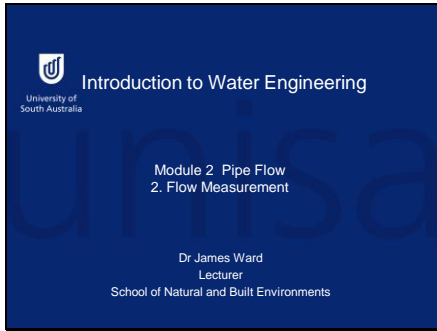


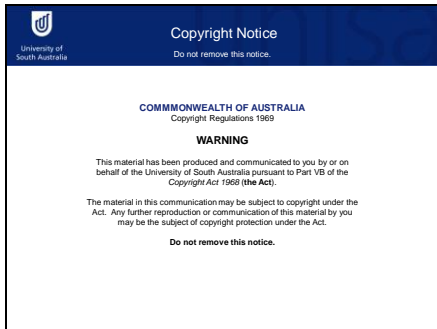
Introduction to Water Engineering

Slide 1



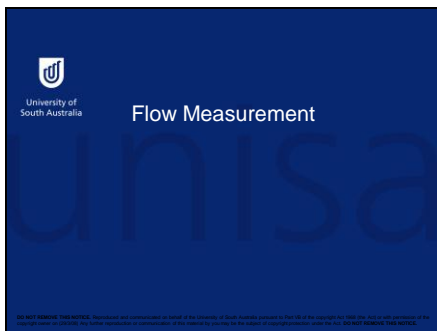
Welcome to Module 2 Pipe flow

Slide 2



Please note

Slide 3



Let's start by looking at flow measurement.

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Intended Learning Outcomes

At the end of this section, students will be able to:-

- Understand how the energy equation can be used to measure flow from pressure differences
- Calculate flow from measurements using devices including venturi meter, pitot tube, orifice and sharp crested weir.

The learning outcomes are presented here – we'll look at how the flow or discharge of a stream of liquid in a pipe or open channel can be calculated with the energy equation and an observed pressure difference using a variety of devices including the venturi meter, pitot tube, orifice and sharp crested weir.

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Energy Equation re-cap

$$z_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = z_2 + \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + \text{energy head losses}$$

• 1 Elevation z_1
Velocity V_1
Pressure P_1

Elevation z_2 • 2
Velocity V_2
Pressure P_2

Just re-capping from the last presentation, we introduced the energy equation. Remember that the sum of elevation head, Z, velocity head (V squared on $2g$) and pressure head (P on ρg) at point 1 is equal to sum of those at point 2

plus we also mentioned that energy head losses need to be considered because there's friction between the fluid and the pipe as it moves from point 1 to point 2.

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Measuring flow $\rightarrow Q = VA$

- Assuming we know pipe size, we know area A
- What we **really** need is velocity, V

$$z_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = z_2 + \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + \text{energy head losses}$$

We should know the elevations (probably horizontal)

A • 1 Elevation z_1
Velocity V_1
Pressure P_1

Elevation z_2 • 2
Velocity V_2
Pressure P_2

Generally we should be able to expect

that we'll know the cross sectional area of a pipe A

Then what we really need, to measure flow rate, is actually the velocity V because $Q = VA$ from the continuity equation.

Based on the geometry of the system we should also be able to assume we know the elevations at two different locations, Z1

and Z2 and if it happens that the pipe's horizontal, then

The Z terms cancel out.

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Inferring V from P

- Start by assuming **energy head losses** are negligible (don't worry, they come back!)

$$\cancel{\frac{V_1^2}{2g}} + \frac{P_1}{\rho g} = \cancel{\frac{V_2^2}{2g}} + \frac{P_2}{\rho g} + \text{energy head losses}$$

Just while we're getting familiar with the concept, we can start by assuming that our pipe's horizontal, and that energy head losses

are negligible in the energy equation. So the remaining components in the energy equation suggest an important relationship between velocity and pressure. This forms the basis of using pressure measurements to work out the velocity in a pipe.

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Inferring V from P

- In this system $P_1 = P_2$ and $V_1 = V_2$, so we can't solve anything!

$$\cancel{\frac{V_1^2}{2g}} + \frac{P_1}{\rho g} = \cancel{\frac{V_2^2}{2g}} + \frac{P_2}{\rho g} \quad \begin{matrix} ?? \\ ? \end{matrix}$$

For a uniform pipe,

the pressures at point 1 and point 2 are equal, and the velocities at point 1 and point 2 are equal. So in that case, we can't solve anything. It wouldn't do any good to measure the two pressures here because we know they're going to be equal.

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Aha!

- What about this?

But check this out.

What if we induce a change in velocity by forcing the flow into a smaller pipe. Now the velocity V_2 is going to be higher than V_1 .

So now it makes sense to take a reading of upstream pressure

And downstream pressure. We should expect P_2 to be lower than P_1 , because the energy equation has to balance. What's happening is point 1 has lower velocity head but higher pressure head, whereas point 2 has higher velocity head and lower pressure head.

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Inferring V from P

$$\frac{V_1^2}{2g} + \frac{P_1}{\rho g} = \frac{V_2^2}{2g} + \frac{P_2}{\rho g}$$

Large P_1
Small V_1

Small P_2
Large V_2

Velocity V_1
Pressure P_1

Velocity V_2
Pressure P_2

So we use the fact

that there's relatively large pressure and small velocity upstream

And the opposite situation downstream with high velocity and low pressure.

Then we use the principle of total energy

and the fact that the upstream energy equals the downstream energy to relate the velocities to a measurable pressure differential.

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Inferring V from P

$$\frac{V_1^2}{2g} + \frac{P_1}{\rho g} = \frac{V_2^2}{2g} + \frac{P_2}{\rho g}$$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$$

Velocity V_1
Pressure P_1

Velocity V_2
Pressure P_2

In order to infer velocity V from the pressure differential, we need to rearrange the energy equation.

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A couple of extra tricks...

- Trick 1:
Define H as the measured **head difference**
$$H = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$$
- Trick 2:
Use the continuity equation and the known cross sectional areas A_1 and A_2
$$Q = A_1 V_1 = A_2 V_2 \implies V_1 = \frac{A_2 V_2}{A_1} \text{ or } V_2 = \frac{A_1 V_1}{A_2}$$

Here's a neat trick –

define head difference H as the measured pressure difference, so we're not interested in the absolute values P1 and P2 anymore. This simplifies two of the variables in the equation to a single variable, which is easy to measure in practice. Also, it's easy to measure this in metres of water.

But the most important trick is to use continuity equation to relate the two velocities to one another based on the two different cross-sectional areas of the pipes.

Rearranging the continuity equation, the upstream velocity is equal to downstream velocity multiplied by the ratio of the two cross-sectional areas,

or vice versa. Now as long as we know the two pipe cross-sectional areas, which should be straightforward, then the two unknowns, V1 and V2, can be simplified to a single unknown.

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Inferring V from P

$$V_2 = \frac{A_1 V_1}{A_2} \text{ and } H = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$$

• 1 Velocity V_1 Pressure P_1

• 2 Velocity V_2 Pressure P_2

Putting these tricks to work,

we can substitute for the downstream velocity V_2 and the pressure head difference, or H .

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Inferring V from P

$$\frac{\left(\frac{A_1 V_1}{A_2}\right)^2}{2g} - \frac{V_1^2}{2g} = H$$

• 1 Velocity V_1 Pressure P_1

• 2 Velocity V_2 Pressure P_2

So we can simplify the 4 unknowns in the energy equation to a single unknown, V_1 , as a function of the pipe sizes and a measured head difference H .

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Derivation on page 128

$$\frac{\left(\frac{A_1 V_1}{A_2}\right)^2}{2g} - \frac{V_1^2}{2g} = H$$

$$\Rightarrow \frac{V_1^2}{2g} \left[\left(\frac{A_1}{A_2}\right)^2 - 1 \right] = H$$

$$\Rightarrow V_1^2 \left[\left(\frac{A_1}{A_2}\right)^2 - 1 \right] = 2gH$$

$$\Rightarrow V_1^2 = \frac{2gH}{\left(\frac{A_1}{A_2}\right)^2 - 1}$$

$$\Rightarrow V_1 = \sqrt{\frac{2gH}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

So now we just need to do a bit of fancy footwork to rearrange this into an expression for velocity

Start by taking out a factor of V_1 squared on $2g$

Put $2g$ on the other side

Get v_1 squared by itself

And finally take the square root. So as long we can measure the head difference H , and we know the two pipe areas A_1 and A_2 , we can quite easily use a sudden pipe contraction to measure the water velocity.

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Flow measuring devices

- Several devices utilise the change in cross sectional area in this manner
 - Venturi meter ($C_D = 0.97$ approx.)
 - Orifice plate (small orifice) ($C_D = 0.59$ to 0.66)

$$V_1 = \sqrt{\frac{2gH}{\left(\frac{A_1}{A_2}\right)^2 - 1}} \Rightarrow Q = A_1 V_1 = A_2 \sqrt{\frac{2gH}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

Correcting for **energy head loss** $Q = C_D A_2 \sqrt{\frac{2gH}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$
loss: $C_D =$ coefficient of discharge

Several devices including

venturi meter

and orifice plate utilise this process, where they induce a change in cross sectional area to measure the flow rate.

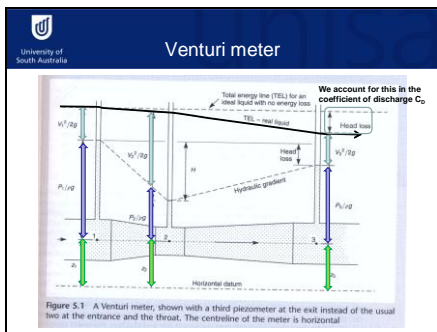
To find flow rate we just use $Q = VA$

Now, remember how we started out by ignoring head losses? Well, obviously that assumption could throw off the accuracy of our measurement so we have to correct for the loss of energy as water gets squeezed through the device.

We just use a coefficient of discharge C_D for correcting the energy head. If C_D is equal to 1, it means there's no head loss. The values of C_D are found by experiments, and for a Venturi meter, which involves a gentle contraction and expansion,

it's pretty close to 1, about 0.97. On the other hand an orifice plate involves squeezing the fluid through a hole and is pretty disruptive to the overall flow.

C_D values for the orifice meter are in the realm of 0.6 as a result.



Here's a somewhat complicated drawing that shows what's happening inside a Venturi meter. Fluid's flowing from left to right, and comes in with a particular

elevation head, pressure head and velocity head. The Venturi meter forces the water through a contraction.

Now assuming the system's horizontal, the elevation head hasn't changed, but we've induced a higher velocity so now the total head is made up of a higher velocity head and a lower pressure head. Notice that the total energy is a little bit lower than before, because already there's been some friction and as a result there's been a loss of energy. At the top of this picture, there's a total energy line for an ideal fluid with no energy losses.

So the fluid keeps trundling along the Venturi meter and the device is designed to slowly expand back out to release the water at its original diameter. When this happens, we've got the same elevation head and because it's returned to its original diameter we've got to have the same velocity head as we had at the start. But overall energy is lower due to energy head losses and this is accounted for in a lower pressure head than we had going into the Venturi meter.

If we track the total energy line and account for energy losses due to friction, it'll go down something like this. Different parts of the system might induce more loss and others might induce less, so the rate of energy loss or the slope of the line changes along the way. But it

always goes down, because we can't gain energy out of nowhere.

Like we said on the previous slide, we account for real-world energy losses over the whole meter by throwing in a coefficient of discharge that we get from experimental data.

Image source- Les Hamill 2011, Understanding hydraulics.

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Example 5.1

- $C_D = 0.97$

- $P_1/\rho g$ measured = 950mm
- $P_2/\rho g$ measured = 200mm

$$Q = C_D A_1 \sqrt{\frac{2gH}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

Alright, here's an example of a Venturi meter where we know the two diameters are 100 mm and 60 mm.

Let's assume a coefficient of discharge of 0.97, fairly typical for a Venturi meter

And we've been told the upstream pressure head has been measured as 950 mm of water. Expressing the measurement in this way might mean that they didn't measure the pressure in Pascals, but they actually measured the height of water using a piezometer or manometer. So the measurement given is for P_1 on ρg .

Likewise P_2 on ρg has been measured at 200 mm, so we can work out the head difference H from these two values.

Using the equation we derived before, you should be able to work out the flow rate in the pipe. The workout procedure's in the text book if you need it.

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Pitot tube

$$z_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = z_2 + \frac{V_2^2}{2g} + \frac{P_2}{\rho g}$$

$$\frac{V_1^2}{2g} = \frac{P_2}{\rho g} - \frac{P_1}{\rho g} = H$$

$$V_1^2 = 2gH$$

$$V_1 = \sqrt{2gH}$$

$$V_1 = C \sqrt{2gH}$$

Figure 5.3 (a) Separate static tube and Pitot tube in a pipe. (b) Combined Pitot-static tube. The outer holes measure the static pressure, while the inner tube measures the combined pressure. When connected to a suitable manometer this enables the differential head, H , to be measured without the need for a separate static tube as in (a)

A Pitot tube is a totally different physical method to measure the flow rate, but it also involves measuring two different pressures. Have a close look at the diagram here, and look at points 1 and 2.

Assuming it's been put in on the horizontal, the elevation head cancels. Now, Point 1's located in the pipe where fluid's flowing along, so it's going to have both a velocity head component and a pressure head component. It's important to realise that if we use some pressure-measuring device, be it a piezometer, manometer, or a Bourdon gauge, it only tells us the pressure head component at that point. The diagram here shows a piezometer with water up to a level corresponding to P_1 on ρg , which is the pressure head. Now have a look at Point 2. This is at the entrance to the Pitot tube itself, which is usually a nice, streamlined tube designed for minimum flow disruption, with a little hole in the end. Water enters the Pitot tube but once it's filled up, it becomes stationary. So in the tube, and all the way out to the entrance,

the velocity's zero. So what's actually happened now is that we've converted all of the velocity head that we had at point 1 into straight pressure head at point 2.

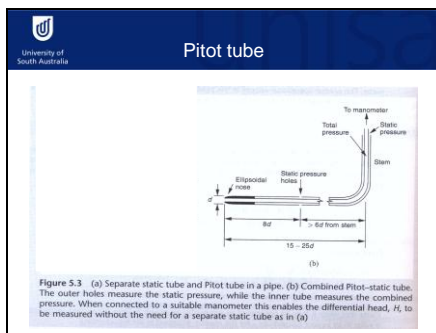
Rearranging we get the velocity head expressed in terms of the difference in observed pressure head, and again we can simplify this to a single reading, H.

Finishing it off we get V equal to root 2 g H, which is a very common relationship found between head and velocity.

To account for any disturbance caused by the device, we introduce a coefficient C, which is just like the coefficient of discharge we used before. Typical values for a Pitot tube are very close to 1, about 0.98 or 0.99.

Image source- Les Hamill 2011, Understanding hydraulics.

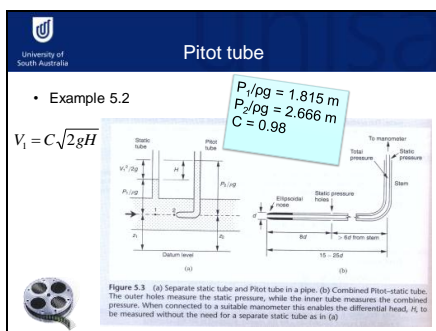
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In a clever Pitot tube, with a tube inside a tube, the outer holes measure the static pressure, which was P1 in the previous version, while the inner tube measures the combined pressure, P2. Apart from the physical layout this type of Pitot tube it's no different from the previous one we looked at.

Image source- Les Hamill 2011, Understanding hydraulics.

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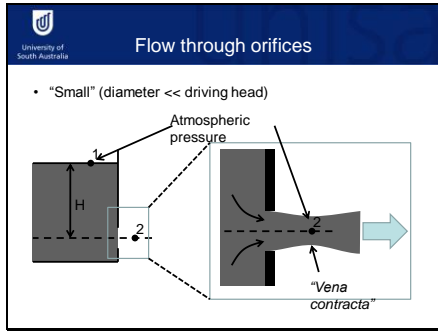


Here's a really easy example where the two measurements have been taken.

Just be careful with these sorts of problems because the wording can be a bit confusing. The P1 on ρg measurement is called the "static pressure head", which means it's the static component of the combined pressure and velocity heads, and the P2 on ρg measurement is called the "stagnation pressure head", which sounds similar but it's talking about the pressure developed inside the Pitot tube, where the flowing water has stagnated and become stationary.

You shouldn't need to consult the textbook to work this out, but check your answer if you get stuck. Image source- Les Hamill 2011, Understanding hydraulics.

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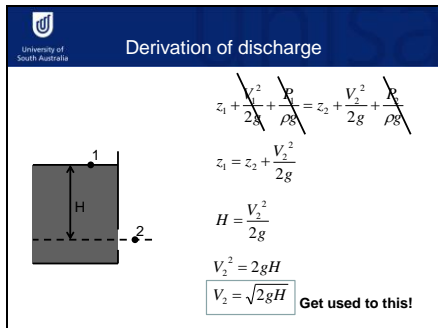


Orifice just means "hole". Let's consider water flowing out of a tank through a small orifice. By "small orifice", we just mean the diameter is much smaller than driving head.

Let's take a close up view of this orifice. Water rushes out through the hole and contracts slightly before expanding again. You might have seen water coming out of a tap doing the same thing. The contraction is called the "vena contracta".

Before we apply the energy equation, we'll select two points. Point 1 can be at the top of the water surface, where there's atmospheric pressure so we know that'll simplify our calculation slightly. We choose point 2 at the vena contracta and there's atmospheric pressure here too, so that's both our pressure terms eliminated from the energy equation.

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Okay so putting the energy equation together, our pressure terms are gone straight away,

And assuming the water in the tank is basically stationary, V_1 is close enough to zero to disregard.

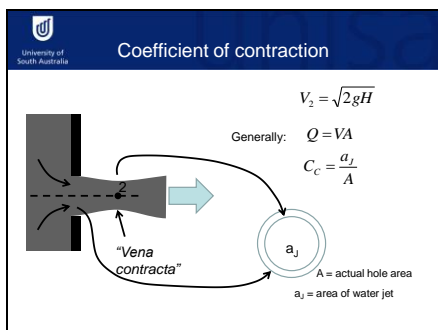
So we've got 3 terms in our equation

And if we let H be the driving head, which is the difference between Z_1 and Z_2 ,

Then this starts to look a bit familiar

We end up with the velocity being equal to root 2 g H, just the same as the Pitot tube, and as I said, this is a pretty common head-velocity relationship.

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If we want to convert that velocity into a flow rate, we need to know the cross sectional area

because $Q = VA$. This means we need to know how much the water contracts at the Vena contracta.

If we have an orifice of area A

And water contracts to a cross-sectional area a_j at the vena contracta

Then we can express the coefficient of contraction C_c as a_j on A.

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Coefficient of velocity

- Actual velocity through orifice < theoretical
 - Coefficient of velocity → C_v
 - Accounts for energy losses
- Overall **coefficient of discharge** $C_D = C_c \times C_v$
- $Q = C_D A V$**
- But C_v typically close to 1, e.g. 0.95-0.99
- So C_D is mostly influenced by C_c

What we find in practice is that

the actual velocity coming through an orifice is less than the theoretical value we get by the previous calculation.

A coefficient of velocity C_v is used

to account for energy losses.

Then, an overall coefficient of discharge is found by multiplying the coefficient of contraction by the coefficient of velocity.

So the discharge Q through an orifice is this coefficient of discharge C_D times area times velocity, which we found by the previous equation.

As it happens, the value of the coefficient of velocity is pretty close to 1, so the main factor influencing C_D is the coefficient of contraction.

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Types of orifice & C_D values

Figure 5.5 Types of orifice and their approximate C_D values

Like the coefficient of discharge for a Venturi meter or the coefficient of a Pitot tube, experimental work has found a range of different values of C_D for different types of orifice. The coefficient can be as low as 0.5 or for a nice smooth orifice it could be close to 1.

Image source- Les Hamill 2011, Understanding hydraulics.

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Example 5.3

- Orifice diameter: 50mm
- $C_D = 0.62$

$$Q = C_D A \sqrt{2gH}$$

- What is the discharge if head in tank is maintained at 2.5 m ?
- What is the **% reduction in discharge** if head is reduced by 50% to 1.25 m ?

Here's an example of a sharp orifice on the side of a tank, with a coefficient of discharge of 0.62. The hole's circular with diameter 50 millimetres.

The first question asks what the discharge would be if the tank level is 2.5 metres, and assume it's being continuously refilled so it stays constant.

The second question asks what the reduction in discharge would be for a 50% reduction in driving head. Do you think it would halve the discharge? Or more? Or maybe less. You'll have to crunch the numbers to find out.

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Jet trajectory

- We can use the observed distance travelled by a jet of water to determine the actual velocity, and hence the **coefficient of velocity, C_v** .
- Derivation** p. 135:

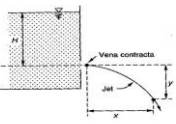


Figure 5.6 Trajectory of a jet leaving an orifice

$$v = x \sqrt{\frac{g}{2y}} \quad C_v = \frac{x}{2\sqrt{yH}}$$

If there's a small hole in the tank,

the stream of water flowing out will follow a fairly predictable jet trajectory, covering a distance X while falling through a height Y. Studying the jet trajectory can be used to help work out the coefficient of velocity, CV.

We won't go into the derivation here, but you can look it up in the textbook if you like.

The two equations we have are velocity equal to the distance X times root G on 2 Y, and

The coefficient of velocity CV equal to X on 2 root Y H.

Image source- Les Hamill 2011, Understanding hydraulics.

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Example 5.4

- Orifice D = 25 mm
- H = 1.42 m
- Jet horizontal distance x = 1.25 m
- Jet vertical distance y = 0.3 m
- Jet diameter at vena contracta = 20 mm

FIND C_c and C_v

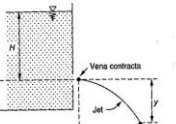


Figure 5.6 Trajectory of a jet leaving an orifice

$$C_c = \frac{a_j}{A} \quad C_v = \frac{x}{2\sqrt{yH}}$$

Here's a jet trajectory example.

Say we've got a 25 millimetre, or one inch, hole.

We've got a driving head of 1.42 metres

And we observe the jet shoots out a distance of 1.25 metres

while falling through a height of 30 centimetres.

Let's say we can observe the diameter of the vena contracta where the flow's reduced to 20mm

The first thing to work out is Cc, which is very straightforward since we've got the diameters and can easily convert these into areas. Next up we want to use the X and Y measurements to calculate the coefficient of velocity. Again, this shouldn't be too difficult for you. See how you go.

Image source- Les Hamill 2011, Understanding hydraulics.

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Submerged small orifice

$$z_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = z_2 + \frac{V_2^2}{2g} + \frac{P_2}{\rho g}$$

$$z_1 - z_2 = H_1$$

$$P_2 = \rho g H_2$$

$$H_1 = \frac{V_2^2}{2g} + H_2$$

$$\frac{V_2^2}{2g} = H_1 - H_2$$

$$V_2 = \sqrt{2g(H_1 - H_2)}$$

In a submerged orifice, we're talking about a situation where one tank discharges water into another one via an orifice.

Obviously we can apply energy equation here, just like anywhere else;

If we take point 1 at the water surface, we can neglect the velocity

and pressure here.

Now, let's define the elevation difference Z_1 minus Z_2 as H_1 , as the diagram shows here.

Next up we express the pressure head at point 2 in terms of the depth of water, H_2 .

Chucking these into the energy equation and rearranging

we get the velocity through the orifice as a function of the difference in tank water levels

which becomes an expression fairly similar to the equation we derived for orifice flow earlier, except this time the driving head is the head difference between the two tanks. This should make some sort of intuitive sense, because we can see that as water flows from one tank to the other, the water levels are going to equalise, and as they do that, the flow rate will drop. Eventually the tanks will have equal water levels and there won't be any flow between them.

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Coefficient of discharge

- C_D typically 0.6-0.62

$$V_2 = \sqrt{2g(H_1 - H_2)}$$

$$Q = C_D A \sqrt{2g(H_1 - H_2)}$$

As always we need to insert a coefficient of discharge for actual flow rate, and typical values for submerged orifices are around 0.6.

So our velocity equation

Becomes a general discharge equation for flow between two tanks.

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Orifice meter

- Used just like a Venturi meter, including same discharge equation

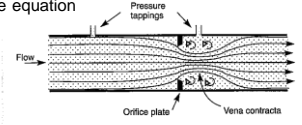


Figure 5.8 An orifice meter, consisting of an orifice plate installed at a flange in a pipeline, and two pressure tappings. Note that the pipe is full of water, with a sudden expansion of the flow downstream of the orifice from the width of the vena contracta to the full pipe width

An orifice meter consists of a flat plate inserted into a pipe, with an orifice in the middle of the plate. Pressure is measured upstream and downstream of the hole, where the downstream measurement captures fluid moving through a vena contracta. The equations for calculating flow from these pressure measurements are identical to those of a Venturi meter. As we said earlier, this type of meter tends to disrupt flow pretty severely and as a result the coefficient of discharge is a lot lower than that of a Venturi meter.

Image source- Les Hamill 2011, Understanding hydraulics.

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Large orifice flow

- "Small" (diameter << driving head)
- "Large" (diameter similar to driving head)
- With a small orifice, **head could be assumed not to vary** over the height of the orifice
- Problem now is that head varies

Okay, so we covered small orifice flow, where the size of the orifice was small relative to the head driving the flow.

Now in a large orifice, the size of the hole is big enough that it's on a similar order to the driving head.

with a small orifice, head could be assumed not to vary over the height of the orifice, which is why the equation had a single H value.

In the large orifice, the problem is that hole is big enough that driving head actually varies significantly from the top of the hole to the bottom.

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Large orifice flow

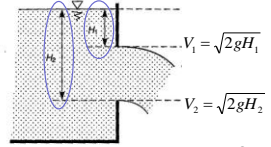


Figure 5.9 Discharge through a large orifice

$$\delta Q = V(h) \times \delta A$$

$$\delta Q = \sqrt{2gh} \times b \delta h$$

So now we need to consider the driving head at the bottom,

H2

and the driving head at the top, H1.

This causes a different fluid velocity at the top and bottom, and obviously it'll change across the whole height of the orifice too.

What we need to do is consider the orifice as a stack of little incremental areas, delta A

So at each depth, there's a portion of flow, delta Q, which is the product of the velocity at that depth times delta A.

Substituting the "V equals root 2 G H" equation for velocity, and letting delta A equal the width of the orifice times an incremental height delta H, we get a differential equation that we can integrate over the whole height.

Image source- Les Hamill 2011, Understanding hydraulics.

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A little friendly integration...

$$\delta Q = \sqrt{2gh} \times b \delta h$$

$$Q = b\sqrt{2g} \int_{H_1}^{H_2} \sqrt{h} dh$$

$$Q = \frac{2}{3} b\sqrt{2g} \left[h^{\frac{3}{2}} \right]_{H_1}^{H_2}$$

Theoretical $\rightarrow Q = \frac{2}{3} b\sqrt{2g} (H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}})$

Actual $\rightarrow Q = \frac{2}{3} C_d b\sqrt{2g} (H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}})$

It's not the hardest integration in the world

Assuming a constant width, which implies a rectangular orifice, "B root 2 G" comes out the front and we're left integrating "root H dH" over the interval from H1 to H2.

Hopefully you know all about how to integrate powers like this, so we end up with two-thirds out the front and H to the 3/2 in the integrated bit.

This expands to Q equals two-thirds times the width b, times root 2 G, times H2 to the 3/2 – H1 to the 3/2.

But that's the theoretical discharge

Of course we have to throw in a coefficient of discharge CD.

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Example 5.6

Theoretical calculation – ignore C_d

Large orifice: $Q = \frac{2}{3} b\sqrt{2g} (H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}})$

& compare with small orifice: $Q = A\sqrt{2gH}$

Now let's do an example where we've got a large orifice

2 metres high and 4 metres across

With the top of the orifice 90 centimetres below the water surface.

Let's compare the answer we get if we treat it as a large orifice

Compared with a small orifice.

Because it's a theoretical comparison we can ignore CD in these equations.

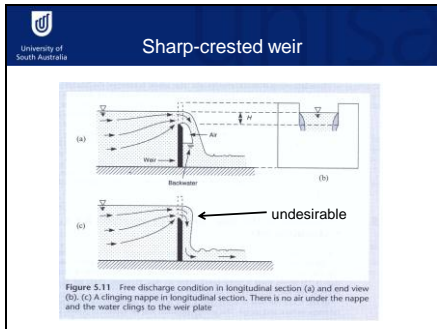
Image source- Les Hamill 2011, Understanding hydraulics.

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A weir is a flow measurement device used for open channel flow, so it's physically totally different to things like the Venturi meter and Pitot tube. The figure here shows a sharp-crested weir.

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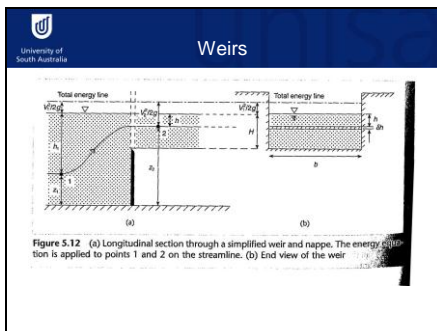


Weir discharge measurement involves carefully measuring the depth of water flowing over a sharp crest. The equations we're going to use for this all assume that water flows over the crest freely, with air directly below the stream of water as it leaves the crest.

Under certain circumstances, especially if the crest spans across the entire channel, it's possible to get the water clinging to the crest, which is undesirable as the equations no longer correspond to the measurements being taken.

Image source- Les Hamill 2011, Understanding hydraulics.

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As usual we need to view this situation through the lens of the energy equation, considering a point 1 upstream of the weir and a point 2 just after the crest. This picture is obviously not very realistic because water discharging over a crest doesn't horizontally flow through the air like this!

Image source- Les Hamill 2011, Understanding hydraulics.

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Recall → large orifice flow

- Discharge equation for large orifice:

$$Q = \frac{2}{3} C_d b \sqrt{2g} \left(H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}} \right)$$

- If $V_1 = 0$, then discharge equation for sharp-crested weir:

$$Q = \frac{2}{3} C_d b \sqrt{2g} \left(H^{\frac{3}{2}} \right)$$

The simplest way to view a weir is if you assume the upstream velocity is zero,

so it becomes basically a tank discharging through a large orifice, where the orifice happens to be located right up at the top of the tank with no wall above it.

In this simple case, we can just use the large orifice equation

And because the top of the orifice is the water surface, H_1 becomes zero

So under the idealistic assumption that upstream velocity is zero, we can get an approximation of discharge

Just by measuring the water depth H over the crest and applying a simplified orifice equation.

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But what if $V_1 \neq 0$?

- After all, this is supposed to be a **flow measurement device!**

Figure 5.12 (a) Longitudinal section through a simplified weir and nappe. The energy equation is applied to points 1 and 2 on the streamline. (b) End view of the weir

Derivation on p. 146

That's all well and good, but we might not be totally comfortable assuming the velocity upstream is zero.

I mean after all, the purpose of the weir is to measure discharge, which implies that there is some flow taking place.

So let's go back and build up the energy equation again.

You can find the full derivation in the textbook

The main thing to watch here is the clever way we build up the energy equation. To start with, take a look the elevation head, Z_2 . This is built up of the elevation head Z_1 , plus H_1 , which gets us to the water surface, minus the depth h which is because we're considering some arbitrary point located somewhere in stream flowing over the weir.

Now upstream, considering some arbitrary depth in the channel, we've got a pressure head, which is P_1 on rho G , equal to h_1 . So we can see that h_1 is going to appear on both sides of the energy equation, which should mean it cancels out. And the elevation head at point 1 is Z_1 too, so that's going to appear on both sides. It's starting to look good.

Okay, now going back downstream to point 2, here we can make a simplifying assumption that pressure P_2 is zero. Remember how I told you these equations assume there's air underneath the flow as it leaves the crest? That's important here, so we can assume the discharging stream of water is at atmospheric pressure. So P_2 equals zero.

From previous slide: $\frac{P_1}{\rho g} = h_1$
 $P_2 = 0$
 $z_2 = z_1 + h_1 - h$

Alright, let's get into it.

Here's our energy equation. Upstream we've got elevation head Z1 plus velocity head V1 squared on 2 G plus pressure head, which we said was equal to h1. Downstream we've got all that mess from before, Z1 plus h1 minus h, plus the velocity head V2 squared on 2 G. And remember we assumed P2 was zero.

So first things first, cancel out the Z1 and h1 terms.

Now we've just got the two velocity heads and the height h, which remember is just an arbitrary depth in the discharge stream.

Rearranging this gives V2 as a function of V1 and h, and we're going to use this in the same style of integration as we used when we built up our expression for large orifice flow.

So if we consider a little portion of the overall flow, delta Q, this is equal to the expression for velocity V2, times the width b, times the incremental height, delta h.

Integrating...

I know this might look daunting, but the integration really isn't that much harder than the orifice equation.

Again assume a uniform width b,

And all that happens is we end up with the upstream velocity head, V1 squared on 2 G, stuck in our integration.

This time we're integrating over the interval from 0 to big H, because we're going from the water surface down to the weir crest as shown in the picture here.

So it comes out into an equation that looks a little unpleasant, but it's not really too bad.

Iterative approach...

- To begin with, we don't know V_1 so we just assume $V_1 = 0$
- This gives an **approximate** flow rate, which we use to calculate an **approximate** V_1 ($V_1 = \frac{Q}{A}$)
- Then substitute your approx. V_1 to the equation and repeat
 - New $Q \rightarrow$ new $V_1 \rightarrow$ substitute \rightarrow repeat
 - STOP** when consecutive Q values are within an acceptable tolerance (e.g. Correct to 3 sig. fig.'s)

Okay, great, we've got this huge equation for measuring discharge but one of the parameters is velocity – so it sort of looks like a self-defeating situation, doesn't it? I mean, if we knew the value of V1, then we'd know the value of discharge, right?

The way forward is a process called iteration, which involves starting with a guess. Let's start by assuming V1 is zero.

Now we can calculate an approximate flow rate

Just from the measurement of the height of water over the weir, which is convenient.

This isn't the real flow rate, but it's probably a reasonably good approximation depending on how fast the upstream water is travelling.

Now we can use this approximate Q value, via the $Q = VA$ relationship, to give us an approximate value for V_1 . It's still not going to be correct, but it'll be a little bit closer than the original guess of zero.

Now with our slightly better guess of V_1 , we can plug this into the discharge equation,

which will get us a new, even better value of Q, which we use to get a new value of V_1 , new Q, new V_1 , new Q, and so on. Obviously you could go on repeating this procedure forever without stopping – you'll never quite get the true answer, whatever that is, using this method. So you need to set some realistic level of tolerance, and then you can stop when you get two consecutive values of V_1 that are within that tolerance. Maybe this would be when you get two values that are within a particular number of significant figures, corresponding to the level of precision in the data you've been provided in the problem at hand. You'll find we use this sort of trial-and-error approach a fair bit for problems in water engineering.

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Here's an example of water flowing through a small channel, and going over a sharp-crested weir. What we're going to do here is calculate the flow rate first with the assumption that V_1 is zero, and then go through the iteration procedure we just learnt, to find a more accurate value.

The two fundamental pieces of info we need are the length of the weir crest, b

And the height of water over the crest, H . With the coefficient of discharge given as 0.62, you've got enough data to calculate an approximate flow rate assuming V_1 is zero. But when we go through the iterative procedure, we need a little bit more information

Because we calculate the approximate upstream velocity by $Q = V$ on A ,

We need to know the dimensions of the cross-sectional area upstream

Fortunately we've been given the channel width, which is 40 centimetres

And an additional height, which we can use to work out the overall water depth to give us the cross-sectional area of flow. See how you go with the iterative approach, and remember to stop when you reach an appropriate level of convergence between your Q values. The textbook can help guide you through this example if you need help.

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But in practice...

- It's common to ignore V_1
- Why?
 - Velocity in an open channel is *usually* quite small (i.e. $< 1 \text{ ms}^{-1}$) $\rightarrow V_1^2$ even smaller divided by $2g$ ($=19.62$) \rightarrow even smaller
 - Typically affects **total upstream energy head** by several millimetres so *usually* not a big error if we neglect it

Okay, we needed to go through that iterative process to show how to get the true answer, and in some cases we might really need this high level of accuracy. But in reality we can often neglect the upstream velocity.

The reason it's not such a problem to ignore upstream velocity is that typical speeds in channel flow are pretty slow, like less than a metre per second. When we turn this into a velocity head, we square it, so it gets even smaller,

and then we divide it by almost 20

so it ends up that typical values of velocity head in a channel are on the order of just a few millimetres, and in a lot of cases this is small enough to ignore.

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Triangular (V-notch) weirs

- Similar derivation as before, but now b varies with h

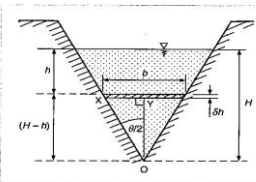
$$\delta Q = \delta h b \sqrt{2gh}$$


Figure 5.14 Triangular weir or V notch

Another common type of weir is the triangular, or V-notch, weir.

The derivation of the discharge equation for a V-notch weir

starts out the same as the rectangular weir

Except now the width varies depending on the depth of water flowing across the weir

We can use geometry to express the width b as a function of the weir angle and water depth, which feeds back into the integration

And the key thing to note now is that we end up with an extra h term in the integration

Image source- Les Hamill 2011, Understanding hydraulics.

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Triangular weir equation

- Skipping the integration...

$$Q = \frac{8}{15} C_d \tan\left(\frac{\theta}{2}\right) \sqrt{2g} \times H^{5/2}$$

constants

Rectangular: $Q = \frac{2}{3} C_d b \sqrt{2g} \left(H^{3/2}\right)$

So without boring you with the integration steps, we end up with this equation for discharge over a V-notch weir, assuming negligible upstream velocity. I know it looks complex but it's really not.

For a start, all this stuff is just a bunch of constants

So the main thing going on here is that because we started out the integration with an extra h term, the final result is that flow is proportional to H to the 5 on 2,

which is exactly one power higher than we had in the rectangular case.

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Example 5.8

Figure 5.14 Triangular weir or V notch

$Q = \frac{8}{15} C_D \tan\left(\frac{\theta}{2}\right) \sqrt{2g} \times H^{5/2}$
 $Q = 0.053 \text{ m}^3/\text{s}$
 $\theta = 60^\circ$
 $C_D = 0.60$

Repeat for a rectangular weir with $b = 0.3 \text{ m}$:

$Q = \frac{2}{3} C_D b \sqrt{2g} \left(H^{3/2}\right)$

Let's do an example where we've got a particular flow rate and we want to know what the corresponding height of water over the weir crest would be.

This means you're going to have to rearrange the discharge equation to solve for h.

Water's flowing at 53 litres a second and the V-notch weir has a total angle theta of 60 degrees. Assume a coefficient of discharge of 0.6

Once you've got your answer, repeat the calculation to see what the height of water would be over a rectangular weir 30 centimetres wide. The textbook will help if you get stuck.

Image source- Les Hamill 2011, Understanding hydraulics.

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Calibration

- Calibrating what?
 - Coefficients
- Example 5.9 (Excel)

All of the devices we've looked at today have some sort of coefficient in them to correct for real-world effects like energy head losses. In most of the examples we've covered, you were just given a value of coefficient of discharge. But you might wonder how we actually go about finding those coefficients. Imagine, for example, that you're developing your own flow measurement device based on the pressure-velocity relationships we've looked at here. Let's use an Excel example to look at how you might go about calibrating the coefficient of discharge for a given flow meter, based on experimental measurements. The example we'll look at is example 5.9 from the textbook. This type of analysis is also the basis of the calculations you'll have to do for your practical on flow measurement.

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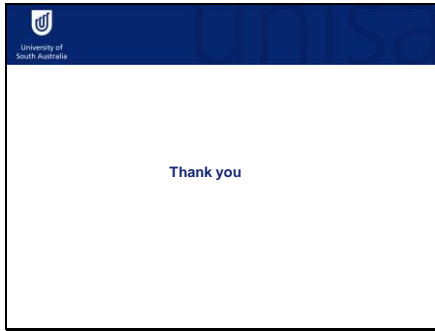
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Summary

Variety of discharge measuring devices:

- Venturi meter, Pitot tube, orifice and sharp crested weir
- all exhibit a power relationship between **velocity** and **pressure head**
- $V \propto h^{0.5}$, or $h^{1.5}$, or $h^{2.5}$...

So in summary, we've looked at a variety of discharge measuring devices including venturi meter, pitot tube, orifice and sharp crested weir. These are all related to one another by the fact that each device exhibits a power relationship between velocity and pressure head. In some cases like the Venturi meter and Pitot tube, velocity's proportional to the square root of a pressure head difference. In weirs the velocity's proportional to head to the power of 3 on 2 for a rectangular weir, and the power goes up to 5 on 2 for a V-notch weir.



If you've got any questions or need some further clarification please post a question or comment on the Discussion Forum.