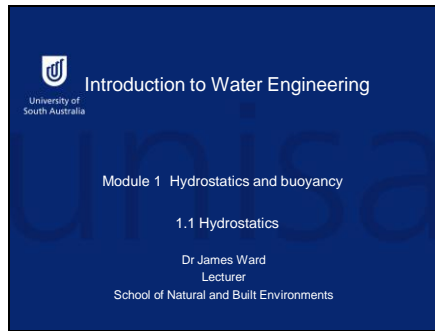


Introduction to Water Engineering

Slide 1



University of South Australia

Introduction to Water Engineering

Module 1 Hydrostatics and buoyancy

1.1 Hydrostatics

Dr James Ward
Lecturer
School of Natural and Built Environments

Welcome to Module 1 and Hydrostatics and buoyancy

Slide 2



University of South Australia

Copyright Notice

Do not remove this notice.

COMMONWEALTH OF AUSTRALIA
Copyright Regulations 1969

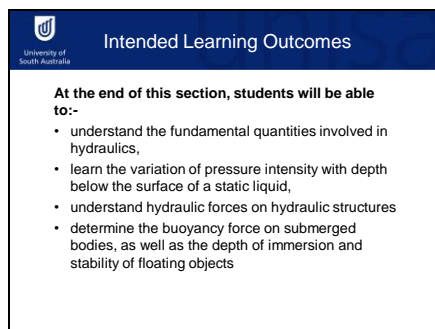
WARNING

This material has been produced and communicated to you by or on behalf of the University of South Australia pursuant to Part VB of the Copyright Act 1968 (the Act).

The material in this communication may be subject to copyright under the Act. Any further reproduction or communication of this material by you may be the subject of copyright protection under the Act.

Do not remove this notice.

Slide 3



University of South Australia

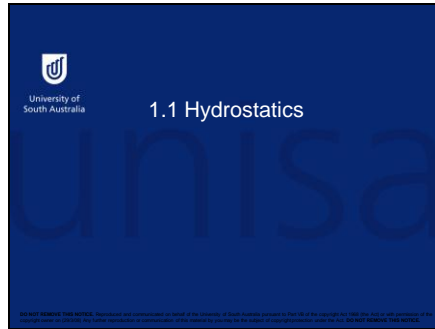
Intended Learning Outcomes

At the end of this section, students will be able to:-

- understand the fundamental quantities involved in hydraulics,
- learn the variation of pressure intensity with depth below the surface of a static liquid,
- understand hydraulic forces on hydraulic structures
- determine the buoyancy force on submerged bodies, as well as the depth of immersion and stability of floating objects

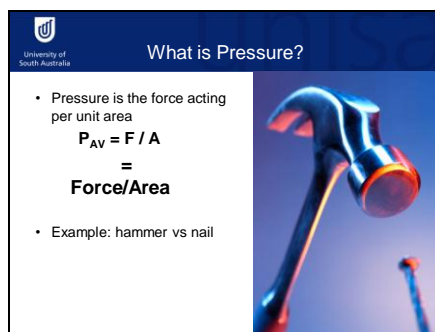
The learning outcomes are presented here – we'll look at hydraulic quantities, pressure intensities and the impact of hydraulic forces on hydraulic structures including buoyancy effects.

Slide 4



Let's start by looking at hydrostatics

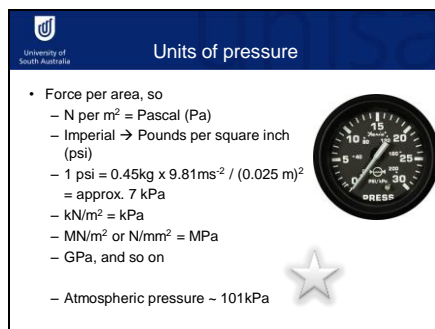
Slide 5



Pressure is defined as the force per unit area.

For example, imagine hitting a piece of wood with a hammer – you might make a small dent in the wood. But apply the same force to a nail and it will easily penetrate the wood. This is because the force is distributed over a much smaller area, creating a high pressure.

Slide 6



Because we're dealing with force per area

The basic unit is Newton per square metre, but we more commonly call this a Pascal.

You might be familiar with the imperial units of pounds per square inch or PSI, which is another way of expressing a force (pound) per unit area (square inch)

Let's do a conversion from imperial to metric units. A pound's about 0.45 kilograms, which we multiply by gravity to get force, then divide by a square inch converted into square metres. One PSI comes out to about 7000 Pascals, or 7 kilopascals. You can check it with a calculator.

A kilopascal is a kilonewton per square metre

A megapascal is a million newtons per

square metre, or could be newtons per square millimetre

A gigapascal is a thousand megapascals etcetera

By the way, atmospheric pressure's around 101 kilopascals.

Image source
<http://blueseas.com/products/1024B>

Slide 7

University of South Australia

Pressure & force

In hydrostatics, the **force** is the **weight** of the water above

- $F = mg = \text{mass} \times \text{gravity acceleration}$
- **Mass = density \times volume:**
- $m = \rho V$

$$\rho = \frac{m}{V}$$

In hydrostatics, the force at a given depth is due to the weight of the water above.

As you know force is mass times acceleration due to gravity.

Mass is density times volume

Which we know because density is mass divided by volume.

Image source -
<http://www.xlerplate.com.au/go/case-study/wineglass-tank-a-towering-achievement>

Slide 8

University of South Australia

Consider the water column

- Cross-sectional area = A
- Height of water column = h

- **Volume $V = Ah$**
- $m = \rho V = \rho Ah$
- $F = mg = \rho Ahg$

- **Pressure = Force / Area**
- $P_{AV} = F/A = \rho gh$
- **Pressure does not depend on cross-sectional area, it depends on height of water.**

Alright, so let's say we're interested in the pressure at the bottom of a column of water.

The volume of the column is the cross-sectional area times its height

We know mass is density times volume

And force is mass times gravity, so the force from the weight of that water is $\rho A h g$

But remember that pressure is force divided by area

So it turns out that the average pressure at the bottom of the column is just $\rho g h$

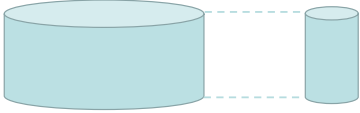
The take-home message here is that pressure only depends on the height of water above the point of interest.

Slide 9

University of South Australia

Pressure & depth

- Pressure increases linearly with depth, i.e. zero pressure at the top and maximum at the bottom of the tank.
- Does not matter what the **volume** of water is above – only the **height** (and density).



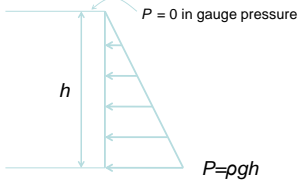
From the previous equation for pressure we can see that pressure increases linearly with depth, from zero pressure at the top to a maximum value at the bottom of a water body. Pressure does not depend on what the volume of water is above, it depends on height of water column. So if we've got a huge wide tank and a skinny little tank with the same height of water inside, they're going to have exactly the same pressure distribution. This shouldn't really come as any surprise to anyone who's been swimming – the pressure you feel when you're diving, say, two metres underwater, is the same whether you're in a little pool or a great big lake.

Slide 10

University of South Australia

Gauge Pressure

- **Gauge pressure** is the pressure measured relative to atmospheric



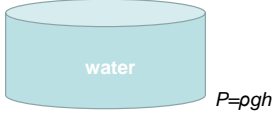
Okay, now a little bit of terminology. "Gauge pressure" is the pressure measured relative to atmosphere. According to gauge pressure, at the top of a body of water, the pressure is zero and it rises to pgh at the bottom.

Slide 11

University of South Australia

What about the weight of air?

- Gauge pressure neglects the weight of air



Gauge pressure neglects the weight of air. If we include the weight of the air sitting top of the water, that's known as "absolute pressure".

Slide 12

University of South Australia

Absolute pressure

- $P_{ATM} = g \cdot (\rho h)_{air} \sim 101,000 \text{ Pa}$

P_{ATM}
 $+ \rho g h$
 $= P_{abs}$

Absolute pressure is the sum of atmospheric pressure

Or the weight of the air

and the hydrostatic pressure from the weight of the water.

Slide 13

University of South Australia

Absolute Pressure

- **Absolute pressure** is the total pressure measured *including* atmospheric pressure

$P_{abs} = \rho g h + P_{ATM}$

The absolute pressure distribution in a water body is similar to gauge pressure, it's just the addition of constant atmospheric pressure (about 101 KPa) all the way down.

Slide 14

University of South Australia

Why we need Hydrostatic Force

Figure 1.5 Typical examples of situations where the hydrostatic force may have to be calculated

- $P_{AV} = F / A \dots$
- \dots so $F = P_{AV} A$

Okay, we've been talking a lot about pressure. But we also need to know how to convert this back into a force if we want to design hydraulic structures like dams or gates.

Since the average pressure is force divided by area,

we get force on a given surface by multiplying average pressure by the area of that surface.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 15

University of South Australia

Direction of Hydrostatic Force

- Perpendicular to all surfaces in contact with liquid

The diagram illustrates two scenarios. In the first, a vertical rectangular surface is shown with a horizontal arrow pointing to the left, perpendicular to the surface. In the second, a horizontal rectangular surface is shown with a vertical arrow pointing downwards, perpendicular to the surface. Right-angle symbols are used to indicate the perpendicularity of the force vectors to the surfaces.

It's important to note that the hydrostatic force acts exactly perpendicular to the surface, irrespective of the orientation of the surface.

Slide 16

University of South Australia

What is P_{AV} on a vertical surface?

- See Appendix 1 for full derivation
- For now, it is the pressure P acting at the centroid, G

The diagram shows two shapes. On the left is a rectangle with a height D and a centroid G located at a distance $G = D/2$ from the bottom. On the right is a triangle with a height D and a centroid G located at a distance $G = D/3$ from the base. A vertical double-headed arrow labeled D indicates the height of both shapes.

See Table 1.1

Before, we said force equals average pressure times surface area. The average pressure is the pressure at the centroid of the surface shape. For a rectangular shape like a rectangular gate or maybe a simple dam wall, the centroid is located at the middle. For other shapes, the centroid might not be in the middle and in the case of the triangle shown here, the centroid's located a third of the way up from base. Table 1.1 in the text book gives a few different equations of centroids for different shapes. Once you've got the centroid, that's the depth you use to calculate average pressure.

Slide 17

University of South Australia

Force on a vertical surface

The diagram shows a cross-section of a sewer pipe with a vertical gate at the end. The gate is hinged at the top. The water level in the sewer is indicated by a dashed line. The gate is labeled 'Gate' and the hinge is labeled 'Hinge'. The distance from the hinge to the centroid of the gate is labeled G . The diagram is labeled (a).

Figure 1.10 A vertical gate at the end of a sewer which discharges to a river. The gate hangs from a hinge at the top: (a) side view, (b) front view, (c) pressure intensity diagram. Note that only the part of the pressure intensity diagram at the same depth as the gate contributes to the hydrostatic force acting on it.

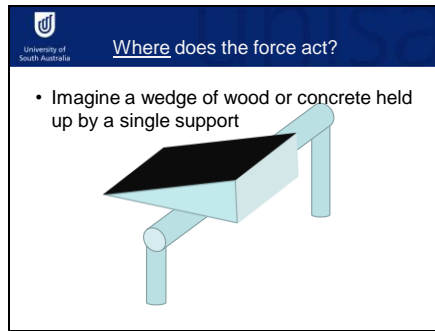
Figure 1.10 shows a vertical gate at the end of a sewer which discharges to a river. Hopefully this is discharging properly treated water so it doesn't pollute the river! Anyway, this is the sort of situation where you'd need to know the force acting on the gate to make sure you can actually open it properly.

hG is the depth to the centroid, in this case it's a rectangular gate so that'd be halfway up the rectangle.

At the top of the gate, the pressure is ρgh_1 and at the bottom, the pressure has gone up to ρgh_2 . The average pressure, which we'd use to calculate the force on the gate, is ρghG

Image source- Les Hamill 2011, Understanding hydraulics.

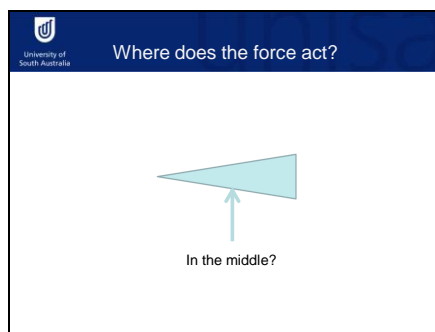
Slide 18



If you remember your basic mechanics, you'll know that it's not enough to just have the magnitude of the force – you also need to know where it acts so you can calculate moments and reaction forces.

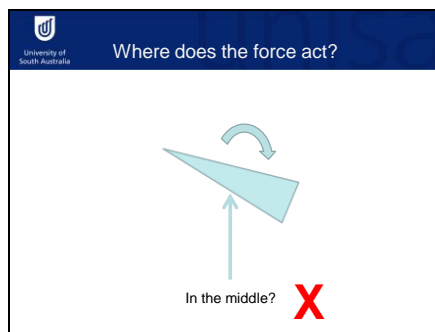
Imagine a wedge of wood or concrete held up by a single support.

Slide 19

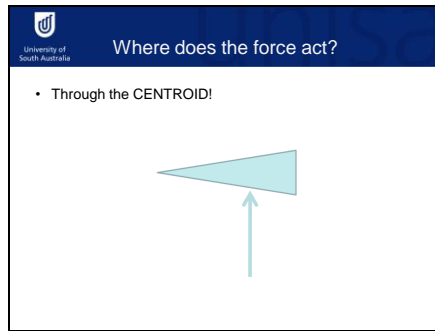


Hopefully you know it couldn't balance if the support was exactly in the middle.

Slide 20

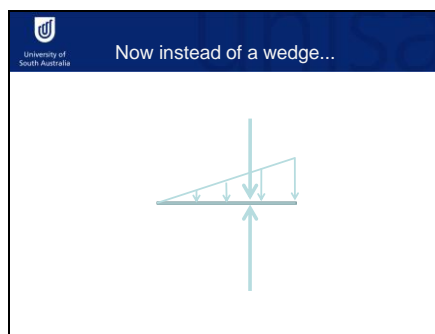


Slide 21



The force acts through the centroid, so to balance the wedge with a single support it'd need to be through the centroid too.

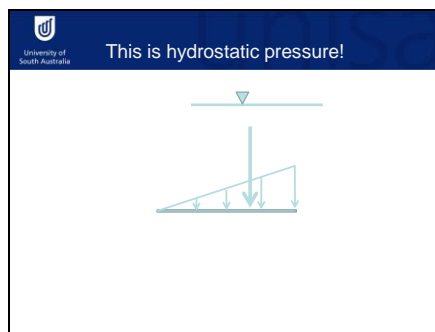
Slide 22



So now imagine a linearly-increasing (distributed) load supported by a single force acting at the centroid of the load distribution. This is just a sort of schematic version of the wedge.

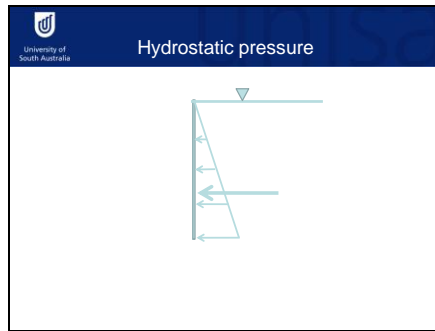
This distributed load could be represented by an equivalent point load acting through the centroid.

Slide 23



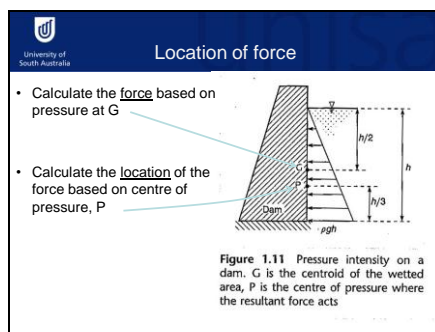
There's nothing really different between this example and what we're considering with hydrostatic forces

Slide 24



The distributed pressure is effectively a linearly increasing load, which is equivalent to single force acting through centroid of the pressure distribution.

Slide 25



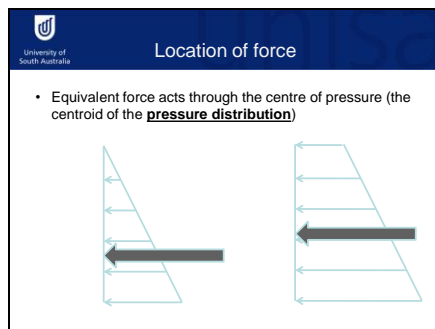
It can get a bit confusing because we talking about two different centroids.

We determine the magnitude of the force based on the pressure calculated at the centroid of the physical surface, which we refer to as G

and we calculate the location of the force based on the centre of the pressure distribution, which we refer to as P. This typically sits a bit below G but it depends on the shape of the submerged surface.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 26

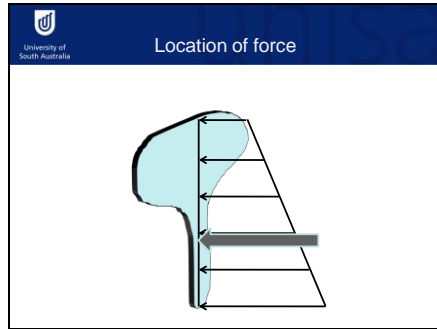


So our equivalent force acts through the centre of pressure (the centroid of the pressure distribution). Two different examples of pressure distributions are shown here.

This has zero pressure at the top, which would be typical for a submerged plane extending all the way up to the water surface, like a dam wall.

This one has nonzero pressure at the top, which would be expected if the entire submerged surface is located some depth underwater, like the sewer discharge gate we looked at earlier.

Slide 27

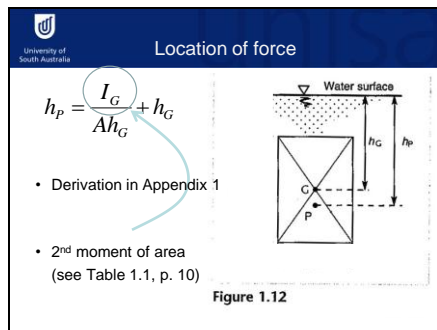


Actually, the location of the force depends on both the pressure distribution and the shape of the submerged object. Here's a totally arbitrary shape with a pressure distribution.

We can tell intuitively that the force isn't going to act here

But it's going to act somewhere up here instead, because that's where the majority of the surface is in contact with the water. We just need a general way of working this out that brings the shape of the object together with the pressure distribution.

Slide 28

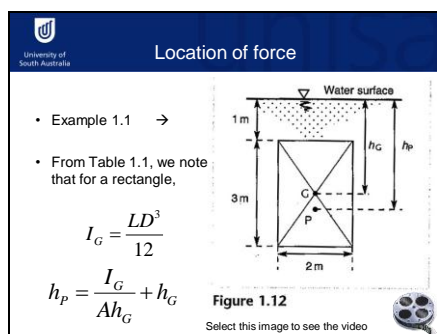


Here's the general equation we use to calculate the depth h_P where the hydrostatic force will act on a submerged flat surface of any shape. You can find the derivation at the back of the textbook if you want to see where it comes from.

The way this equation accounts for the shape of the object and its depth underwater is by considering three things: the depth to the centroid h_G , the area of the shape A , and the 2nd moment of area I_G .

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 29



Alright, here's a calculation example we can do. We've been asked to determine the location h_P and magnitude of the hydrostatic force on this rectangular surface. We're given the relevant dimensions to calculate the centroid of the rectangle h_G .

And here's the equation for I_G for a rectangle.

See how you go. If you get stuck you can follow the worked example in the text book.

Pause this presentation to see a video of this worked example

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 30

University of South Australia

Force on an inclined surface

Figure 1.16 (a) Force on an inclined surface. (b) When the surface is inclined always use the dimensions L_c and L_p with equation (1.13) (never the vertical dimensions h_c and h_p with equation (1.12))

$$L_p = \frac{I_G}{AL_G} + L_G$$

So far we've been thinking about vertical surfaces. For an inclined surface, we use slightly modified formula. We replace L instead of h , and then we just do all the same calculations as we did before, but do them in the sloping plane of the surface as shown on the right here.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 31

University of South Australia

Force on an inclined surface

- Example 1.3

Figure 1.17 An inclined, circular gate at the end of a sewer

$$L_p = \frac{I_G}{AL_G} + L_G$$

Okay, let's look at a slightly more complicated example.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 32

University of South Australia

Force on an inclined surface

• Example 1.3

Figure 1.17 An inclined

$$I_G = \frac{\pi r^4}{4}$$

This time we've got an inclined, circular gate.

Actually it's not really that complicated at all – so long as you use the correct IG value for a circle.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 33

University of South Australia

Force on an inclined surface

• Example 1.3

Figure 1.17 An inclined, circular gate at the end of a sewer

$$L_p = \frac{I_G}{AL_G} + L_G$$

$$I_G = \frac{\pi r^4}{4}$$

Don't forget to do all your calculations in the plane of the gate, which means first you need to calculate the length LG.

See how you go working this out and you can always check the text book for the worked solution.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 34

University of South Australia

Force on a curved surface

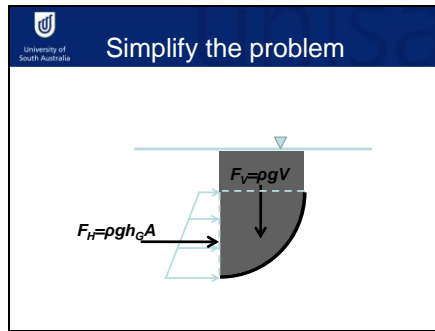
Yuck!

Okay, so now we've mastered hydrostatic forces on flat surfaces it's time to look at curved surfaces. This could be a dam wall, or a drum gate.

Of course the linear distribution of pressure with depth is still correct, but remember that hydrostatic force always acts perpendicular to the surface.

This means whatever the resultant force is on the curved surface it's going to involve the summing up of many contributing forces, all with different magnitude and direction.

Slide 35



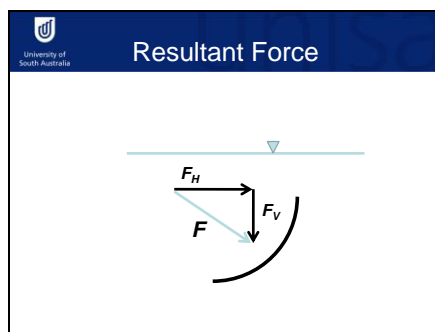
It's actually not as difficult as it looks.

What we have to do is consider a vertical column of water on top of the curved surface.

We can conceptualise this column of water as having a horizontal force acting on it, just like the force on a vertical surface

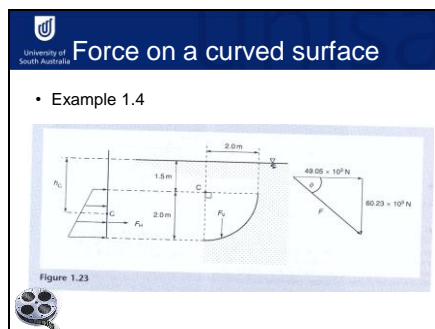
Then it also has the weight of all the water in the column generating a vertical force

Slide 36



The resultant force on the curved surface is the vector sum of these two forces.

Slide 37

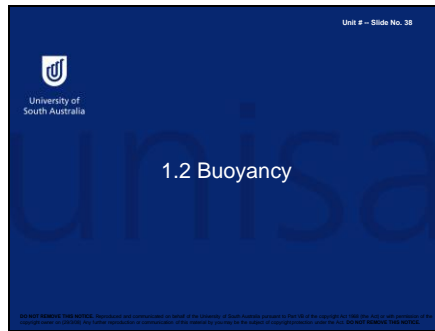


Here's another example from the textbook.

See if you can calculate the correct horizontal and vertical components of the force on the curved surface.

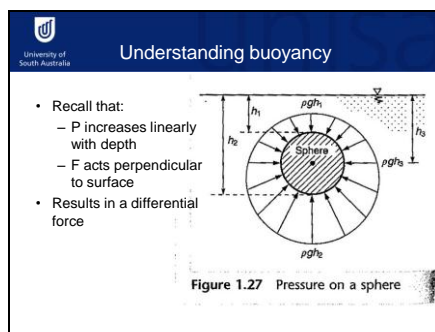
Image source- Les Hamill 2011, Understanding hydraulics.

Slide 38



Now let's look at buoyancy. This is actually just a very brief extension of the principles of hydrostatics and force we've just been learning about.

Slide 39



By now you know that

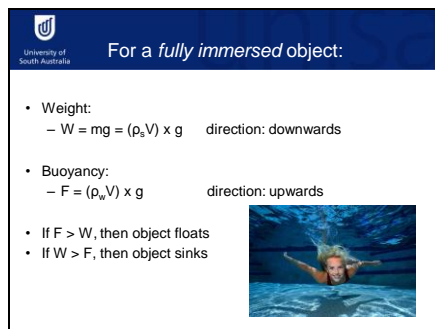
pressure increases linearly with depth,

and force acts perpendicular to all surfaces. In Figure 1.27, we see a sphere submerged underwater and hydrostatic forces are plotted all round its surface. See how at the bottom of the sphere the forces are bigger than at the top?

This is creating a differential net force pointing upward.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 40



We know weight is mass times gravity, and it acts downwards.

Written another way, weight is the density of the object, times its volume, times gravity.

The buoyancy force equals the density of the fluid times the displaced volume of the object times gravity. So the only difference between buoyancy and weight is the density term.

Buoyancy acts upwards. So if the buoyancy force is greater than the weight of an object, it will float upward through the water.

If the weight's greater than the buoyancy force, it'll sink.

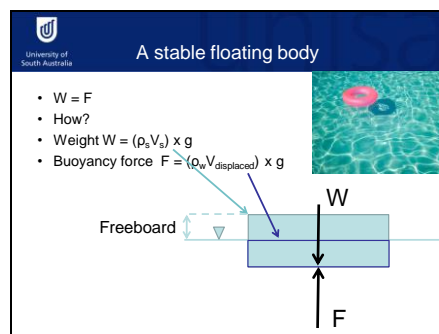
Image source - cilipart

Slide 41

University of South Australia

A stable floating body

- $W = F$
- How?
- Weight $W = (\rho_o V_o) \times g$
- Buoyancy force $F = (\rho_w V_{\text{displaced}}) \times g$



If the weight of a floating body equals the buoyancy force, it'll be stable. We see things floating in water every day. How do the forces balance?

Well, let's assume the weight of the object can't change. Then it's got to be the buoyancy force adjusting itself so that it equals the weight.

The key thing here is that buoyancy force depends on the displaced volume, which means the volume underwater. This means a floating object will naturally rise up out of the water until the submerged volume, that's the bit left underwater, is displacing a weight of water equal to the body's own weight.

The height that the object pokes up out of the water is called the freeboard.

Image source: Clip art

Slide 42

University of South Australia

Calculating Freeboard

Example 1.7

Mass: 50 tonnes
Length: 10 m
Fluid: seawater ($\rho_w = 1025 \text{ kg/m}^3$)
 $F = (\rho_w V_{\text{displaced}}) \times g$

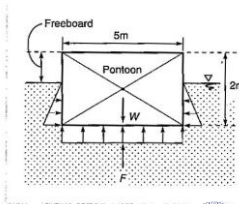


Figure 1.33

Here's a buoyancy example from the text book. We've got a pontoon 10 by 5 metres and 2 metres high, with a mass of 50 tonnes. It's floating stably in seawater so we know the buoyancy force has got to equal the weight. Note that the density of seawater is slightly higher than freshwater, which is usually about 1000 kg/m³. We need to work out how much water this pontoon is going to displace, and then use that to determine the depth of immersion and the freeboard.

Give it a go and check the video or textbook for the worked solution.


Image source- Les Hamill 2011, Understanding hydraulics.

Slide 43

University of South Australia

Summary

- Hydraulics measurements
- Water Pressure
- Hydraulic forces
- Buoyancy



So in summary, we've looked at the fundamental quantities involved in hydraulics, the variation of pressure with depth, calculation of the magnitude and location of hydraulic forces, and we've looked at buoyancy forces.

Slide 44

University of South Australia

Thank you

If you have any questions or desire further clarification please post a question or comment on the Discussion Forum.