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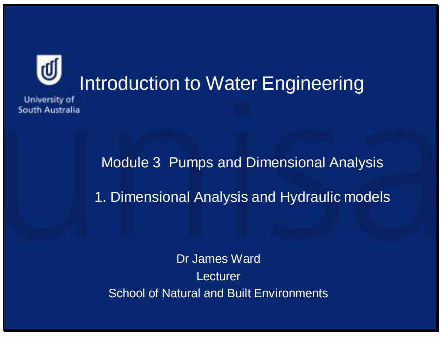

Welcome to Module 3, pumps and dimensional analysis. Today we're introducing dimensional analysis and hydraulic models.

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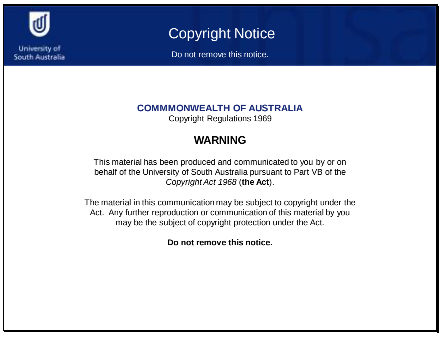

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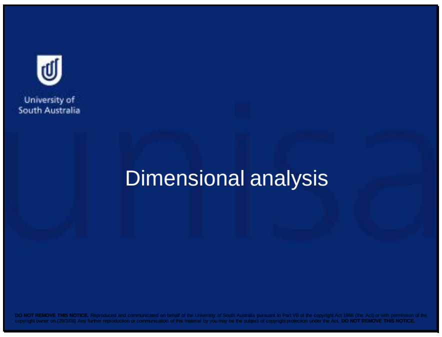

Alright, Let's start by looking at dimensional analysis.

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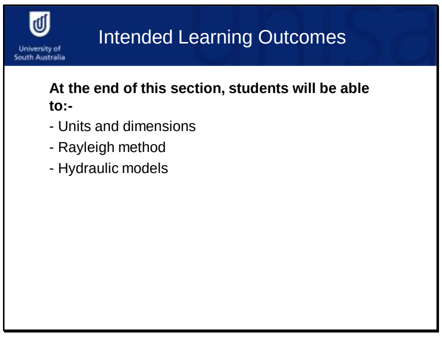

The learning outcomes are presented here – we're going to start by looking at units and dimensions, then go through the Rayleigh method of dimensional analysis, and look at how different hydraulic models can be designed based on dimensionless groups.

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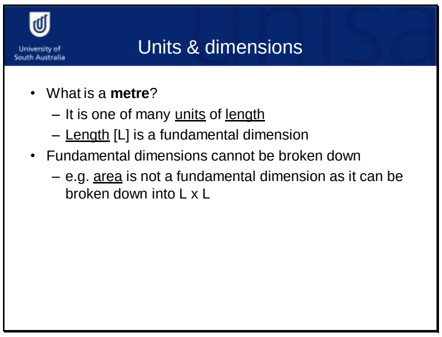

Okay so first things first – let's look at units and dimensions.

If ask you what a metre is, you'll be able to say it's a measure of length

And hopefully you know it's just one of many different length units – think about centimetres, kilometres, and so on. Even light-years are a unit of length.

The thing that ties all of these units together is the fact that they're all expressing the same dimension, which is length. We call this a fundamental dimension.

Essentially what makes it a fundamental dimension is that you can't break it down into anything,

Whereas other units, like area, aren't fundamental because they can be broken down into constituent parts. In the case of area, you get there by multiplying two lengths together so you'd express area in fundamental dimensions as a "length squared".

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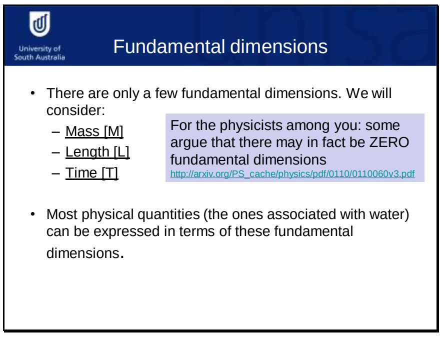

We're going to consider three fundamental dimensions:

mass,

Length

and time.

These cover most physical quantities and certainly they're sufficient to look at the sorts of things we encounter in hydraulics. For more extended work you'd need to include a fourth dimension which is temperature, and if you got into really complex physics you might need to include things like electric charge

Actually there's an intriguing debate between physicists about how many truly fundamental dimensions there are, because depending on what assumptions you make you can use theoretical physics to argue for more or fewer dimensions. Some physicists even argue for zero dimensions! But that aside, for practical purposes we'll stick to the big three.

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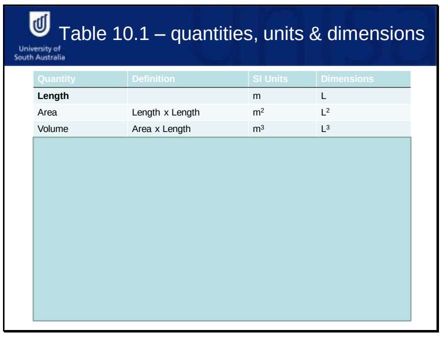

So here are some basic physical quantities you're all familiar with. Let's start with the length-based properties. After basic length, you've got area that you get by multiplying two lengths together, which as we said before ends up with fundamental dimensions of L squared, and if you multiply it by another length you get a volume, which has the dimensions L cubed. It's important to keep your wits about you with these sorts of things, because as you know, a common unit for volume is the litre, which has the symbol of capital L and obviously that could get really confusing!! So make sure you know whether you're dealing with units or dimensions.

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Table 10.1 – quantities, units & dimensions

Quantity	Definition	SI Units	Dimensions
Length		m	L
Area	Length x Length	m^2	L^2
Volume	Area x Length	m^3	L^3
Time		s	T
Discharge	Volume / Time	m^3/s	L^3T^{-1}
Velocity	Length / Time	m/s	LT^{-1}
Acceleration	Velocity / Time	m/s^2	LT^{-2}

Bringing in the time dimension now, let's look at some quantities based on length and time. The obvious one's velocity, which is distance over time, so expressing that in fundamental dimensions is L by T-1. Acceleration is velocity over time so the dimensions have T-2 instead of T-1. But we've also looked at discharge as a volumetric flow rate, so that's a volume over time, or L cubed by T-1.

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Velocity	Length / Time	m/s	LT^{-1}
Acceleration	Velocity / Time	m/s^2	LT^{-2}
Mass		kg	M
Force	Mass x Acceleration	N	MLT^{-2}
Work / energy	Force x Distance	Nm or J	ML^2T^{-2}
Power	Work / Time	J/s or W	ML^2T^{-3}
Pressure	Force / Area	N/m^2	$ML^{-1}T^{-2}$
Mass density	Mass / Volume	kg/m^3	ML^{-3}
Weight density	Force / Volume	N/m^3	$ML^{-2}T^{-2}$

Last of all we bring in mass and this really opens things up. The first one we can do is force, which is mass times acceleration. It's the physical relationship (mass by acceleration) that we use to build up the fundamental dimensions for force – M by L by T-2. Using what we know about energy, being work done or force times distance, again we can figure out what the fundamental dimensions are going to be – in this case we take the dimensions for force and multiply them by another length which is why there's an L2 term in there. And you can see how the other quantities are built up. You're probably thinking this seems like a lot of unnecessary complication – for instance why bother expressing energy as M by L2 by T-2 when it's so much simpler just to call it a joule?! So let's move on and see why we need to be able to boil everything down to fundamental dimensions.

Slide 10

Dimensional homogeneity

- Suppose I have three quantities A, V and Q.
- If A is area [L^2], and V is velocity [LT^{-1}], and we know Q = AV, then <u>what are the dimensions of Q</u>?

→ **According to dimensional homogeneity, both sides of the equation must have the same fundamental dimensions**

→ It follows that <u>we can only add or subtract quantities of the same dimensions</u>.

$$L^2 \times L \times T^{-1} = L^3 \times T^{-1}$$

We're going to explore a concept called "dimensional homogeneity". We'll start with an example.

We've been given some information about a pipe –

area, velocity and flow. We know the dimensions of area and velocity and we know the equation that brings them together, Q = AV, but we can't remember what the dimensions of the flow are.

This is where the principle of dimensional homogeneity comes in handy. It basically says that dimensional quantities on either side of an equals sign have to be the same – that means that all of the fundamental dimensions have to have the same power on both sides of the equation.

Because equations can be rearranged, an outcome of dimensional homogeneity is that you can't add or subtract quantities of unequal dimensions. Obviously you know you can't take a distance and add it to a time – for instance the distance to the nearest doorway might be, say, 3 metres, and you can't add that to the number of hours you slept last night – it just wouldn't make sense.

So going back to the Q = AV question, we know that the dimensions of Q have to equal the dimensions of AV for that relationship to hold true, so we can figure out the dimensions of Q by multiplying the dimensions together. Taking the area, which is L2, and multiplying it by the dimensions of velocity, L by T-1, we end up with L3 by T-1. And that means it's volume over time, so we know Q must be referring to a discharge.

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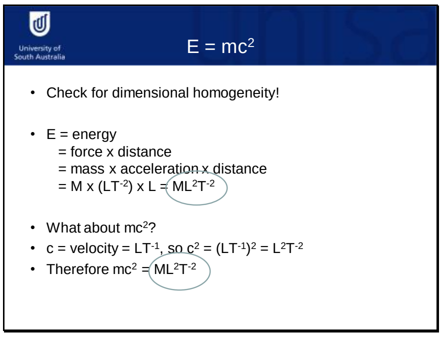

Here's another example you might be familiar with – the famous E = mc2 equation.

Let's check that the fundamental dimensions match on both sides of the equation. Obviously we don't seriously expect them not to, but we'll do the exercise anyway.

So remember we said energy can be written as force times distance, because it refers to work. Force was mass times acceleration so the whole thing becomes mass times acceleration times distance, which is M by L2 by T-2.

Now we'll do the other bit, mc2.

Hopefully you know the "c" here stands for the speed of light, which is a velocity so it's got dimensions of L by T-1. But it's squared in the equation so we have to square the dimensions. That becomes L2 by T-2.

Multiplying by the mass term we get mc2 expressed in terms of its fundamental dimensions, M by L2 by T-2

And as expected, this matches the

dimensions of energy perfectly.

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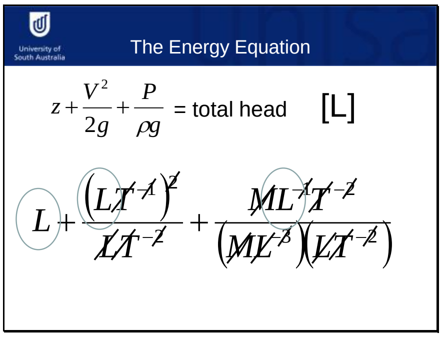

Now we'll bring it into something we're all really familiar with, the energy equation. Remember how we said a consequence of the dimensional homogeneity principle is that you can't add or subtract quantities unless they've got the same fundamental dimensions? Well, for the energy equation that means the three terms, elevation head, velocity head and pressure head, and actually the total head too, all have to have the same dimensions. Let's check to make sure they do.

Here's what you get if you replace each symbol with its fundamental dimensions, so Z just becomes L, V becomes L by T-1 all squared, and so on. I know it looks a bit ugly but you get the idea. Also you should be able to see that a lot of stuff's going to cancel. In fact, looking at the L term right at the start, you can see that for this equation to be dimensionally balanced, the other two terms are actually going to have to reduce to plain L terms. So that means all the mass and time terms need to cancel out along with most of the length terms.

Cancelling out terms isn't that difficult really, and eventually it emerges that yes, the velocity head and pressure head terms are actually reducible to basic lengths.

So head has dimensions of length, which is no surprise since it's usually expressed in units of metres.

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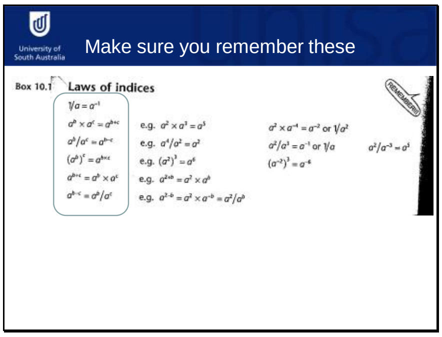

For dimensional homogeneity, we're always interested in the overall power or exponent attached to each individual dimension. Since most equations involve a number of terms being multiplied together, along with all of their dimensions, we need to make sure we're familiar with the laws of indices, which are shown here

These are very important in dimensional analysis and you'll need to keep these committed to memory.

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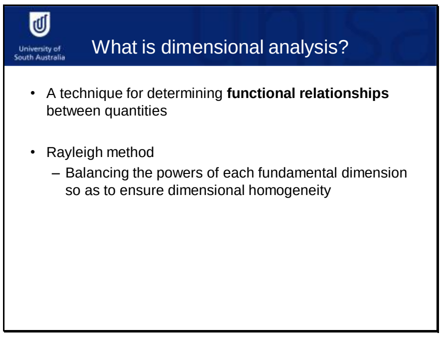

So far, we've always been working with known relationships between physical quantities – in other words you're always given an equation that relates quantities together – like Q = VA, P = ρgH and so on. But what happens if you've got some physical quantities that seem to be related to one another – perhaps you've been doing some experiments and you've got some data – but you don't know the equation relating them? Well, we can use dimensional analysis to determine the functional relationships between quantities.

There are two basic methods for dimensional analysis: Rayleigh and Buckingham Π. We're only going to look at the Rayleigh method, which involves balancing the power of each fundamental dimension to satisfy dimensional homogeneity. The Buckingham Π method does the same thing, but it's a more convoluted process and we're only going to look at the Rayleigh method in this lecture.

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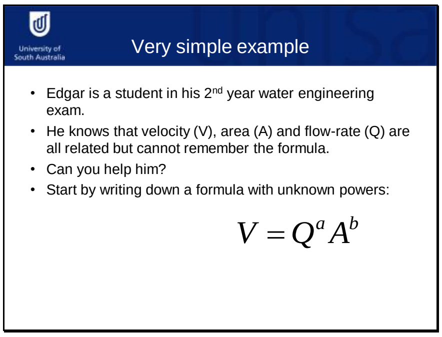

Let's go through a simple example of the Rayleigh method.

Edgar's student in his 2nd year of water engineering and he's found himself stuck during the exam.

He forgot to write the continuity equation on his formula sheet. He knows it relates velocity, area and flow rate but he can't remember what order they go in the equation.

Fortunately we can help him out by using dimensional analysis.

We start out writing a formula as V=Qa times Ab.

In this case the order of the symbols doesn't matter; you could have put Q = Va times Ab or whatever. Next we're going to solve the exponents in order to balance the dimensions.

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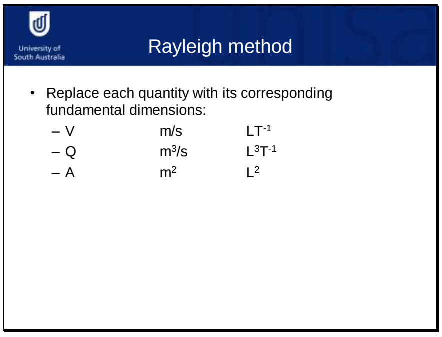
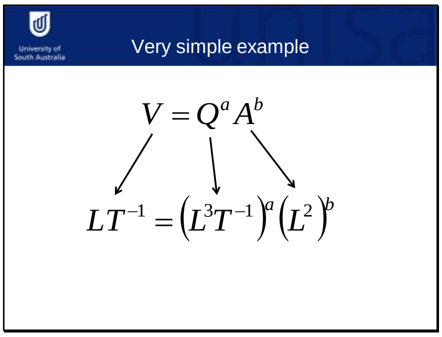


The next step is to replace the quantities (V, Q and A) with appropriate dimensions. Obviously Edgar's doing pretty well if he can remember all this, and you do tend to wonder why he had so much trouble remembering the formula.

Slide 17



Okay

so we've got our arbitrary formula

And we replace each parameter with the corresponding dimensions. You can see this is where those laws of indices are going to be important, right? Now we're going to figure out what value of the exponents, a and b, you need in order for the dimensions on the left of the equals sign to equal the dimensions on the right. You can see just by looking at it that if the exponents were both 1, then you'd have the right power of TIME on both sides, but there'd be way too much LENGTH on the right hand side because you'd wind up with L5. And obviously you know the answer anyway, but let's go through the motions.

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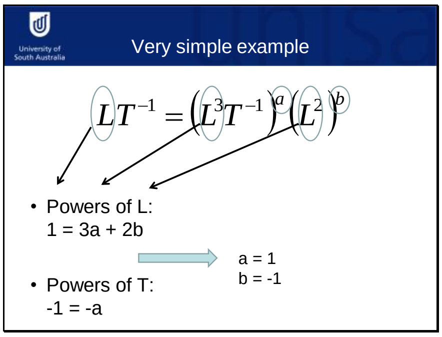

So

here's our dimensional equation

The first thing to do is look at the powers of L

And we've got just L on the left hand side, so that's L to the power of 1

Now on the right hand side once we multiply and add the exponents according to the laws of indices, the total result is going to be L to the power of 3a

Plus 2b.

The powers of T are really easy, just "minus 1" on the left and "minus a" on the right.

So we've got a=1 and substituting that into the first equation we can find b, which is "minus 1".

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Very simple example

- a = 1
- b = -1

$$V = Q^a A^b$$

$$V = Q^1 A^{-1}$$

$$V = \frac{Q}{A} \quad \text{or} \quad Q = AV$$

Putting these values of a and b back into the original formula

We get this

Which is the same as this

Or in its more familiar form, our friend the continuity equation. Great! Now Edgar can continue on with his exam, if he's still got time.

Slide 20

Slightly bigger example (p.355)

- Suppose we want to estimate discharge (Q) from a pump.
- Start by guessing the influential parameters:
  - Impeller diameter (D)
  - Rotational speed (N)
  - Pressure added by the pump (P)
  - Density of the liquid (ρ)
- Assume Q is a function of all these parameters raised to various powers:

$$Q = KN^a D^b P^c \rho^d$$

Well that example was pretty simple since you already knew the answer. Let's try one where you probably don't already know the result.

Say we've got a pump and we want to develop a physical relationship between the pump's discharge and the other significant parameters in the system.

First off we need to guess at what those parameters are going to be. Okay, it needs to be a fairly well-informed guess and possibly this comes from having done some experiments where we changed a few things around, and found that different parameters seemed to influence the discharge. Anyway, at this point we need to at least know enough about the physical system to guess what parameters to include.

So we've got the impeller diameter – don't worry if you're not 100% sure what an impeller is; we learn more about pumps in the next lecture.

Next up's the rotational speed, as in it stands to reason that if you run the pump faster, it probably affects the discharge

Then we expect pressure's going to have something to do with it

As well as the density of the fluid we're pumping

And so we set out by assuming the discharge is an arbitrary function

made up of all four of these parameters raised to various powers. We chuck a K out the front as a dimensionless constant to allow the function to scale up and down if it needs to.

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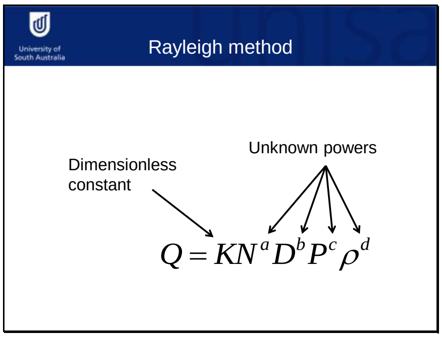

So

we've got our equation here

And all these are the unknown powers that we're going to have to solve to make sure the dimensions are properly balanced.

Like I said, the K is just a scaling factor that we include – for instance, that might be used as a fitting parameter to calibrate the function against experimental data.

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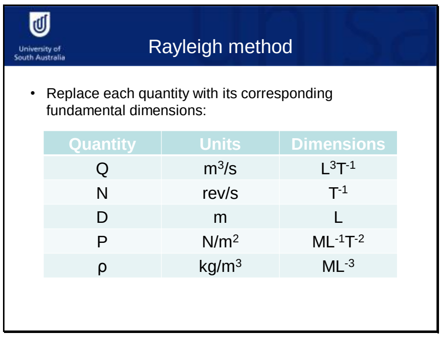

Just like we did with the simple example before, the first step is writing down the dimensions of Q, N, D, P and ρ like this. It helps to write down the units first, and then you might have to refer back to the table of dimensions for different quantities. If you don't have a table of dimensions for different quantities, you can always build up the dimensions from first principles – so for example Newtons refer to a force, and force is mass times acceleration, acceleration is length over time squared and so on. Maybe the trickiest one here is the pump rotational speed, N. That'd be measured in units of revolutions per second (or per time generally – like revs per minute and so on). Now because a "revolution" doesn't actually have a dimension, it's only the "per second" bit that contributes to the dimensions – so that's just T-1.

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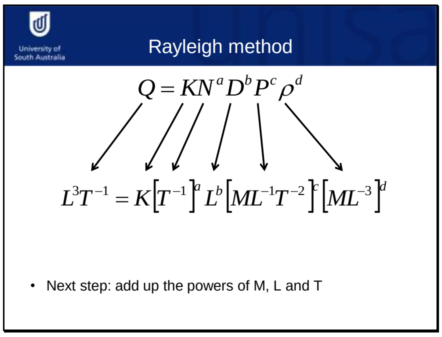

Same as before, we replace each quantity in our function with its corresponding dimensions. Next comes the bit where we figure out the values of the exponents, a, b, c and d to make sure the powers of each dimension balance on both sides of the equals sign.

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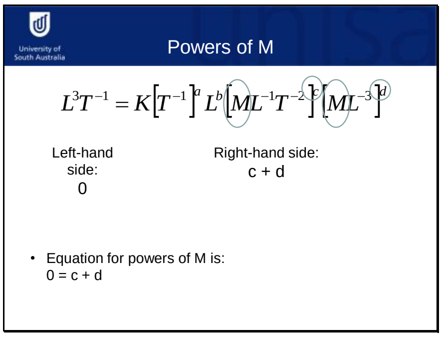

It doesn't matter what order we do it in; let's start with the powers of M. There's no M on the left-hand side so it's got a power of zero.

Here's one on the right hand side, which is a single M being raised to the power c.

Next up we've got another single M raised to the power d.

So the total mass dimension on the right hand side is "M to the c + d".

And now our equation is 0 = c + d.

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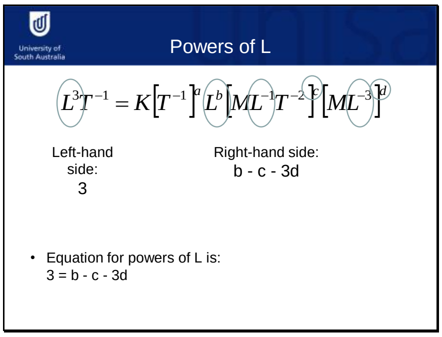

Next up let's do it for the powers of L.

This time on the left hand side there's an L cubed so the left-hand side is 3.

Going through the right-hand side you've got a b

Then a "minus 1" to the c, so that's a minus c when we apply the laws of indices

And then there's a "minus 3" to the d, which is minus 3d.

So pulling that all together we've got b – c – 3d on the right there.

And that all looks like this now, 3 = b – c – 3d.

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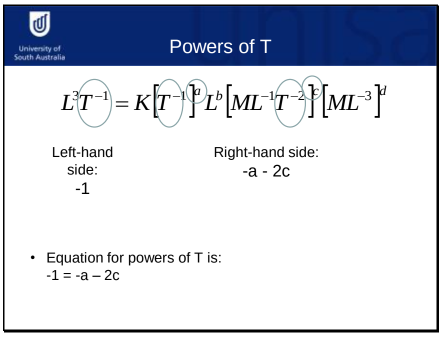

Last of all we do the powers of T,

Where on the left-hand side there's a -1

and on the right hand side there's a "minus a"

And a "minus 2c"

So that's "a minus 2c", obviously

And our overall equation's -1 = -a – 2c.

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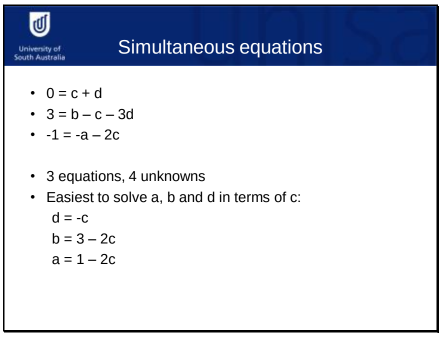

Cool, so now we've got 3 equations that need to be solved to make the dimensions balance.

Now we've got a problem because there are 4 unknowns – so we're not going to get an answer for all of them. Oh well, the best we can do is figure out a way to express three of our unknowns in terms of the fourth, so we should be able to get from the original four unknowns down to 1.

It doesn't really matter which variable we pick to leave unsolved but if you've got one appearing in all of the equations, that's a logical one to use. In this case "c" appears in all 3 equations so we'll try to express a, b and d in terms of c.

The first one's the simplest, just rearrange and it becomes d = -c

Subbing –c in for d in the second equation and rearranging, we get b = 3 – 2c

And lastly we rearrange the third equation to get a = 1 – 2c

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Slightly bigger example (p.355)

- d = -c
- b = 3 – 2c
- a = 1 – 2c
- Substitute into original equation for Q:

$$Q = KN^{a}D^{b}P^{c}\rho^{d}$$

$$Q = KN^{1-2c}D^{3-2c}P^{c}\rho^{-c}$$

The next step is chucking these values

back into the original equation

So that looks like this

The a, b and d terms have all been replaced now by something that just has c as the unknown.

Slide 29

Rearrange the equation

- Separate out the powers of c:

$$Q = KN^{1-2c}D^{3-2c}P^{c}\rho^{-c}$$

$$Q = KNN^{-2c}D^{3}D^{-2c}P^{c}\rho^{-c}$$

We're nearly there – just need to use those laws of indices again to separate out the constant powers from the unknown powers

Starting with the equation with all the messy powers in it

We expand this

so each term's now the product of the parameter raised to the constant part of the exponent

Multiplied by the variable, or unknown, part of the exponent

And we do that for the 3 – 2c bit as well. We don't have to worry about the last two terms because they've got nothing to separate out.

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Rearrange the equation

$$Q = KNN^{-2c}D^{3}D^{-2c}P^{c}\rho^{-c}$$

$$Q = \left(ND^{3}\right)K\left(N^{-2}D^{-2}P\rho^{-1}\right)^{c}$$

$$\frac{Q}{ND^{3}} = K\left(\frac{P}{N^{2}D^{2}\rho}\right)^{c} = f\left(\frac{P}{N^{2}D^{2}\rho}\right)$$

where $f(x) = Kx^{c}$

So now we're going to tidy up the equation – just rearranging it,

by gathering up the constant power parts and separating them from the variable power parts. We can conveniently then take out a common exponential factor, which is "c", so we've got now a cluster of parameters in brackets, all being raised to the power of c.

There's nothing wrong with leaving it there but for neatness it tends to be rearranged a little bit more so you put all the non-variable exponent terms together, in this case Q on ND3 on the left hand side and P on N2D2ρ all to the power of c on the right.

If we want to, we can then just say it's equal to some function "f" of that stuff inside the brackets, where we specify that the function's an arbitrary power relationship. That's as far as we can get with this one – there's no way to solve for all four exponents so you're always going to end up with some unknown power. But it's not a bad outcome – at least you know the form of the relationship now, it's some sort of basic power relationship between those two groups.

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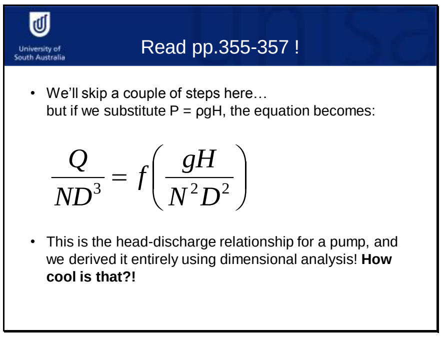

Again, we'll dive into pump stuff in the next lecture, but basically if you sub in the expression for pressure,

it turns into a nifty function relating discharge to head, along with the rotational speed and diameter of the rotor.

This is actually a fundamental physical property of pumps, known as the head-discharge relationship, and here we derived it entirely using dimensional analysis. You still might be concerned that we don't know exactly what the equation is, because we've got this sort of undefined function "f" there. But what we know from the previous slide is that it's a particular type of function – it's a power function – so we're not completely in the dark.

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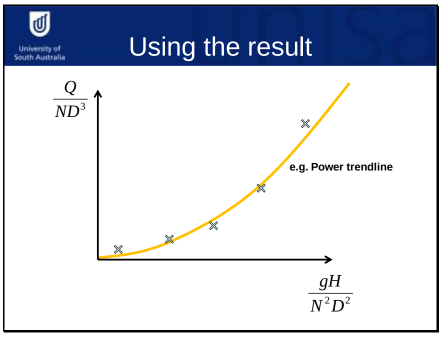


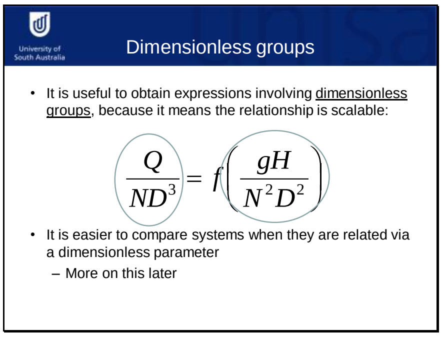
Actually, you can do a lot with the relationship we just derived. For instance,

if you've got some experimental data from testing a particular type of pump, perhaps where you tried it at different speeds and with different sized rotors, and measured the discharge and head, then you can plot it all up on a single set of axes like this

it's quite useful to be able to do that, because otherwise you'd have 1, 2, 3, 4 different parameters, Q, N, D and H, that you'd have to graph somehow against each other. And once you've graphed these, you at least have confidence that it's going to look like a power-type curve. If it doesn't look like a power type function, you know something's gone wrong and perhaps that'd mean your dimensional analysis was incomplete or there was something wrong in your experiment.

So assuming it plots up okay, you could work out the actual relationship that corresponds to the particular pump you've tested, by applying a line of best fit to your data. In a spreadsheet program like Excel this'd be as simple as clicking "Power" in the trendline box and it'd do the rest for you. But without the dimensional analysis to begin with, you wouldn't have a clue what to do with your data, nor would you know what to expect even if you could plot it up.

Slide 33



The groovy thing with all this dimensional analysis

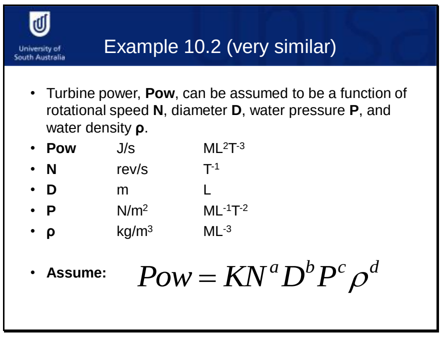
is that to some extent, it lends itself to generating what we call "dimensionless groups" –

in this case the Q on ND3 is dimensionless, and so is the gH on N2D2. That means if you double Q but also ND3, you haven't actually changed anything, so the other side's going to stay the same. It's a bit hard to get your head around, but it's this relationship that allows you to lump together a bunch of different parameters into a smaller number of key functional groups. That's why you could do all sorts of experiments varying the discharge, head, speed and size of the pump and still expect the results to plot up on a single graph like we looked at on the previous page.

Dimensionless groups also make it easier to compare systems across different scales – if you have one pump setup with a particular value of Q on ND3, and you want to know how fast to run a smaller pump to achieve the same discharge, you could use the Q on ND3 relationship to achieve it; that is, figure out what value of N you need with a smaller D value to keep Q the same.

Some of this should become a little clearer in the next lecture.

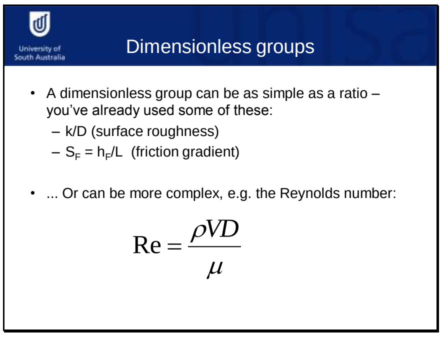
Slide 34



Here's a similar example about turbine power. We can assume that the power produced by a hydroelectric turbine's going to be a function of the speed, diameter, pressure and fluid density – so it's basically the same set of key parameters as the pump discharge, which isn't that surprising since a turbine's effectively just a pump working in reverse. The units and dimensions are all given here

So you can kick things off in basically the same way as with the pump example. The only difference is the dimensions for power, which are different to the dimensions for pump discharge. So once you've subbed in all the dimensions your simultaneous equations are going look different to the ones we got for the pump. Apart from that it's exactly the same method. See how you go.

Slide 35



Just getting back to dimensionless groups

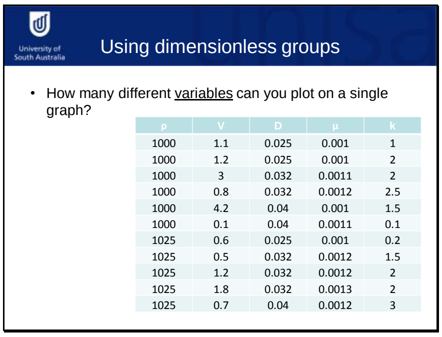
We've already used dimensionless groups in this course, like,

surface roughness which is a length over a length, so that's dimensionless

and although it didn't feature heavily, there was a friction gradient in pipe flow which is dimensionless too since friction head loss is expressed as a length.

And of course our friend the Reynolds number (Re) is a dimensionless number – if you work out the dimensions of the top and bottom terms you find they cancel out.

Slide 36

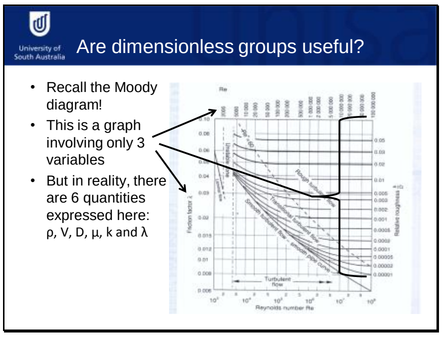


ρ	V	D	μ	k
1000	1.1	0.025	0.001	1
1000	1.2	0.025	0.001	2
1000	3	0.032	0.0011	2
1000	0.8	0.032	0.0012	2.5
1000	4.2	0.04	0.001	1.5
1000	0.1	0.04	0.0011	0.1
1025	0.6	0.025	0.001	0.2
1025	0.5	0.032	0.0012	1.5
1025	1.2	0.032	0.0012	2
1025	1.8	0.032	0.0013	2
1025	0.7	0.04	0.0012	3

Imagine an experiment where you're varying all sorts of different parameters and studying the results

Maybe you've got the ability to run water of different densities and viscosities through a bunch of pipes of different diameter, and with different absolute roughness – so you end up recording everything and you've got this huge amount of data to process. How can you bring all of that together in an understandable way? Do you plot random graphs of density versus absolute roughness, viscosity versus velocity, and wait until something leaps out? Obviously the answer is to use dimensionless groups. And you've actually already seen the result.

Slide 37



It's the Moody diagram!

In general a 2-D graph is limited to plotting 3 quantities –

the quantity on the x-axis, which in this case is the Reynolds number

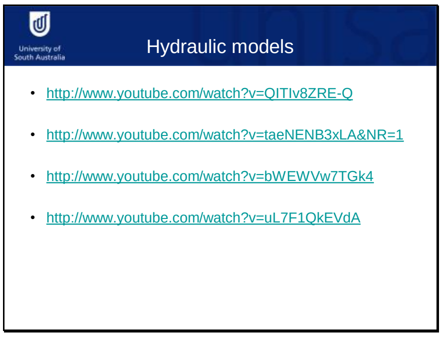
Then you've got the y-axis, which is the friction factor lambda

And then to get a third quantity on a 2-D graph you have to plot multiple lines on the graph, and in this case our different lines are for the relative roughness.

But because we've used the dimensionless groups, in actual fact we've managed to squeeze 6 different physical parameters onto this single graph, which is even one more than what we had on the table on the last slide. So any combination of those 6 individual parameters can be represented on this graph via the dimensionless groups.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 38



We're about to move on to see how we can use dimensionless groups for designing hydraulic models. Before we do that, have a quick look at some of the sorts of models people create. We're talking about real, physical models now, rather than computer simulations.

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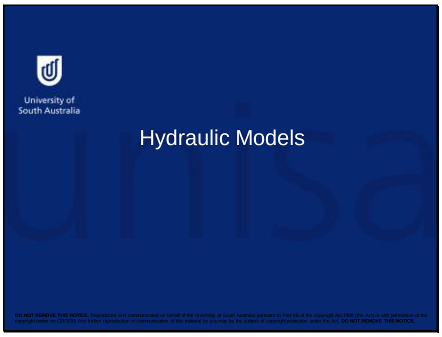
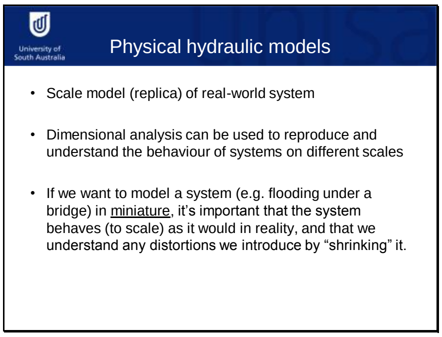


Okay, now to look at Hydraulic models

Slide 40



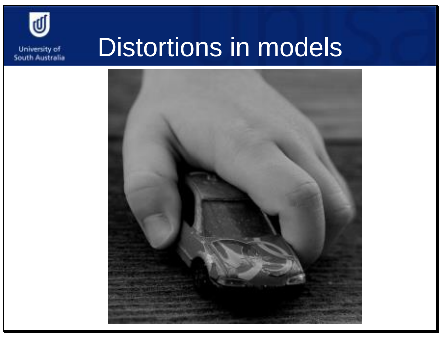
It's common nowadays to employ pretty sophisticated computer simulations of lots of different situations. But no matter how sophisticated your computer program is, eventually it needs to be compared to the real world.

Sometimes you can compare computer models with full-scale real world settings, and other times you might need to build a scaled-down replica, especially if you're trying to test out a structure that hasn't been built yet or you want to simulate a natural disaster like a flood. Or, you might simply be working on a problem that's just too darn complex to set up on a computer and your only option is to head into the lab and build it in miniature.

One really useful application of dimensional analysis, and in particular the dimensionless groups produced by the process, is that we can use it to scale down a real-world system in a physically realistic way.

Most importantly, we need to understand how individual parameters in the model scale down, and whether by shrinking the physical geometry of the system we introduce any distortions that need to be corrected.

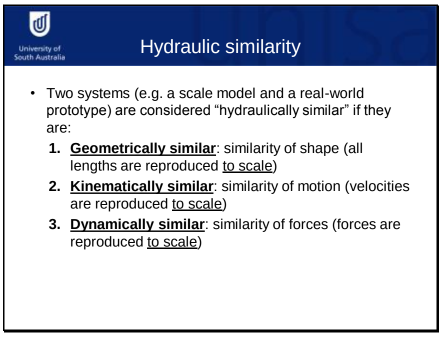
Slide 41



A simple way to imagine distortions in models is to consider a matchbox car. Imagine it's made of metal and is a perfect replica of a real-world car in terms of the exterior geometry, and it's 1/50th of the actual size. Now let's say a grown adult can tread on the car without breaking it. And let's say the adult weighed 70 kilos. So is that a realistic, scale representation of the actual crushing resistance? In other words, if you multiply the 70 kilos by 50 and get 3.5 tonnes, does that mean that the real world car could cope with 3 and a half tonnes bearing down on it? Probably not. And that's because some physical distortions have been introduced by scaling down the geometry and they haven't been accounted for in the design of the crushing experiment. It might be the case, for example, that by scaling down the car's size, for some reason you've greatly increased its shell strength so to simulate a realistic load relative to the strength, you have to distort the load by the same amount.

Image source - http://farm2.staticflickr.com/1332/710807297_9efa641a55_z.jpg?zz=1

Slide 42



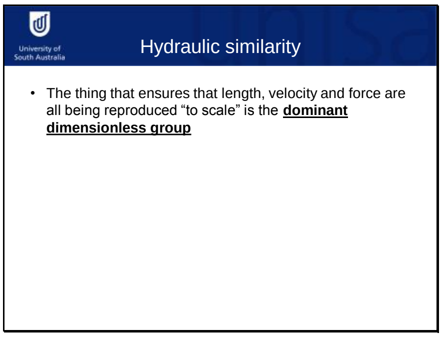
Getting back to hydraulic models, when we’re designing a model, in order to make sure the model behaves like a proper scaled down replica of the real world system, we have to look at what’s called hydraulic similarity.

The first type of similarity refers to geometry and this is the most obvious – we need the physical form of the replica to actually represent the physical form of the real world system. But like the matchbox car example, simply reproducing the geometry isn’t likely to be sufficient.

So the next type of similarity looks at motion, and it’s called kinematic similarity. This means you’ve got to figure out how to make the velocities in the model actually work at an appropriate scale to be physically consistent with the velocities in the full-sized system. We’ll look at this in a minute because it’s generally not a matter of scaling down the velocity by the same factor as the length dimensions.

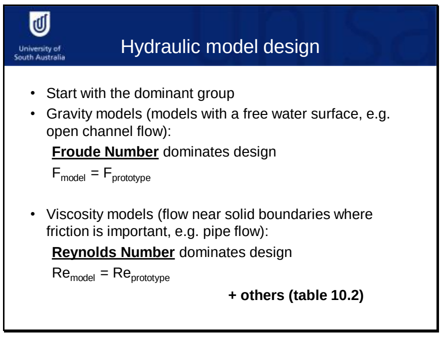
Lastly we look at similarity of forces, or dynamic similarity. This is like the matchbox car example where we need to make sure that for a given geometric scaling, we get the forces scaled appropriately so we’re not over- or under-predicting the performance of the system.

Slide 43



The thing that ensures that length, velocity and force are all being reproduced “to scale” is the dominant dimensionless group, and this depends on the type of model you’re looking at.

Slide 44



So in fact,

When you're designing a model it's important to figure out the dominant dimensionless group first, and work from there.

So-called "gravity models" are models of open channel flow, or other situations with a free surface.

For these models it's the Froude number that governs the model design – you haven't been introduced to the Froude number yet, but it's an important property of open channel flow that dictates the characteristics of the flow.

Which basically means you have to tweak the values in your scaled model to ensure that you've got the same Froude number as you'd have in the real world.

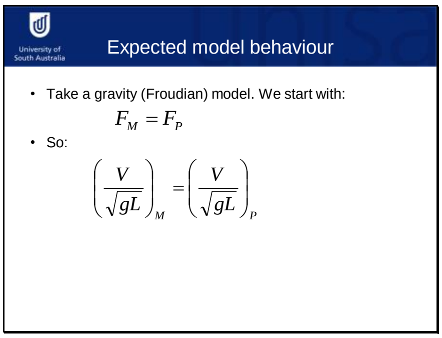
On the other hand if you're trying to simulate a pipeline or some other situation where the important process it the laminar and turbulent flow interaction between the fluid and a solid surface,

Then you're going to need to use the Reynolds number

And like in gravity models, you need to work things out so that your model gives the same Reynolds number as the real world setting.

There are other model types listed in the textbook if you're interested.

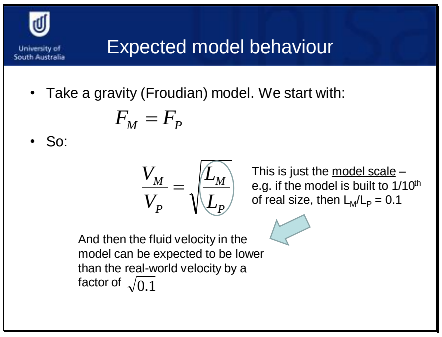
Slide 45



So take a gravity model as an example. As we said, the Froude number of the model needs to be exactly the same as the Froude number in the real world. Oh, by the way, the way the textbook describes the real world scenario is with the subscript "P" here, meaning "prototype" – so they're assuming the model is a scale replica of a full scale prototype. Hopefully that doesn't confuse you too much. Okay so the Froude number is V over root G L and like I said, you haven't been introduced to this yet. But anyway this has got to be equal in the model and the full-scale prototype if we're going to get the model producing the expected behaviour.

By rearranging the equation, starting with equal Froude numbers, we can work out the ratio of the modelled velocities to the real-world velocity as a function of the geometric ratio.

Slide 46

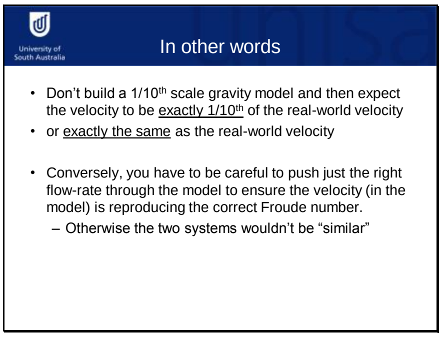


You can see that properties like velocity generally won't scale exactly the same as the geometric scale.

if our scale model was, say, built to 1/10th the physical size of the real-world prototype,

then we'd need to set up the modelled velocity of the fluid so that the velocity ratio is the square root of 1 over 10.

Slide 47



The take home message here is

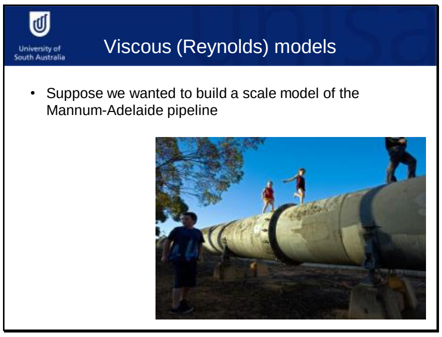
don't build a 1/10th scale gravity model and running a velocity exactly 1/10th of the real-world velocity.

And definitely don't use the real-world velocity!

The idea is that a key part of hydraulic model design involves figuring out the appropriate flow rate that preserves the Froude number

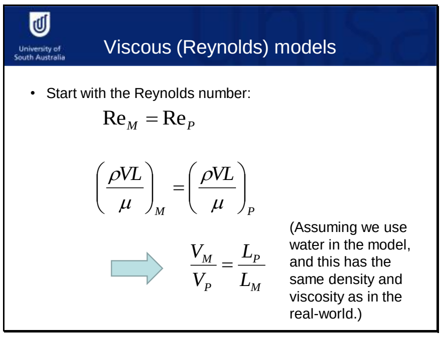
And this gives us the hydraulic similarity we need to get useful model results.

Slide 48



By way of another example, imagine we're interested in building a scale model of the Mannum-Adelaide pipeline in South Australia

Slide 49



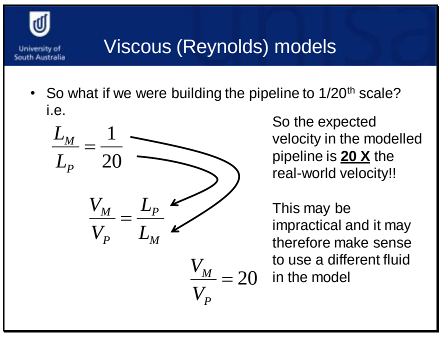
This times it's a pipe flow model, so we need to kick things off with the Reynolds number. Again, the Reynolds number in the model has to equal the Reynolds number in the full-scale prototype, or in this case it's actually the real system.

So we plug in the values that make up the Reynolds number of the model and prototypes.

Let's say in the first instance, we're planning to pump water through our model – this is going to effectively fix both the density and the viscosity, making them equal in both the model and the real-world case

So then, rearranging the equations, the velocity ratio is actually the inverse of the geometric scale.

Slide 50



That could be a real practical challenge – I mean, what if we're building the pipeline to 1/20th scale? It's a 1-metre pipe so that's a 50-millimetre pipe when it's scaled down.

So LM on LP is 1 on 20

And VM on VP is the inverse

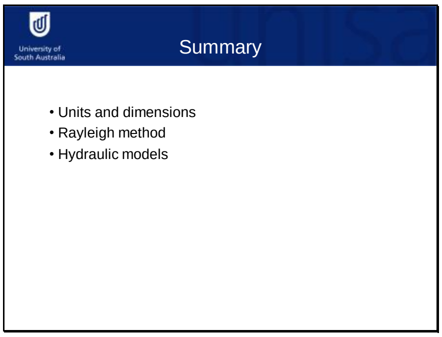
So that means the 20's going to end up on top!

In other words

we need to crank 20 times the real-world velocity through our little model pipeline. It's not quite what you might expect, is it?

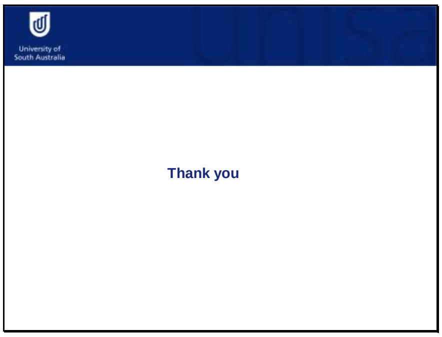
And it’s likely that this would be quite impractical to achieve, which means our plans to use water might need to be revised. Perhaps a different density and/or viscosity fluid could be found that’d satisfy the Reynolds number without inducing ridiculous model velocities.

Slide 51



So in summary, we’ve looked at units and dimensions, learnt how to develop functional relationships using the Rayleigh method, and applied dimensionless groups to hydraulic model design.

Slide 52



If you’ve got any questions need further clarification, please post a question or comment on the Discussion Forum.