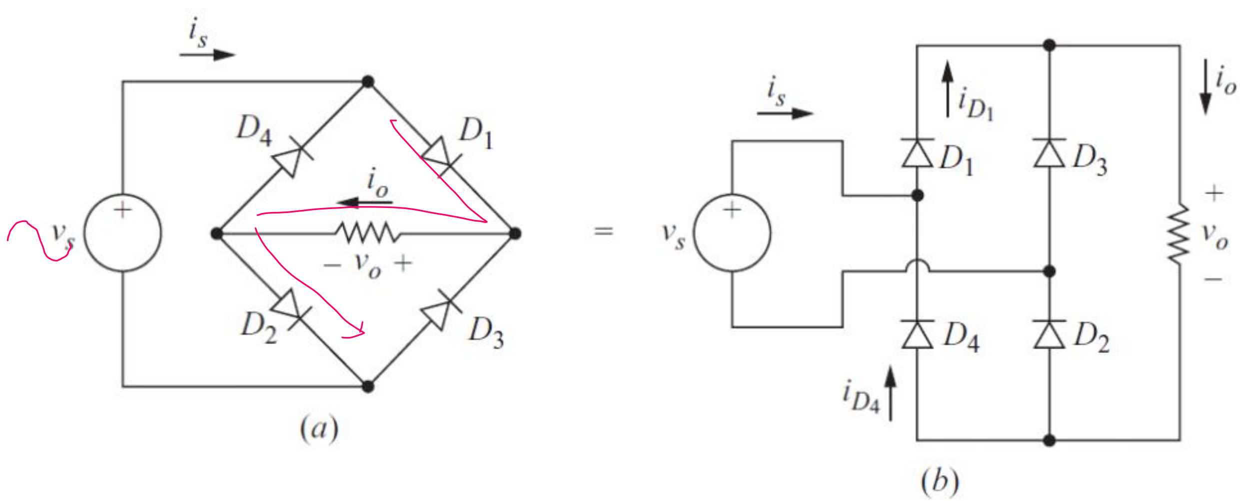
Full-wave rectifiers have some fundamental advantages over half-wave rectifiers:

- The average current in the AC source is zero in the full-wave rectifier, thus avoiding problems associated with nonzero average source currents, particularly in transformers.
- The output of the full-wave rectifier has inherently less ripple than the half-wave rectifier.



Discussion of full-wave rectification is divided into four sections, i.e., Sections 2-3, starting with uncontrolled, single-phase, full-wave rectification.

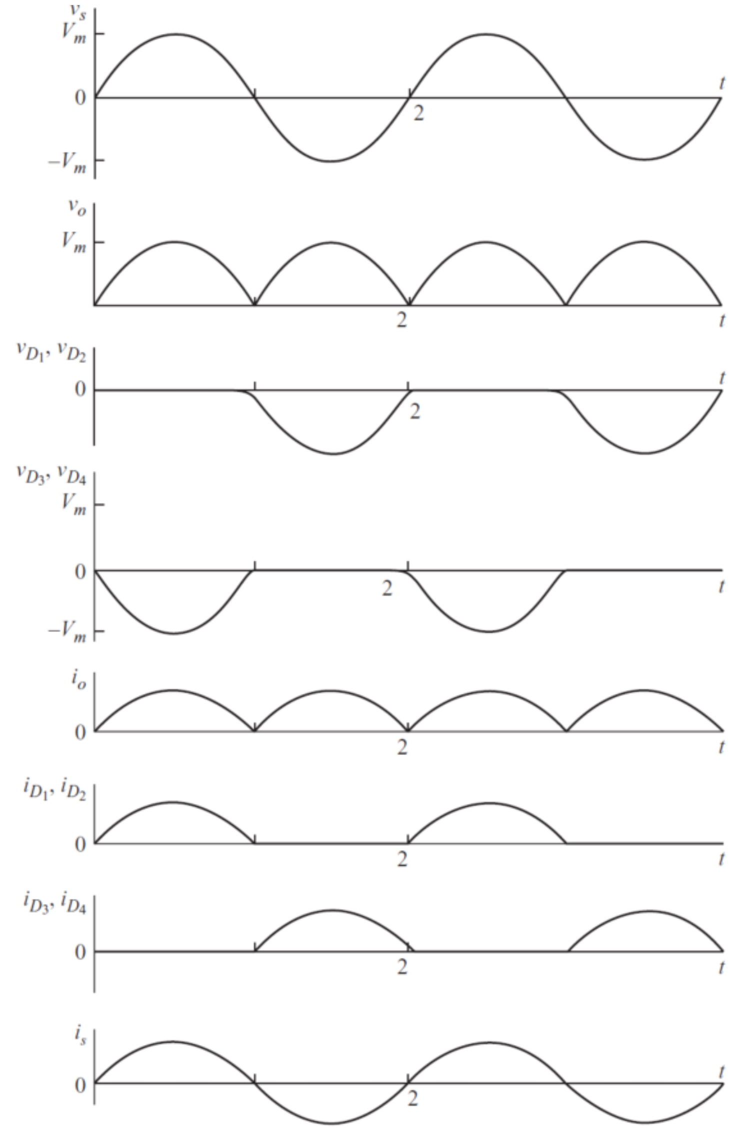
Let us start with the basic circuit:

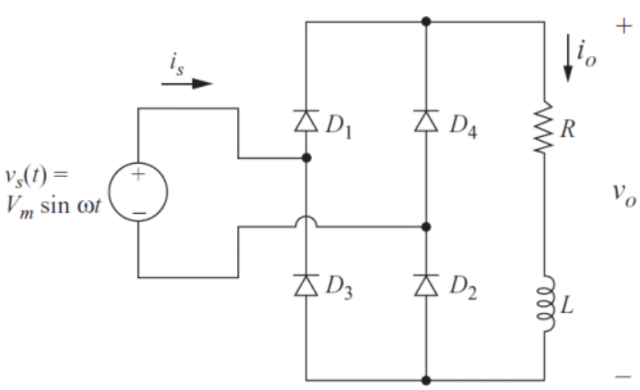


(a) (b)

Characteristics:

- Diodes D_1 and D_2 conduct together, and D_3 and D_4 conduct together.
- By KVL, we can tell D_1 and D_3 cannot conduct at the same time, and D_2 and D_4 cannot conduct at the same time.
- When D_1 and D_2 are on, $v_o = v_s, i_o = i_s$. When D_3 and D_4 are on, $v_o = -v_s, i_o = -i_s$.
- i_o can be positive or zero but can never be negative.
- The RMS source current (denoted $I_{s,\text{rms}}$) is the same as the RMS load current (denoted $I_{o,\text{rms}}$). The squares of the load and source currents are the same, so the RMS currents are equal.
- Since two periods of the output occur for every period of the input, the fundamental frequency of the output voltage is 2ω, where ω is the frequency of the AC input .
- The Fourier series of the output consists of a DC term, and the even harmonics of the source frequency.



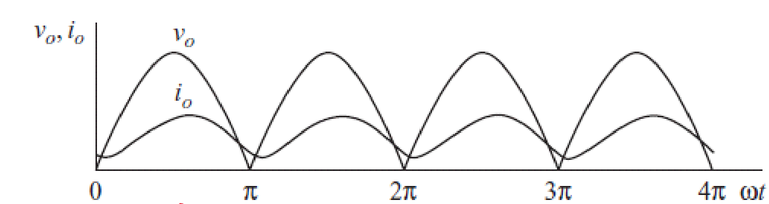


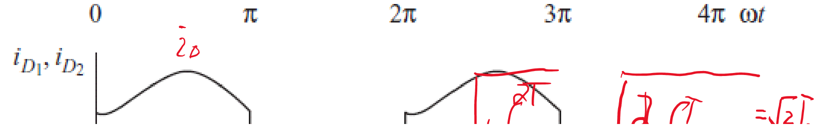
For our analysis, let us consider resistive-inductive load.

The full-wave rectified sinusoidal voltage across the load can be expressed as a Fourier series consisting of a DC term and the even harmonics (see Tutorial 8 for derivation):

(2.1)
$$v_o(t) = \frac{2V_m}{\pi} - \frac{4V_m}{\pi} \sum_{n=2,4,\ldots}^{\infty} \left(\frac{1}{n^2-1}\right) \cos(n\omega t)\,.$$

The current through the RL load is the superposition of the currents associated with each frequency.





Applying phasor analysis,

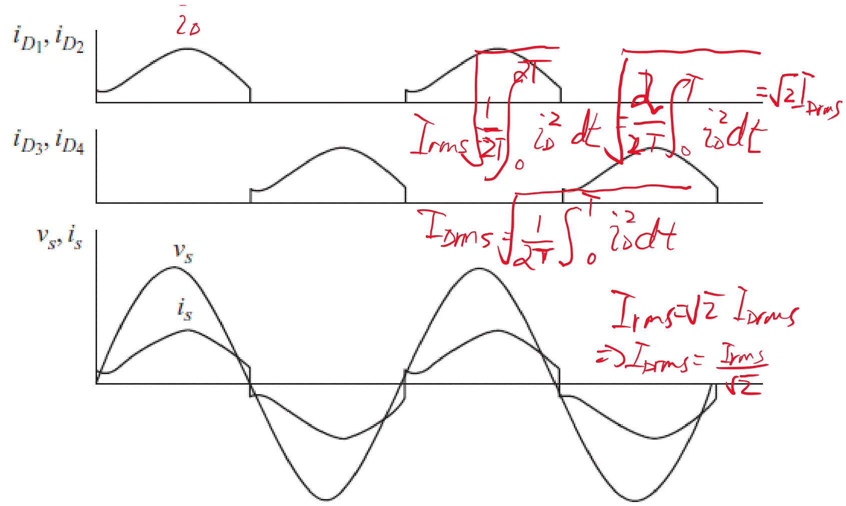
$$I_0 = \frac{V_0}{R},$$

$$I_0 = \frac{V_0}{R},$$

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega L|} = \frac{V_n}{\sqrt{R^2 + n^2\omega^2 L^2}}.$$

Note:

- As n increases, V_n decreases, but the impedance increases, so I_n decreases rapidly with n.
- This means the DC term and only a few, if any, of the AC terms are usually necessary to describe current in an RL load.



The RMS current through the load is thus

$$I_{rms} = \sqrt{I_0^2 + \frac{1}{2}\sum_{n=2,4,\ldots} I_n^2}\,.$$

Example 2.1

This example was adapted from [Har11, EXAMPLE 4-1]. The bridge rectifier circuit of has an AC source with $V_m = 100$ V at 60 Hz and a series RL load with $R = 10\ \Omega$ and $L = 10$ mH.
(a) Determine the average current in the load.
(b) Estimate the peak-to-peak variation in load current based on the first AC term in the Fourier series.
(c) Determine the average power absorbed by the load and the power factor of the circuit.
(d) Determine the average and RMS currents in the diodes.

Solution: Detailed solution is provided in the accompanying Live Script.

(a) The DC load voltage is $V_0 = 2V_m/\pi$ and the DC load current is $2V_m/(\pi R) = 6.3662$ A.

(b) The first AC term has magnitude

$$I_2 = \frac{V_2}{\sqrt{R^2 + 2^2\omega^2 L^2}} = 3.3888 \text{ A}.$$

So, peak-to-peak variation in I_2 is 6.7776 A.

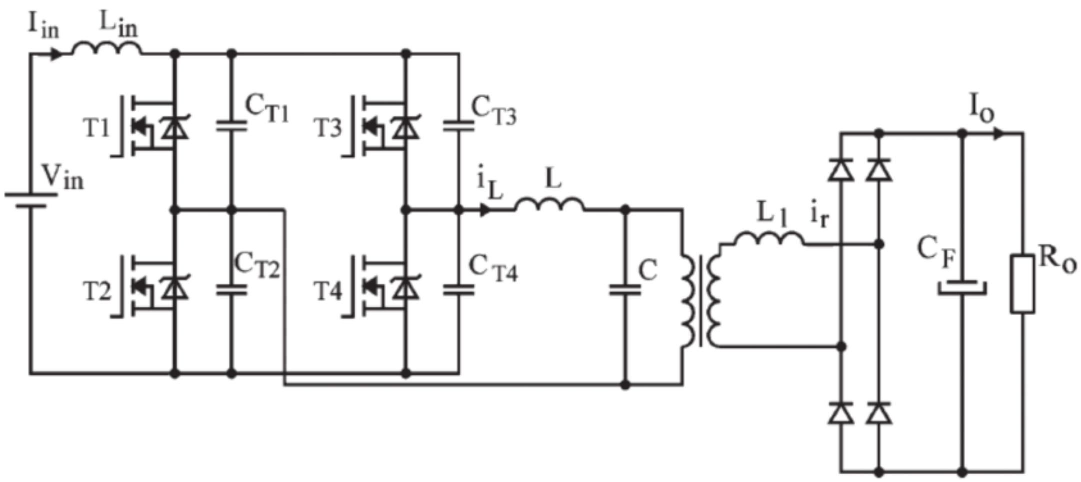
(c) $I_{rms} \approx \sqrt{I_0^2 + \frac{1}{2}I_2^2 + \frac{1}{2}I_4^2 + \frac{1}{2}I_6^2} = 6.8111$ A. (Quiz: Based on the Live Script, why did the computation stop at I_6 and not extend to higher-order terms?) The average power $P = I_{rms}^2 R = 463.9137$ W, and thus the power factor is

$$pf = \frac{P_{av}}{V_{s,rms} I_{rms}} = 0.9632.$$

(d) Since each diode only carries the current half of the time, average diode current is $I_0/2 = 3.1831$ A. For the same reason, RMS diode current is

$$I_{d,rms} = \sqrt{\frac{1}{T}\int_0^{T/2} i^2\,dt} = \sqrt{\frac{1}{2T}\int_0^{T} i^2\,dt} = \frac{I_{rms}}{\sqrt{2}} = 4.8162 \text{ A}.$$

A widely used configuration involves the use of an output capacitor; see the output stage of the current-fed resonant full-bridge boost DC/AC/DC Converter below [JC08].

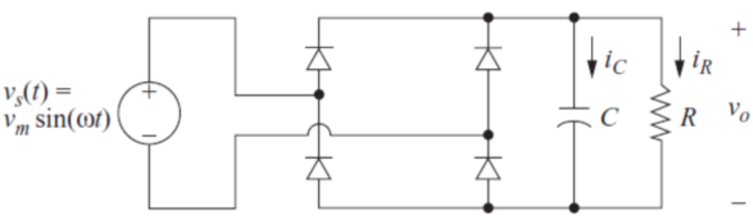


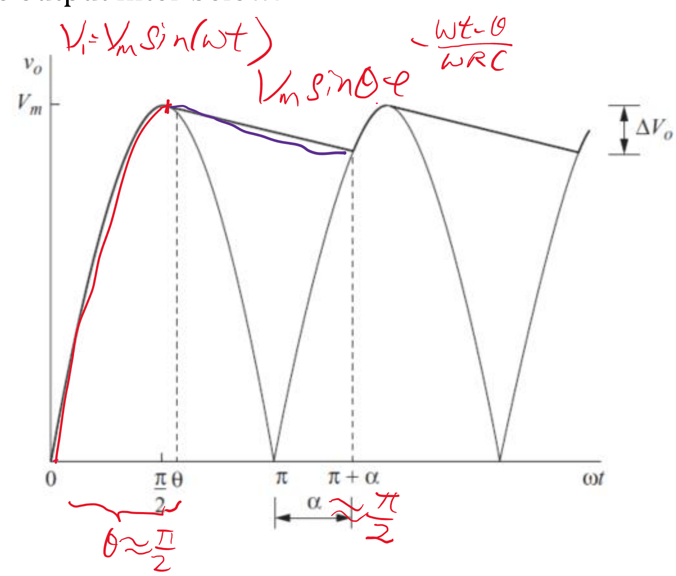
Main advantages of indirect and isolated DC/AC/DC converters [JC08]:

- 👍 Output filter capacitors are less bulky and less expensive than output filter inductors for high-voltage applications [CLL05].
- 👍 Output rectifier diodes can be operated in a discontinuous conduction mode with zero-current switching, minimizing switching losses.

Applications: battery chargers and dischargers, uninterruptible power systems, alternative energy systems, hybrid electric vehicles, and medical X-ray imaging [JC08].

Thus, let us consider the full-wave rectifier with a capacitive output filter below:





The capacitor smoothens out the output voltage.

Take θ as the angle where a diode pair becomes reverse-biased and turns off, then for the first cycle,

$$v_o(\omega t) = \begin{cases} V_m \sin(\omega t), & 0 < \omega t \le \theta; \\ V_m \sin\theta \exp\{-(\omega t - \theta)/(\omega RC)\}, & \theta < \omega t \le \pi + \alpha. \end{cases}$$

- The expression containing an exponential above can be obtained by solving the first-order differential equation associated with a parallel RC circuit. Derivation omitted here.

- The angle $\pi + \alpha$ is when another diode pair becomes forward-biased and turns on.

Let us determine the voltage ripple $\Delta V_0 = V_m - v_o(\pi + \alpha)$, but first we need to determine $\theta, \alpha, v_o(\pi + \alpha)$. Having an expression relating ΔV_0 to circuit parameters R, C allows us to determine what R, C should be, given ΔV_0.

Let us start by determining θ. Define

$$v_1(\omega t) \triangleq V_m \sin(\omega t), \qquad v_2(\omega t) \triangleq V_m \sin\theta \exp\left\{-\frac{\omega t - \theta}{\omega RC}\right\}.$$

At $\omega t = \theta$, clearly we have $v_1(\theta) = v_2(\theta)$, but continuity in rate of change also tells us

$$\frac{d}{d(\omega t)} v_1(\omega t)\bigg|_{\omega t=\theta} = \frac{d}{d(\omega t)} v_2(\omega t)\bigg|_{\omega t=\theta}.$$

Since

$$\frac{d}{d(\omega t)} v_1(\omega t) = V_m \cos(\omega t), \qquad \frac{d}{d(\omega t)} v_2(\omega t) = -\frac{1}{\omega RC} V_m \sin(\theta) \exp\left\{-\frac{\omega t - \theta}{\omega RC}\right\},$$

we have

$$V_m \cos(\theta) = -\frac{1}{\omega RC} V_m \sin(\theta) \Rightarrow \tan(\theta) = -\omega RC.$$

We must be careful when taking arctan, because a negative tangent can be in the second or fourth quadrant, but we know this tangent is in the second quadrant. Therefore,

(2.2) $\theta = \mathrm{atan2}(\omega RC, -1)$.

Now, let us tackle α by applying the same technique as before. Clearly, we must have $|v_1(\pi + \alpha)| = v_2(\pi + \alpha)$, i.e.,

$$|V_m \sin(\pi + \alpha)| = V_m |-\sin\alpha| = V_m \sin\alpha = V_m \sin\theta \exp\left\{-\frac{\pi + \alpha - \theta}{\omega RC}\right\}.$$

In the absence of a closed-form solution to the equation above, we can only say α is the solution to

(2.3) $$\sin\alpha - \sin\theta \exp\left\{-\frac{\pi + \alpha - \theta}{\omega RC}\right\} = 0.$$

Once we have θ and α, the voltage ripple is just

(2.4) $\Delta V_o = V_m - v_o(\pi + \alpha) = V_m - V_m \sin\alpha = V_m(1 - \sin\alpha)$.

In practical circuits with large output capacitance, i.e., $\omega RC \gg \pi$, $\theta \approx \pi/2$, $\alpha \approx \pi/2$, in which case,

$$v_o(\pi + \alpha) \approx V_m \exp\left(-\frac{\pi}{\omega RC}\right),$$

and

(2.5) $$\Delta V_0 = V_m - v_o(\pi + \alpha) \approx V_m\left\{1 - \exp\left(-\frac{\pi}{\omega RC}\right)\right\} \approx \frac{V_m \pi}{\omega RC} = \frac{V_m}{2fRC}.$$

Above, we used the well-known approximation that when $\pi/(\omega RC)$ is small $\exp\{-\pi/(\omega RC)\} \approx 1 - \pi/(\omega RC)$.

Example 2.2

This example was adapted from [Har11, EXAMPLE 4-4]. A full-wave rectifier with a capacitive output filter has a 120 Vrms source at 60 Hz. Additionally, $R = 500\ \Omega$ and $C = 100\ \mu$F.
(a) Determine the output voltage ripple.
(b) Determine the value of C for reducing the output voltage ripple to 1% of the DC value.

Solution: Given $V_m = 120\sqrt{2}$, $\omega = 120\pi$, $R = 500$, $C = 10^{-4}$, the plan is to find θ, α, and then ΔV_o based on (2.4).

(a) From (2.2), $\theta = \mathrm{atan2}(\omega RC, -1) = 1.6238$ rad. By solving (2.3):

$$\sin\alpha - \sin\theta \exp\left\{-\frac{\pi + \alpha - \theta}{\omega RC}\right\} = 0,$$

we get $\alpha = 1.0574$ rad. Now, applying (2.4) gives us

$$\Delta V_o = V_m(1 - \sin\alpha) = 21.8787\ \text{V}.$$

(b) Given $\Delta V_o = 0.01 V_m$, the low ripple implies a high ωRC and hence small $\pi/\omega RC$, making (2.5)

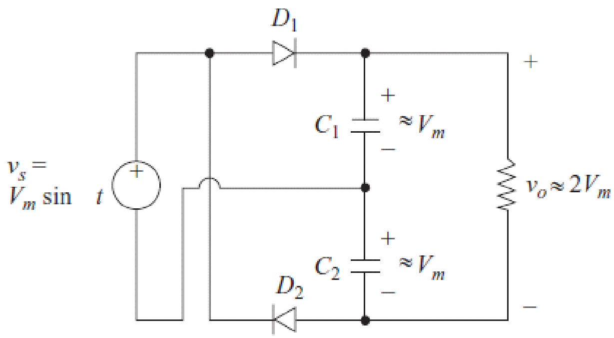
When ΔV_o is given and is small, C can be estimated from (2.5). Detailed solution is provided in the accompanying Live Script.

applicable. So,

$$\frac{\Delta V_o}{V_m} \approx \frac{1}{2fRC} \Rightarrow C \approx 0.0017 \text{ F}.$$

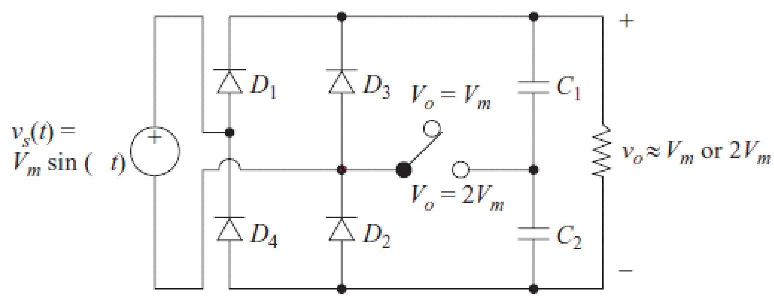
A related rectifier design to the rectifier just discussed is the ***voltage doubler***.

The voltage doubler [Har11, Figure 4-7(a)]:



- C_1 charges to V_m through D_1 when $v_s > 0$.
- C_2 charges to V_m through D_2 when $v_s < 0$.
- v_o is the sum of the capacitor voltages.
- Useful when the output voltage of a rectifier must be larger than the peak input voltage.
- Removes the need for a transformer to step up voltage, saving volume, weight and cost.

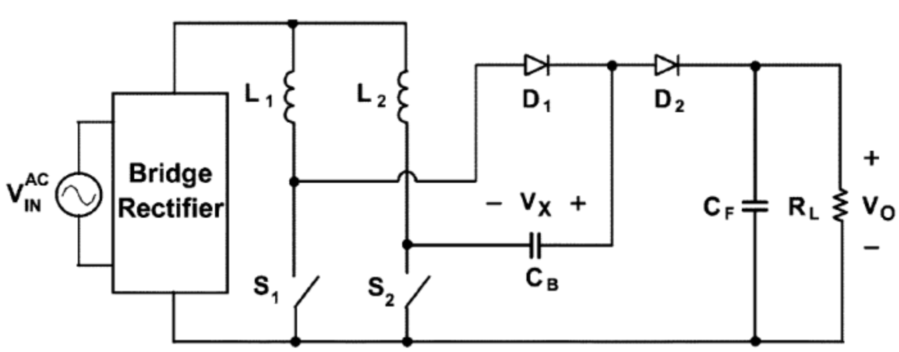
Dual-voltage rectifier [Har11, Figure 4-1(b)]:



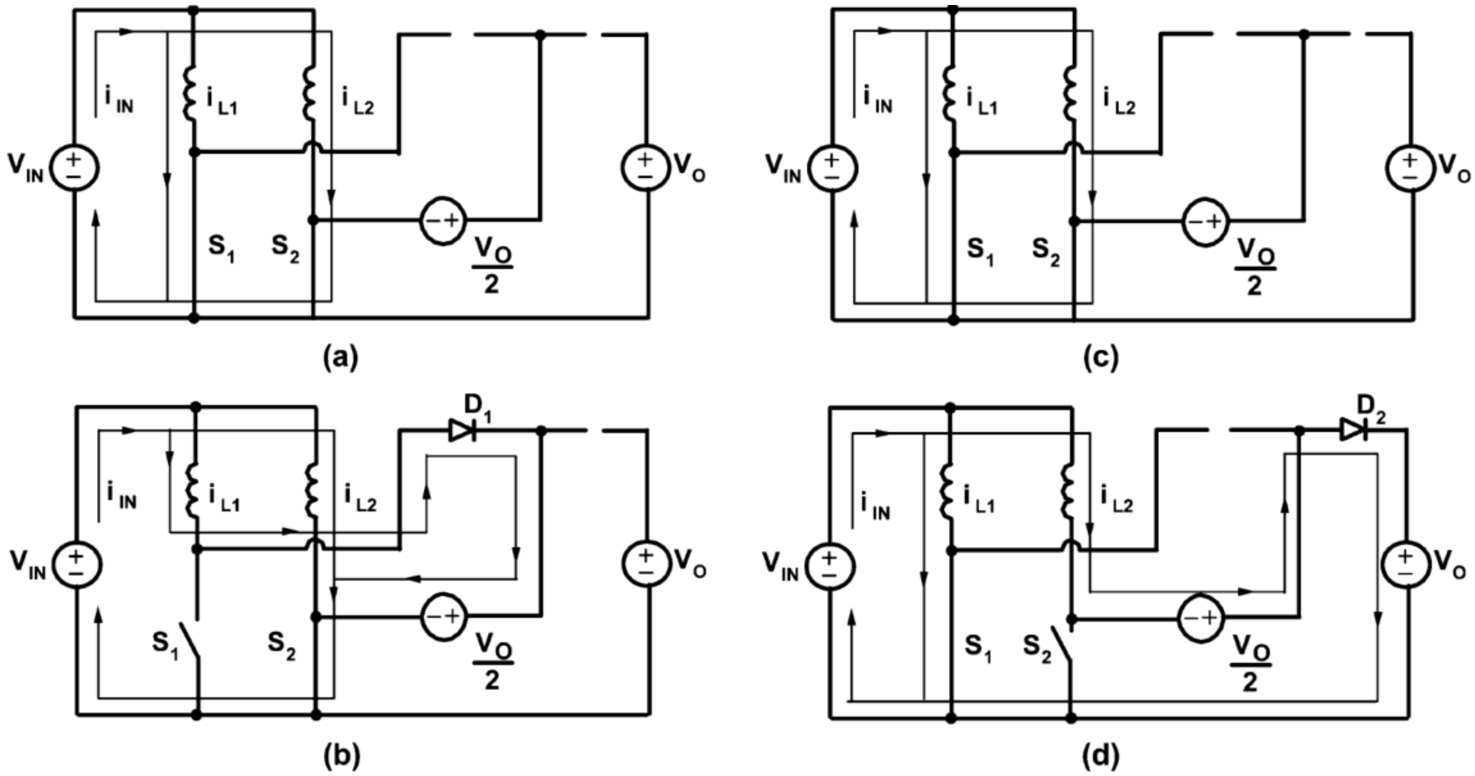
- The full-wave rectifier with a capacitive output filter can be combined with the voltage doubler, as shown above.
- When switch is open, the circuit is similar to the full-wave rectifier, with output at approximately V_m if the capacitors are large.
- When switch is closed, the circuit acts as the voltage doubler:
  - Capacitor C_1 charges to V_m through D_1 when $v_s > 0$.
  - Capacitor C_2 charges to V_m through D_4 when $v_s < 0$.
  - Diodes D_2 and D_3 remain reverse-biased in this mode.
  - $v_o \approx 2V_m$

Applications of the voltage doubler:

- Circuits for catering to two voltage standards, e.g., 240 V for Australia and 120 V for the US.
- Output stage of a more advanced converter, e.g., the power-factor-corrected boost rectifier below [JJ07].



Topological stages for $0.5 \leq D < 1$:



Without derivation [JJ07], $0.5 \leq D < 1, \frac{V_O}{V_{IN}} = \frac{2}{1-D}$, which is double that of the classical boost converter; in this sense, this circuit acts as a voltage doubler.

3. Full-wave controlled rectification

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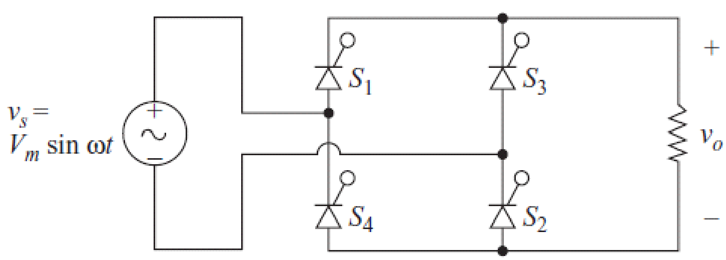
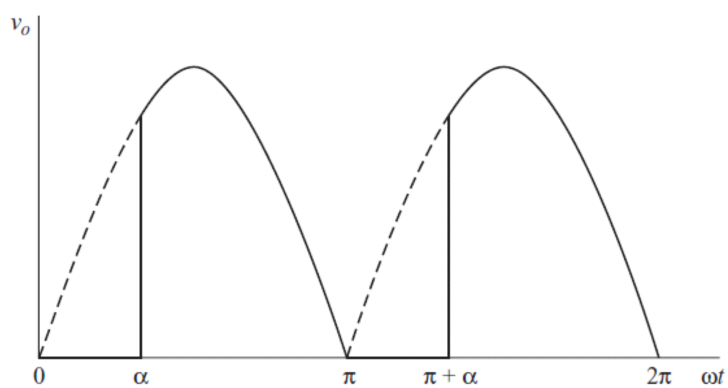
Clearly, being able to control the output of a full-wave rectifier has many advantages, including tracking the desired voltage level.

Substituting the diodes with SCRs gives us a control mechanism based on the delay angle of each SCR.



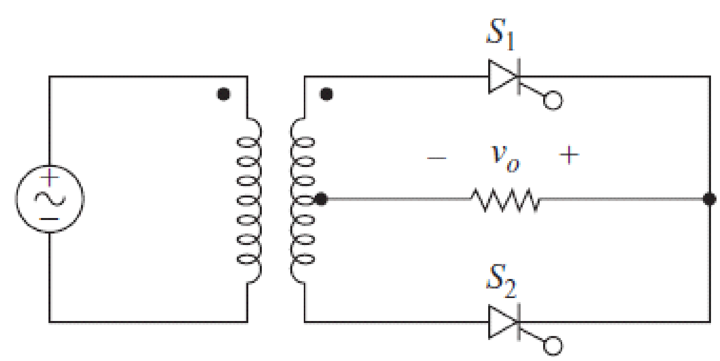


☜ Most common topology: ***bridge rectifier*** [Ras14, p. 107]

- SCRs S_1 and S_2 will become forward-biased when $v_s > 0$, but will not conduct until gate signals are applied.
- Similarly, S_3 and S_4 will become forward-biased when $v_s < 0$, but will not conduct until they receive gate signals.
- The delay angle, α, is the angle interval between the forward biasing of the SCR and the gate signal application.
- If the delay angle is zero, the rectifiers behave exactly as uncontrolled rectifiers with diodes.

Alternative topology: ***center-tapped transformer rectifier*** ☞

- S_1 is forward-biased when $v_s > 0$, and S_2 is forward-biased when $v_s < 0$, but each will not conduct until it receives a gate signal.

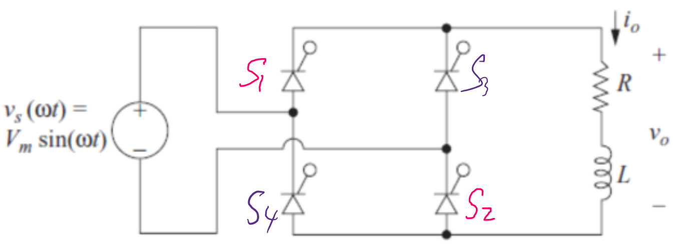


Pros and cons of bridge rectifier and center-tapped transformer rectifier [Ras14, TABLE 3.1]:

	Advantages	Disadvantages
Center-tapped transformer	Simple, only two diodes/SCRs	Limited low power supply, less than 100 W
	Ripple frequency is twice the supply frequency	Increased cost due to the center-tapped transformer
	Provides an electrical isolation	Dc current flowing through each side of the secondary will increase the transformer cost and size
Bridge rectifier	Suitable for industrial applications up to 100 kW	The load cannot be grounded without an input-side transformer
	Ripple frequency is twice the supply frequency	Although an input-side transformer is not needed for the operation of the rectifier, one is normally connected to isolate the load electrically from the supply
	Simple to use in commercially available units	

Quiz

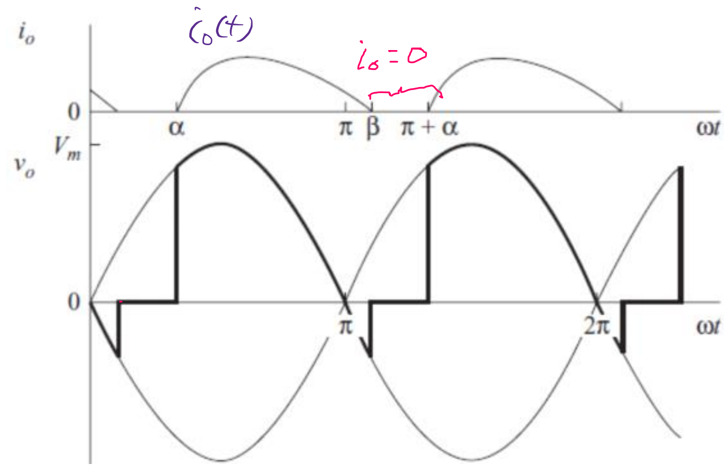
Among "semiconverter", "full converter" and "dual converter", what type of converter are the bridge converter and center-tapped transformer converter?



Load current for a controlled full-wave rectifier with an RL load can be either *continuous* or *discontinuous*, and a different analysis is applicable to each.

Let us consider discontinuous current, then continuous current.

Discontinuous conduction mode (DCM): Analysis of the controlled *full-wave* rectifier operating in the DCM is identical to that of the controlled *half-wave* rectifier (see Sec. 1.2) except that the period for the output current is π rather than 2π rad.



When $v_s > 0$, SCRs S_1 and S_2 in the bridge rectifier are forward-biased, while S_3 and S_4 are reverse-biased.

At $\omega t = \alpha$, S_1 and S_2 are fired and become conducting, causing $v_o = v_s$.

At $\omega t = \beta$, where $\beta < \pi + \alpha$, $i_o = 0$, S_1 and S_2 turn off and become blocking.

Between angles α and β, (1.3) is applicable:

$$i_o(t) = \frac{V_m}{Z}\left\{\sin(\omega t - \theta) - e^{\frac{\alpha - \omega t}{\omega \tau}} \sin(\alpha - \theta)\right\}.$$

β can be determined from (1.4)

Between $\omega t = \beta$ and $\omega t = \pi + \alpha$, i_o remains at zero.

At $\omega = \pi + \alpha$, S_3 and S_4 are fired and become conducting

$$i_o(t) = \frac{V_m}{Z}\left\{\sin(\omega t - \theta) - e^{\frac{\alpha - \omega t}{\omega\tau}}\sin(\alpha - \theta)\right\}.$$

β can be determined from ([1.4](#)).

At $\omega = \pi + \alpha$, S_3 and S_4 are fired and become conducting.

Example 3.1

This example was adapted from [Har11, EXAMPLE 4-7]. A controlled full-wave bridge rectifier has a source of 120 Vrms at 60 Hz. Additionally, $R = 10\ \Omega$, $L = 20$ mH, $\alpha = 60°$.
(a) Determine if the rectifier is operating in DCM.
(b) Determine an expression for the load current.
(c) Determine the average load current.
(d) Determine the power absorbed by the load.
(e) Make a plot of power factor against $\alpha \in [10°, 80°]$.

Solution: Due to the computational nature of this problem, detailed solution exists in the [accompanying Live Script](#). The following is an outline of the solution. Based on the given data, we can straightaway write down $V_m = 120\sqrt{2}$, $Z = \sqrt{R^2 + \omega^2 L^2}$, $\tau = L/R$, $\theta = \text{atan2}(\omega L, R)$.

(a) To determine if the rectifier is operating in DCM, we need to check if the extinction angle β is less than $\pi + \alpha$. Solving ([1.4](#)), we get $\beta = 3.7772$ rad or $216.41692°$, which is less than $\pi + \alpha = 240°$, so the rectifier is operating in DCM.

(b) We have all the ingredients we need to assemble an expression for the current:

$$i_o(t) = \frac{V_m}{Z}\left\{\sin(\omega t - \theta) - e^{\frac{\alpha - \omega t}{\omega\tau}}\sin(\alpha - \theta)\right\}$$

$= 13.55\sin(377t - 0.646) - 21.22e^{-500t}$ A, where $0.0028 \le t \le 0.01$.

(c) The average load current is

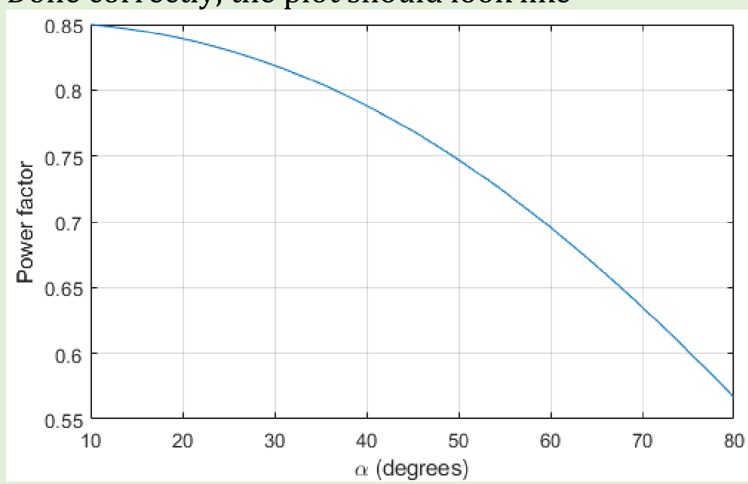
$$I_0 = \frac{1}{T}\int_{\alpha/\omega}^{\beta/\omega} i(t)dt = 7.048\text{ A}.$$

⚠ T is half of the period of the input!

(d) The power absorbed by the load is

$$P = I_{rms}^2 R = \frac{R}{T}\int_{\frac{\alpha}{\omega}}^{\frac{\beta}{\omega}} i^2(t)dt = 697.1287\text{ W}.$$

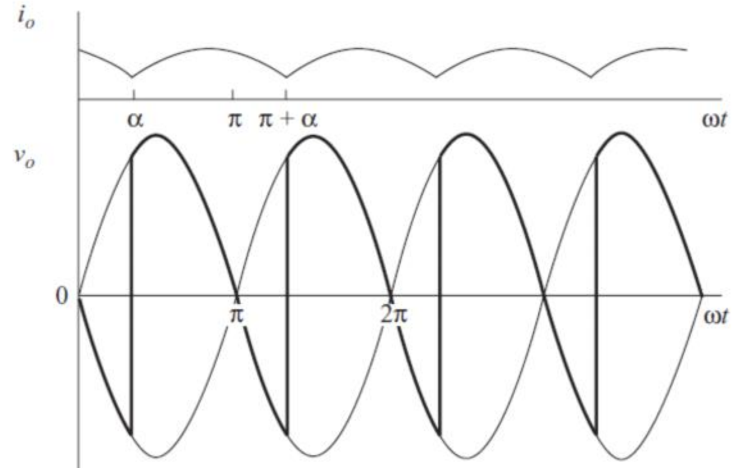
(e) Making a plot of power factors for a range of delay angles requires us to write a "for" loop to repeat the calculations above for the range of angles. Done correctly, the plot should look like



The plot shows increasing the delay angle worsens the power factor.

What we can learn from the previous example: while being controllable, phase-controlled rectifiers have poor power factors. The next lecture will introduce better rectifiers.

Continuous conduction mode (CCM): Load current is still positive at $\omega t = \pi + \alpha$, when S_3 and S_4 are turned on and S_1 and S_2 are forced off.



Since the initial condition for current in the second half-cycle is not zero, the current function does not repeat, i.e., (1.3) does not apply.

So that CCM occurs, $\beta = \pi + \alpha$, and $i(\beta/\omega) \geq 0$, or equivalently, from (1.4),

$$\sin(\beta - \theta) - e^{\frac{\alpha-\beta}{\omega\tau}} \sin(\alpha - \theta) \geq 0$$

$$\Rightarrow \sin(\pi + \alpha - \theta) - e^{-\frac{\pi}{\omega\tau}} \sin(\alpha - \theta) \geq 0$$

$$\Rightarrow \sin(\alpha - \theta)\left(1 + e^{-\frac{\pi}{\omega\tau}}\right) \leq 0$$

$$\Rightarrow \sin(\alpha - \theta) \leq 0$$

We know α and $\theta = \text{atan2}(\omega L, R)$ are in the first quadrant, so $-\pi/2 \leq \alpha - \theta \leq 0$. The inequality $\alpha \geq \theta - \pi/2$ (a negative number) is not useful, so we use the inequality $\alpha \leq \theta$ instead, i.e.,

(3.1) $\alpha \leq \theta = \text{atan2}(\omega L, R)$.

$\alpha \leq \text{atan2}(\omega L, R)$

$-\frac{\pi}{2} \leq \alpha - \theta$

$\alpha \geq \theta - \frac{\pi}{2}$

$\alpha - \theta \leq 0$

$\alpha \leq \theta$

Either $\beta \geq \pi + \alpha$ or (3.1) can be used as a criterion for the CCM.

Without derivation (see [Har11, pp. 136-137]), the output voltage can be expressed as Fourier series:

(3.2)
$$v_o(t) = \frac{2V_m}{\pi}\cos\alpha + \sum_{n=2,4,\ldots}^{\infty} V_n \cos(n\omega t + \theta_n),$$

where

$$V_n = \sqrt{a_n^2 + b_n^2},$$

$$a_n = \frac{2V_m}{\pi}\left\{\frac{\cos((n+1)\alpha)}{n+1} - \frac{\cos((n-1)\alpha)}{n-1}\right\},$$

$$b_n = \frac{2V_m}{\pi}\left\{\frac{\sin((n+1)\alpha)}{n+1} - \frac{\sin((n-1)\alpha)}{n-1}\right\}.$$

An expression for θ_n is not given, because it is not needed for our purpose.

For calculation of the RMS current, power and power factor, the technique of Sec. 2 is applicable.

For an example, refer to Tutorial 8.

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Friday, 14 May 2021 4:01 PM

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