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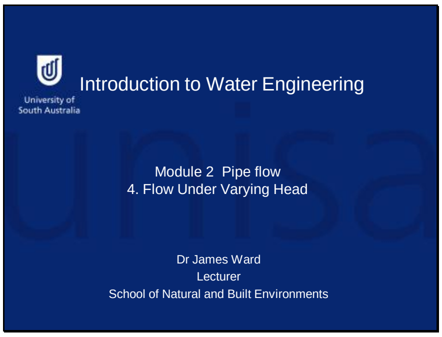
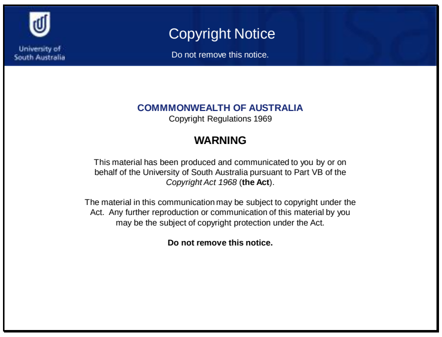


Welcome to Module 2 and Pipe flow.

Slide 2



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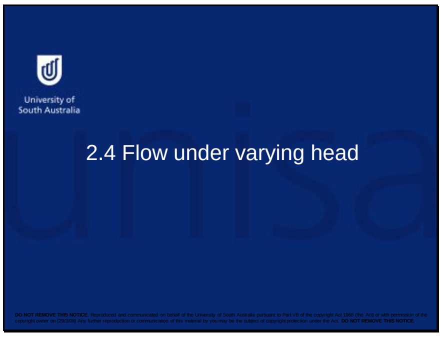
Slide 3



Today we're looking at flow under varying head.

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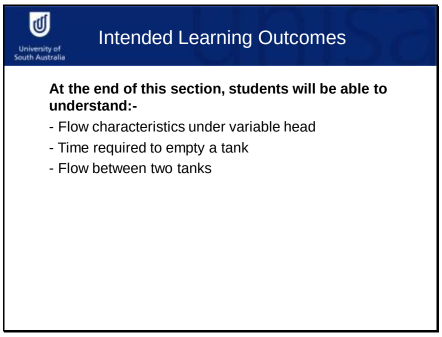

The intended learning outcomes from this presentation are for you to understand how flow changes under variable head, what this means for calculating the time to empty a tank, and how flow moves between two tanks.

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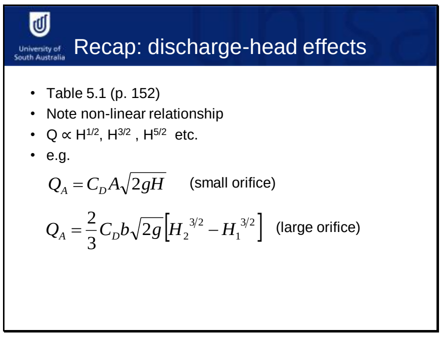

In chapter five, we looked at relationships between head and flow for different situations. Hopefully you remember that there was generally a non-linear relationship between flow and head.

For flow through an orifice the discharge was proportional to the square root of head, for flow over a rectangular weir it was H ^ 3/2 and 5/2 for a triangular weir.

Just looking at a couple of those equations – one for small orifice

And one for a large orifice – we can see that the flow isn't directly proportional to the head, it's proportional to some power of the head. This means if our head is changing – for instance if a tank is draining out of a hole – the change in discharge isn't going to be exactly proportional to the change in head.

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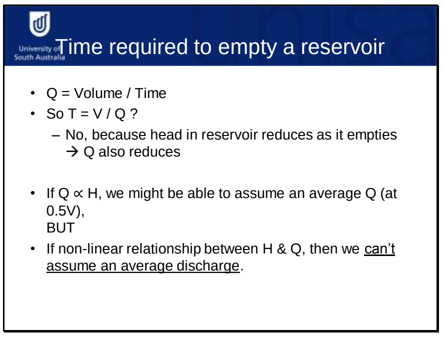

Okay, let's say we're interested in working out how long it'd take for a reservoir to empty. Well discharge is volume over time

and you could say that rearranging this gives time equal to volume over the discharge rate.

But that's not right because the head goes down as the tank empties out, so the flow goes down.

Now, if we had a proportional relationship between flow and head, maybe we could assume an average flow rate and go from there,

But we know we generally don't have a proportional head-discharge relationship so that's not much use to us.

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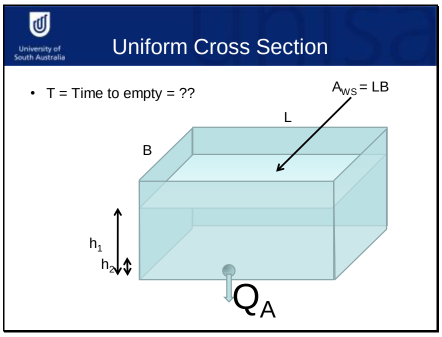

Alright, let's take this tank as a basic illustration

It's got a length and width here

And that gives us the water surface area

So we're talking about starting out at a particular head of water

Which gives us a high flow rate, which gradually decreases as the driving head gets smaller and smaller. What we're going to need to do is use a bit of Calculus to estimate the emptying time.

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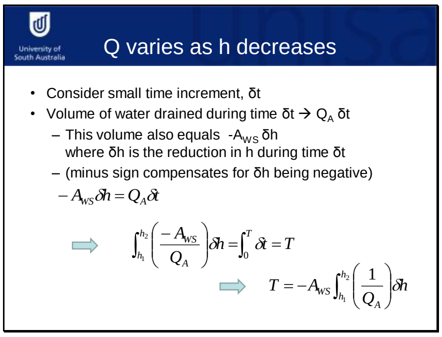

Right, so let's build up the description of emptying time using calculus.

We start out by considering a miniscule increment of time, delta T.

Now, during that little time increment, the tank drained out at a particular flow rate, and if we multiply that flow rate by the time increment we get the volume discharged during delta T.

The other way to work out the volume drained during the little time interval is to look at the physical change in volume in the tank, so that means taking the water surface area and multiplying it by the change in height, delta H. Area times height equals volume.

The negative sign's important because delta H is negative, so we need to put a minus sign in to give the volume as a positive.

So we put these two expressions of volume together and we've got Aws by delta H equal to Q by delta T.

Now we're going to divide both sides by QA, and then integrate. This means taking the H from H1 to H2, which is going from the upper level H1 down to the lower level H2, and on the other side we're just integrating delta T from the start, which is time equals zero, to the end, which is T. So that just becomes T.

So for the special case where there's a constant water surface, which just means a reservoir with vertical sides, the Aws term comes out and we've got the emptying time T equal to Aws by the integral from h1 to h2 of 1 on QA delta H.

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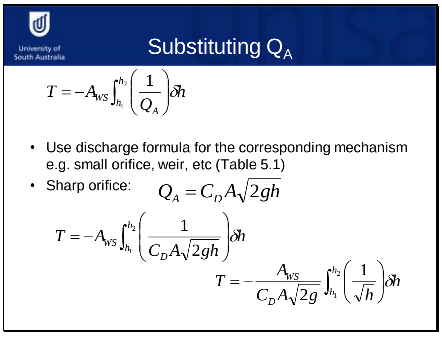

So then you take that basic integration

And sub in whatever discharge equation you need – for example if you've got a small orifice, you chuck in the orifice discharge equation in stead of QA.

Say that'd look like this

And the equation for emptying time'd come out like this. I know it looks a bit exciting but actually most of that stuff inside the integration sign's just constant

So we can pull it outside like this – constant water surface area, constant discharge coefficient, constant orifice area, so in this case all we're having to integrate is a simple "1 on root H". Actually, it's really not that big a deal if any of those parameters happened to not be constant – for instance if the water surface area changes with water depth because the tank's got sloping sides – it just slightly changes what's inside the integration sign but at the end of the day we're not talking about really tough integration.

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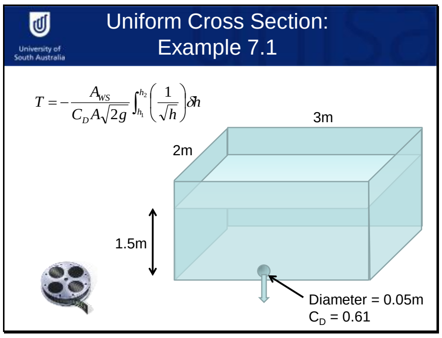

Let's run through a simple example where we've got a small orifice and constant water surface area.

In this case the water level's dropping from 1.5 metres, measured above the centreline of the orifice, all the way down to zero. So that gives you the values for H1 and H2 in the integration and everything else is given. See if you can work it out and head to the textbook if you need any help.

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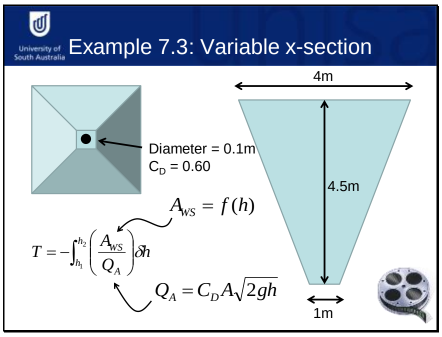

Okay, now let's cut our teeth on a more involved example. In this case we're still considering a small orifice but we're going to move away from the constant water surface area. Now it's a pyramid shaped tank with all four sides sloping out.

So this time we've got to take a step back and make sure we keep the AWS term inside the integration because it's not a constant.

AWS is going to be a function of the height,

getting larger as you go further up the tank.

Luckily we've been given a bunch of dimensions we can use to work out the function AWS – this might mean going back to basic geometry for you though.

The equation for QA gets plugged in the bottom, same as before, and anything that stays constant can get taken out of the integration.

Give it a shot. If you need help – especially when it comes to remembering how to work out a geometric relationship like this – the textbook goes through it, but just remember there are different ways you can go to get the same answer and the way they do it might not be the way you intuitively get there. See how you go, anyway.

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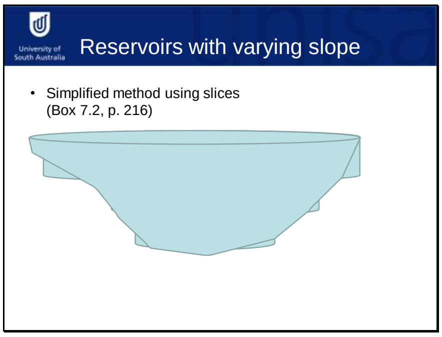

Alright, it's all well and good if you've got a nice constructed tank where you can either assume the sides are vertical or you can derive a neat equation for water surface area as a function of depth, but what are you supposed to do if you've got a dam or reservoir in a more natural landscape where there's no easy relationship between the surface area and the depth of water?

The answer is that you need to simplify it down to slices that behave in a mathematically predictable way, and just put up with a pretty inaccurate answer as a result. What we do in the simplified method is divide the reservoir into slices that are preferably of equal thickness, and we assume each one is a big flat thing with vertical sides.

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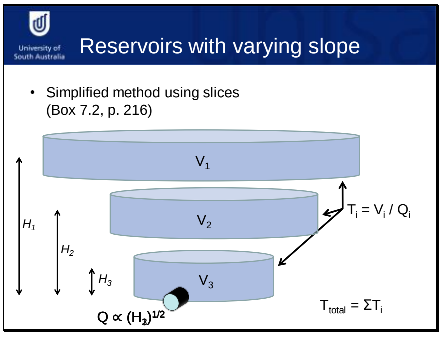

Providing we've got some sort of contour map we can work out the area of each slice in plan view, and convert that to a volume by multiplying it by the slice thickness. Then we can make a rough assumption that the time taken for each slice to empty is going to be the volume divided by an average flow rate, which we work out using whatever discharge equation is appropriate – in this case let's assume the reservoir's emptying through an orifice so the discharge is proportional to H ^ (1/2).

So the first slice empties with a discharge proportional to some sort of average head. Or we might work out the discharge based on the head at the top and bottom and take the average.

For the next slice, we jump to a different discharge because we're at a lower elevation

And likewise for the next slice. For each slice, the flow rate's assumed to be constant but it changes from one slice to the next.

Since we've got a reasonable idea of the volume of each slice, we can work out the approximate emptying time by dividing the volume by the average discharge rate

And finally by summing together all the emptying times of all slices, we get an approximate value of total emptying time for the reservoir.

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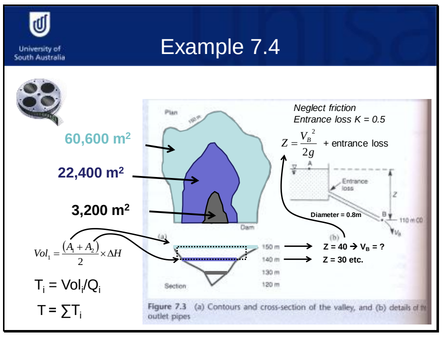


So here’s an example of the slice method. We’ve been given a contour map of the reservoir and someone’s figured out the area of each contour.

The first one’s about 6 hectares, or 60,000 square metres

The next one’s about 2 hectares

And the last one’s about 3,000 square metres. We’ve also got a cross-section view down the bottom showing the elevation of each contour line.

To work out the volume of each of these slices we take the average area from the upper and lower contours

and multiply by the height difference.

To work out the average flow rate for each slice, this time we don’t have a simply discharge equation – we actually need to solve for velocity using the energy equation and then multiply by the pipe’s cross-sectional area to get flow rate. So take the elevation at the top and bottom of each slice and solve for VB.

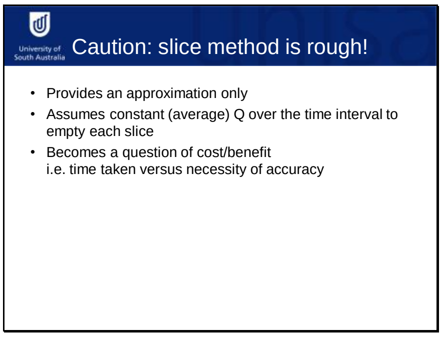
Repeat this for each elevation to get a value of velocity at each depth and convert each one into a flow rate using the continuity equation – we’ve been told it’s a 0.8 metre diameter pipe. Each slice then gets an average flow rate based on the upper and lower values.

Once you’ve worked out the average flow rate for the slice you can get the approximate emptying time for that slice

And sum them together to get the total approximate emptying time. As always check in the textbook if you need help with the worked example.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 15



Just as a point of caution,

The slice method only provides an approximation,

as it assumes constant discharge over the time interval to empty each slice. Depending on the thickness of the slice, this might be relatively accurate or very inaccurate.

It really becomes a question of cost/benefit, meaning the time taken to get a precise and accurate answer versus the benefit that comes from having a high level of accuracy. If all you need is a rough idea, like, whether the emptying time for a dam is going to be closer to three hours versus three days, then the slice method is good because it gives you a quick answer. But if you need a very accurate prediction, you'll need to move to a more sophisticated model.

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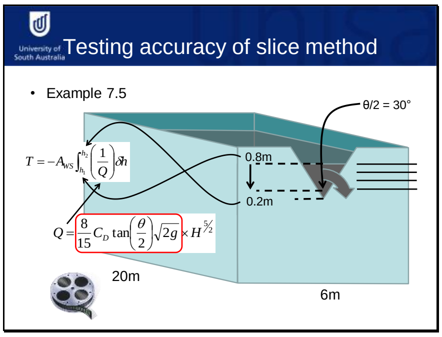

By way of an example, let's take a situation where we know we can get the true answer using integration and then use the rough slice method to see how close we get.

Here we've got water discharging out of a tank via a V-notch weir.

It's a pretty big tank, 20 metres long and 6 metres wide

Theta / 2 for the weir's 30 degrees

And we're interested in the time taken to get from an initial height of 0.8 metres

Down to a height of 0.2 metres above the bottom of the triangular weir.

So since we've got vertical sides, we can assume the water surface area's a constant and our basic integration looks like this

And we're going from H1 = 0.8 metres to H2 = 0.2 metres in the integration.

The final thing to do is chuck in the appropriate discharge equation, which for a V-notch weir looks like this. I know it's a little scary when you first look at it, but don't forget you can take anything out that's a constant,

which basically means all of this – so you're only left with a simple H ^ (5/2) term that you have to integrate.

After you've integrated that and found the true value of the emptying time, work out the approximate emptying time using the slice method. Go with three slices, each one 0.2 metres thick – so you'll have four elevations to consider – 0.8, 0.6, 0.4 and 0.2 metres. You can build up your areas, volumes, flow rates and discharge times for each slice just the same as last time and work out whether there's any significant difference between the true method by integration versus the approximate method. Check the textbook if you need help.

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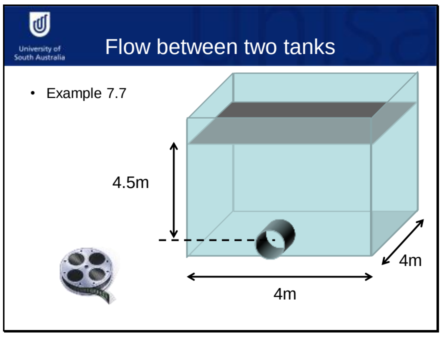

So far so good I hope! Now let's step it up and look at how you work out the flow between two tanks. If we start with a tank like this, discharging to the atmosphere, it's just like the other examples we've looked at.

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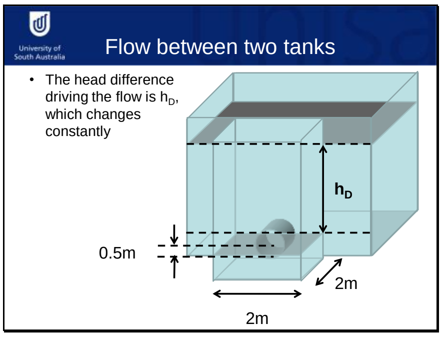

But what happens if instead of discharging to the atmosphere, it's actually draining into another tank? In this example it's draining into a narrower tank, with a smaller water surface area – this means that for a given water level drop in the big tank, it'll rise a lot more in the smaller tank.

The driving head causing the flow is the difference in water level between the two tanks – as the water level drops in the big tank and rises in the small tank, the overall head difference reduces

and eventually as the water levels equalise, the driving head becomes zero and there's no longer any flow between the tanks.

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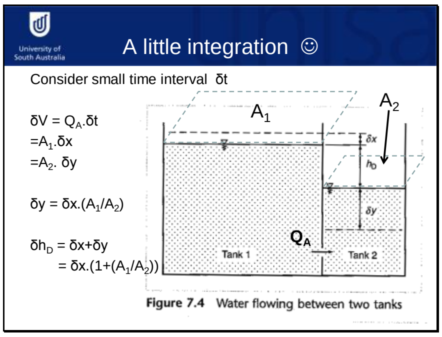


Right, so in case you hadn't picked it, the way to work this out comes back to integration. The thing that makes this one a bit trickier to work out than the first lot of integration we did is that in this case, we don't actually know what the final water level's going to be. What we do know, though, is that the end point is when the driving head HD drops to zero. So we need to set up the integration with that end point in mind.

Let's kick things off by considering a tiny increment of time, delta T.

Now, just like the first lot of integration, we've got a change in volume equal to the flow rate times delta T. And obviously we've got the same problem as always, which is that the flow rate changes because of the change in driving head.

So just like before, we can also express the change in volume as the water surface area times the change in height. In this case we'll call the change in height of the large tank "delta X"

And because the volume leaving tank 1 is entering tank 2, we can also express the same volume as the area of water surface in tank 2 times the rise in water level, delta Y.

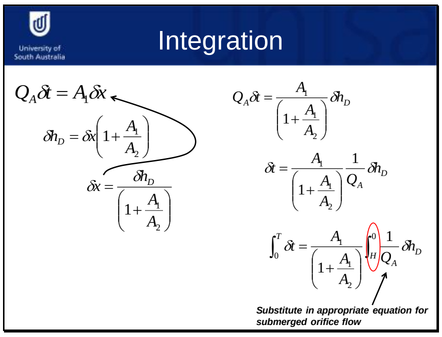
Rearranging these gives us delta Y in terms of delta X and the two water surface areas.

Now the overall change in the driving head, delta HD, is equal to the sum of the change in water level in both tanks.

We can use the relationship with the surface areas to get this all in terms of delta X.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 20



Alright, so going back to the first part where we equate the two different expressions for the change in volume

And using our expression for delta HD

Rearranging to give delta X instead

We can substitute this into the equation

That gives us an expression like this,

and now it's just a matter of rearranging

into something we can integrate. So we integrate the time side which gives us the time taken to equilibrate the two tank levels, and on the right hand side we end up with a whole lot of constant areas and an integral of 1 on Q by delta HD.

The important thing here's the interval over which we're integrating – remember we've reorganised this to be in terms of the head difference, HD, which is what's driving the flow. So the integration goes from the initial head difference, which we could call H, to the final head difference, which for equilibration's just going to be zero.

As with the other examples, you substitute the appropriate equation for the type of discharge you've got – bearing in mind in this case it's going to be an equation for submerged flow, so it needs to be expressed as a function of the head difference HD. Check out pages 137 to 138 back in chapter 4 of the textbook for a recap on submerged orifices. Meanwhile go through example 7.7 carefully in the textbook to get your head around the method we've just been through.

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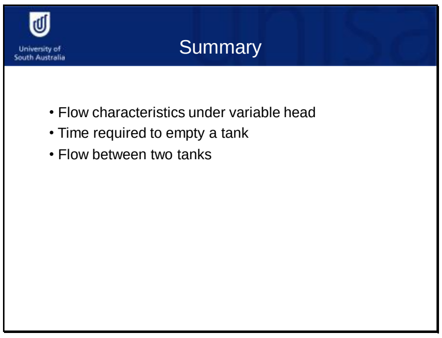
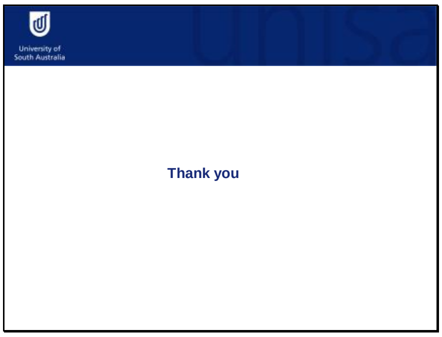


So, in summary, we've looked at flow characteristics under variable head, in particular focusing on how this affects our calculation of the time required to empty a tank or for water levels to equalise between two tanks.

Slide 22



If you've got any questions or need further clarification, please post a question or comment on the Discussion Forum.