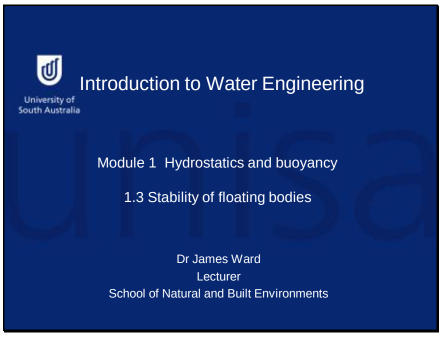
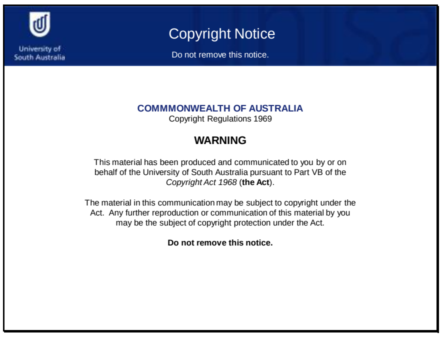
Slide 1



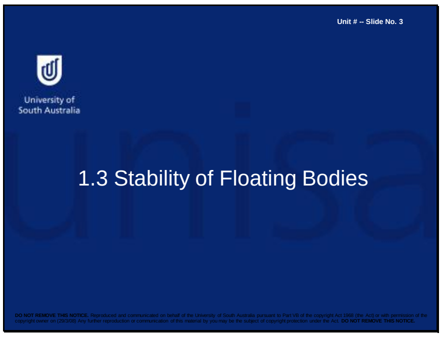
Welcome to Module 1 Hydrostatics and buoyancy

Slide 2



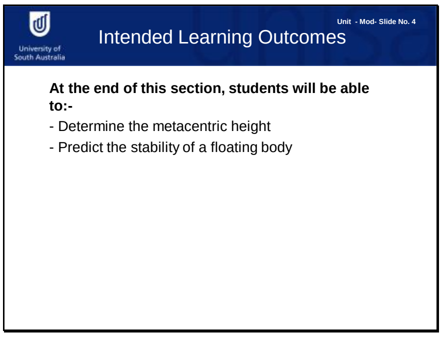
Please note

Slide 3



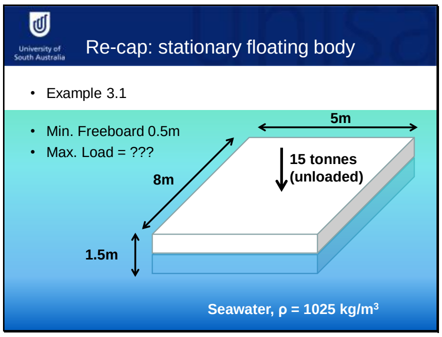
This time we'll look at stability of floating bodies.

Slide 4



The intended learning outcomes from this presentation are to be able to understand the stability of a floating body using metacentric height.

Slide 5

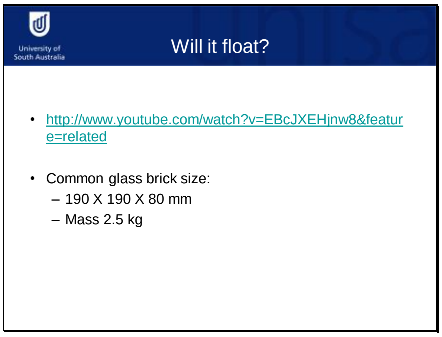


Before we look at stability, let's do a quick recap of buoyancy with an example. This is a pontoon weighing 15 tonnes with the dimensions as shown.

We've been told it needs to maintain a freeboard of at least half a metre, which implies a submerged depth of a metre.

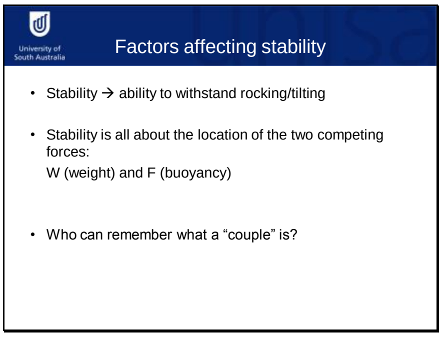
We want to know how much extra load it can support. The workout procedure's in the text book if you need it.

Slide 6



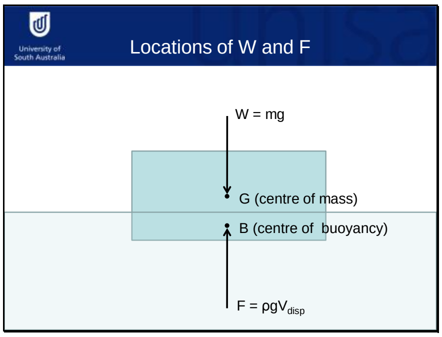
The YouTube video shows a funny debate about whether or not a glass brick will float. I know it's not a really serious example, but if we wanted to we could work out the density of a glass brick to predict whether it would float. Using density equals mass divided by volume you can calculate whether the density's more or less than 1000 kg per cubic metre.

Slide 7



If a body's floating, the buoyancy force is equal to the weight force. Stability is about the location of these two forces. A "couple" is two forces that are equal in magnitude, but separated by a distance.

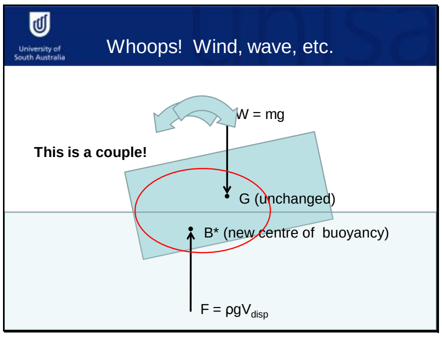
Slide 8



The weight force always acts through the body's centre of mass.

On the other hand, the buoyancy force acts through the centre of buoyancy, which is the centre of the displaced volume.

Slide 9



When a floating body is tilted in the water, like what happens when a wave comes past,

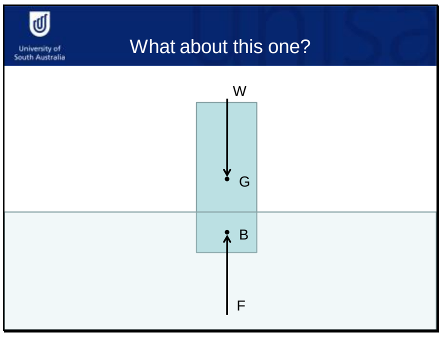
the centre of mass is unchanged.

However, the position of centre of buoyancy will be changed because the shape of the displaced volume has changed.

So now there are two opposite directional forces, equal in magnitude, but separated by a distance. This is a "couple"

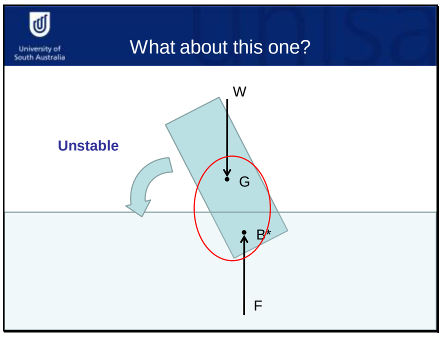
And it'll cause a moment that naturally corrects the tilt of the floating body. This continuously self-correcting moment is what causes floating objects to rock back and forth but not turn over completely. We call these stably floating bodies.

Slide 10



Now let's look at a different floating body that has a much taller geometry.

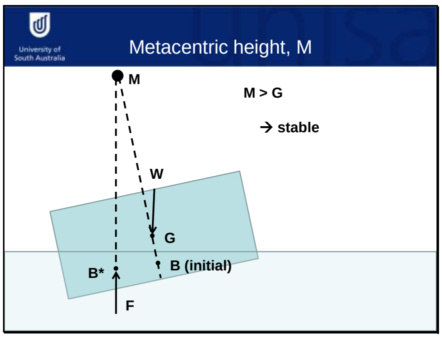
Slide 11



This time, if it tilts over even a little bit,

The couple formed by the weight and buoyancy forces will exacerbate the tilting and it'll fall over into the water. It's pretty intuitive for a simple shape like this, but we need to develop a more general way of describing the stability of a floating body if we're going to consider more complicated systems.

Slide 12



The general way to describe stability is the metacentric height, M. Metacentric height's a hypothetical point above the object. Geometrically, we define the metacentric height

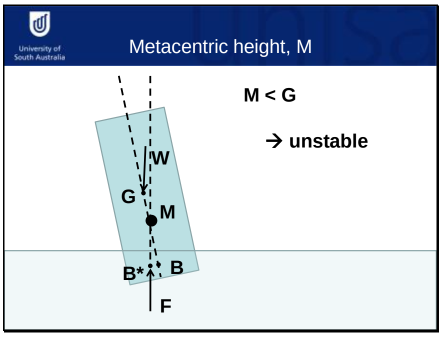
by drawing an imaginary vertical line through the new centre of buoyancy formed when an object is tilted,

And joining it to a line connecting the centre of mass and the initial centre of buoyancy

The intersection of these lines is the metacentric height, M.

For a stable object, the position of metacentric height is above the centre of mass.

Slide 13



If we repeat the procedure for an unstable object

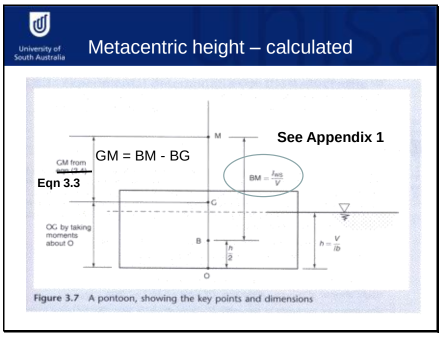
By drawing a vertical line up from the new centre of buoyancy

Connecting a line through B and G

And finding the intersection point,

We can see that in the unstable case, the position of the metacentric height is below the centre of mass.

Slide 14



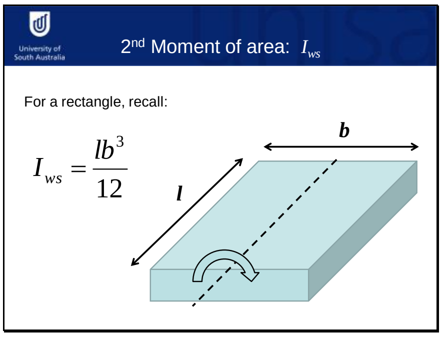
Those examples involved tilting the body and somehow finding the new centre of buoyancy in the tilted case. What we need is a general mathematical way to predict whether a body is stable or not, without having to tilt it over. So what we really want is a formula to calculate metacentric height from the initial centre of buoyancy and the centre of mass.

The textbook gives us an equation – please note the figure from the book shown here refers to the wrong equation number, which should be 3.3. What we've got is the distance GM, which is the distance from the centre of mass to the metacentre, given as the distance BM minus BG. So as long as we know enough about the mass distribution to calculate G and enough about the object's volume and shape to calculate B, all we then need is BM, the distance from the centre of buoyancy to the metacentre.

We're given this simple equation for BM, which relates to the submerged volume V and the second moment of area of the shape. You can chase up the derivation in Appendix 1 of the textbook if you want to know where it comes from.

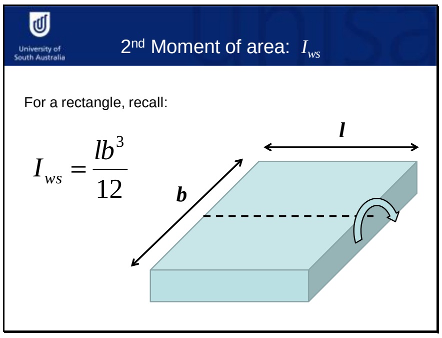
Image source- Les Hamill 2011, Understanding hydraulics.

Slide 15



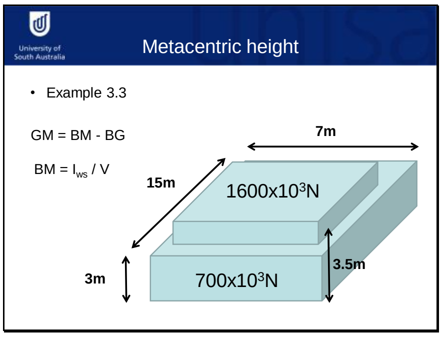
The BM term in the previous slide was found by dividing the second moment of area Iws by the submerged volume. Here, the subscript “ws” refers to the shape in the plane of the water surface, so imagine taking a horizontal slice through the object, perfectly level with the surface of the water. The picture here shows a rectangular object, like a pontoon. You can look up the second moment of area of a rectangle or you might remember it’s L B cubed over 12.

Slide 16



Obviously, depending on which way you orient L and B, you’re going to get two different values of Iws. In practical terms, these are the two different axes of stability – you can imagine the pontoon rocking back and forth, as well as side to side, and it’ll rock more in one direction than the other. Most shapes will have two axes of stability – imagine the hull of a typical ship, which has a long axis and short axis. From the point of view of calculating overall stability, you need to find the worst-case scenario which means you need to use the smallest value of BM. This means use the smallest value of Iws.

Slide 17



Let’s do an example. Here we’ve got a pontoon 15 by 7 metres and 3 metres deep, weighing 700 kilonewtons

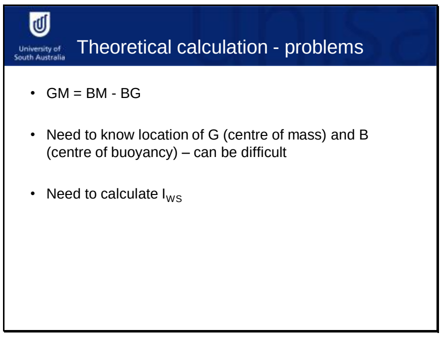
and it’s carrying a weight of 1600 kilonewtons. This is a bit of a tricky one because the you need to calculate the position of the centre of mass based on the weight distribution.

We’ve been told the centre of mass of the 1600 kilonewton load is 3.5 metres above the bottom of the pontoon. You can assume the centre of mass of the base is half way up, so 1.5 metres above the bottom. You might have to go back and look at how to calculate the centroid of a composite shape in order to calculate the overall centre of mass. Then you’re going to have to work out the depth of immersion to figure out the centre of buoyancy and the submerged volume.

Use the formula GM = BM – BG

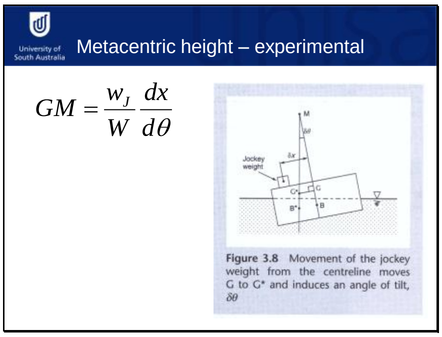
With BM equal to Iws over V to determine the metacentric height and figure out if the object's stable. Follow the example in the text book if you need to.

Slide 18



Alright, so that last example was actually pretty complicated, and it shows one of the weaknesses of the theoretical approach to calculating the metacentric height. Imagine how much harder it'd be with a container ship loaded up with lots of different equipment and cargo. The centre of mass, centre of buoyancy and second moment of area are all so much more complicated in the real world.

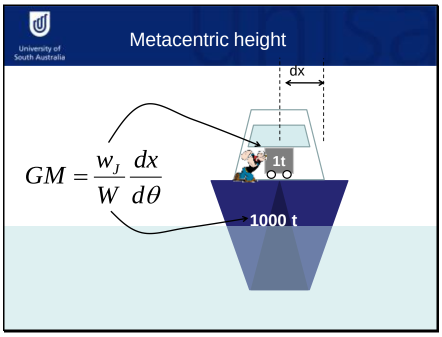
Slide 19



Luckily there's an experimental method to estimate the metacentric height, which is quite accurate and actually pretty simple to do. A jockey weight, which is a mass that's small relative to the overall mass, is placed some distance away from the centre of the object. This induces a small tilting angle, which is technically called "list". So long as we can measure the distance from the centre line and the angle of list, and we know the overall mass of the object, we can determine the metacentric height using this simple equation.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 20



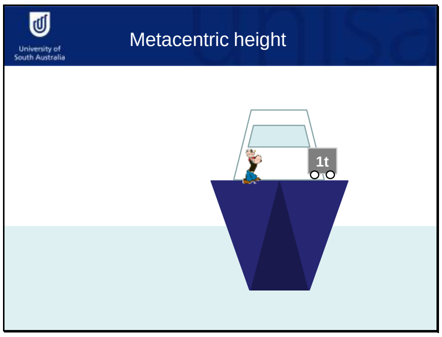
This animation shows the experimental method for calculating metacentric height.

Imagine this ship has a mass of 1000 tonnes

and we place a jockey weight of 1 tonne on board.

We then displace the jockey weight by a known distance dx

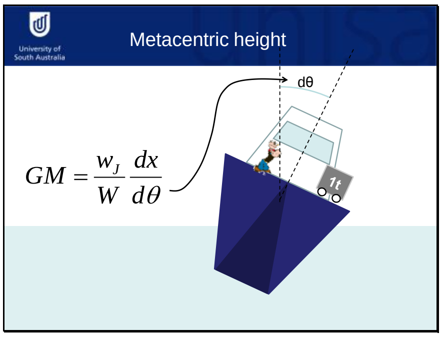
Slide 21



Now the displaced

jockey weight induces a list angle.

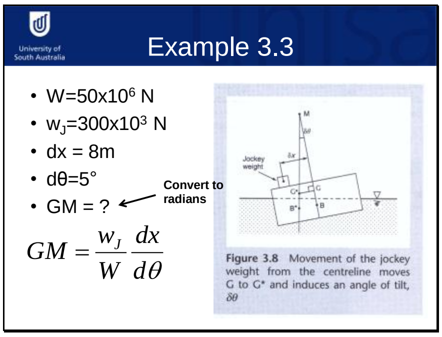
Slide 22



We just measure the angle of list

And the procedure is complete!

Slide 23

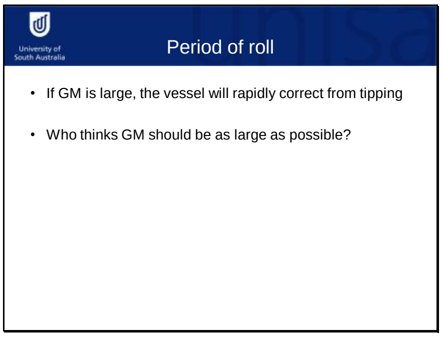


This is a really simple example – it's just a matter of plugging in the numbers.

Well, actually, the one slightly complicating factor is that you need to convert the angle from degrees to radians. Apart from that you shouldn't have any trouble with this method.

Image source- Les Hamill 2011, Understanding hydraulics.

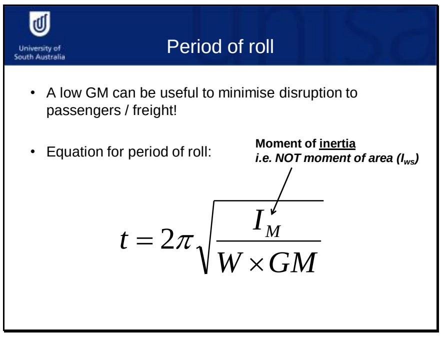
Slide 24



The larger the metacentric height, the more rapidly the vessel's going to correct from tipping because the couple created by the two forces will be larger.

So you might be inclined to think that it's good design to create a floating body that maximises the metacentric height – you could do this by playing around with the shape of the vessel, which would increase the Iws term, or perhaps you could change the internal weight distribution to lower the centre of mass. But it's important to understand that doing this might have an undesirable effect because the body rocks back and forward too quickly. We refer to the speed of rocking as the "period of roll".

Slide 25

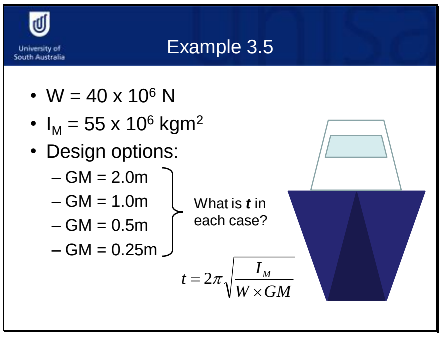


If your vessel is carrying freight or passengers, although you need it to be physically stable, you probably don't want it rocking back and forth too fast. The lower the value of metacentric height, the slower the rocking.

We've got a handy equation for calculating the period of roll here. It's just 2 pi times the square root of second moment of inertia divided by weight by metacentric height. If we want to make sure the rocking motion is slow, we need large values of t, so you can see that the smaller GM is, the larger t will be.

It's really important to note that the IM term here is the moment of inertia, which relates to the size and weight distribution of the vessel, and this is totally different to the Iws term we used earlier.

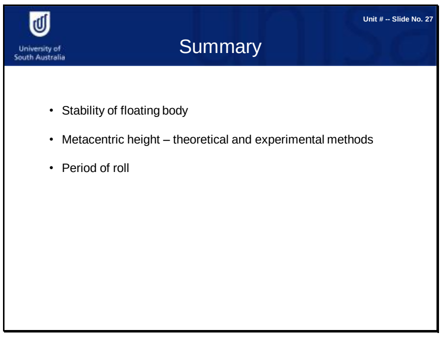
Slide 26



Imagine you've got the ability to design a vessel in different ways that'll result in metacentric heights varying from as high as 2 metres to as low as 25 centimetres. Okay, I don't know how realistic this example is because I suspect the IM term would probably change in each case. But for the sake of practice let's assume it's as it says here, and IM stays constant. You can use the simple formula for period of roll to work out t for each of the different design options and see what difference GM makes to the rocking.

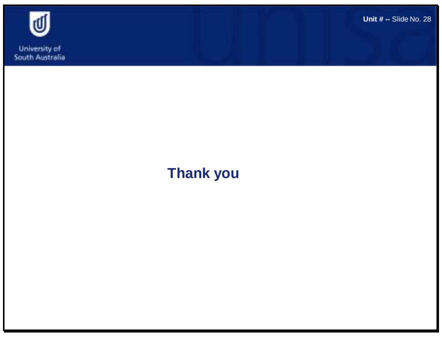
The workout procedure of Example 3.5 is available in the text book.

Slide 27



So, in summary, we have looked at the stability of floating bodies by calculating metacentric height theoretically and experimentally, and we briefly touched on the implications of metacentric height for the speed of an object's rocking motion, using the period of roll calculation.

Slide 28



If you have any questions or need clarification, please post a question or comment on the Discussion Forum.