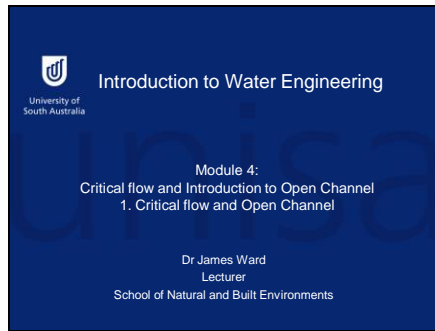


Introduction to Water Engineering

Slide 1

A blue slide with white text. The University of South Australia logo is in the top left. The title 'Introduction to Water Engineering' is centered. Below it, 'Module 4: Critical flow and Introduction to Open Channel' and '1. Critical flow and Open Channel' are listed. At the bottom, the lecturer's name and affiliation are provided.

University of South Australia

Introduction to Water Engineering

Module 4:
Critical flow and Introduction to Open Channel
1. Critical flow and Open Channel

Dr James Ward
Lecturer
School of Natural and Built Environments

Slide 2

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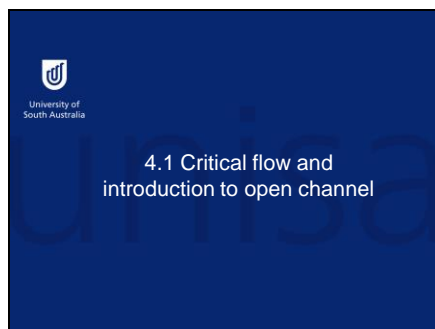
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Slide 3

A blue slide with white text. The University of South Australia logo is in the top left. The title '4.1 Critical flow and introduction to open channel' is centered.

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4.1 Critical flow and introduction to open channel

Let's start by looking at critical flow and introduction to open channel.

Slide 4

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Intended Learning Outcomes

At the end of this section, you will be able to:-

- Predict Critical flow
- Calculate Froude number
- Determine Hydraulic jump

The learning outcomes are presented here – we will look at critical flow, Froude number and hydraulic jump.

Image source:

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Slide 5

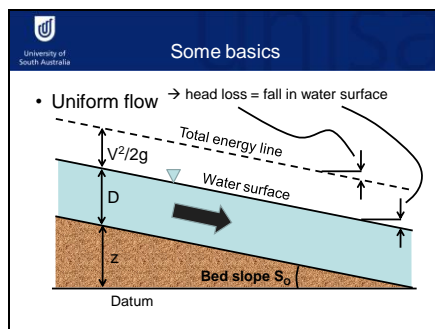


The main difference between open channel flow and pipe flow is that the water surface in an open channel always has atmospheric pressure. So for instance a half-full sewer or stormwater pipe always has atmospheric pressure inside on the water surface. Open channels can be in the form of visible large channels, or partially full pipes. In a large open channel, like a creek or a drain, the cross section might be irregular and can vary significantly along the channel's length. We're going to focus here on subcritical and supercritical flow and what this means for analysing flow in open channels. This forms a foundation for more advanced concepts in civil engineering hydraulics.

Image source -

http://s0.geograph.org.uk/geophotos/02/79/40/2794069_3c647b56.jpg

Slide 6



The driver of flow in an open channel is gravity

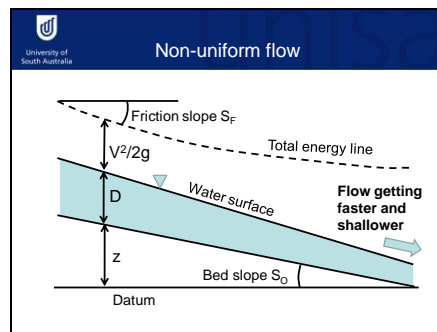
so we need to consider the bed slope
Let's start by considering uniform flow. In uniform flow, the flow depth is constant so the water surface follows the bed slope

In uniform flow we have a constant cross-sectional area which means uniform velocity along the channel length.

As there's no change in velocity head, the total energy line goes down with the bed slope.

Total head loss is equal to the fall in water surface.

Slide 7



Okay, now let's consider non-uniform flow. Again we need to look at the bed height and slope. But in non-uniform flow the water depth is not constant. In this picture, velocity is increasing with bed slope as the flow is getting faster and shallower. As V increases, the square of V increases even more, So the total energy line is a curved line and it doesn't follow the bed slope. The slope of the total energy line is called the friction slope.

Slide 8

Introducing the Froude number

- The Froude number, F , is given by:

$$F = \frac{V}{\sqrt{gD}}$$
- When:
 - $F < 1 \rightarrow$ subcritical flow
 - $F = 1 \rightarrow$ critical flow
 - $F > 1 \rightarrow$ supercritical flow

Flow in open channels can be deep and slow, or shallow and fast. We use the terms "subcritical" and "supercritical" to distinguish these types of flow. To determine whether flow in a particular channel is subcritical or supercritical we use the Froude number, velocity divided by square root of gravity times depth of water. If this ratio is less than one, it effectively means the velocity is smaller and depth is larger, which is subcritical flow. If the value is greater than one, the large velocity and smaller depth imply fast moving and shallow flow, which is supercritical.

Slide 9

What is critical flow?

- We have to start with the energy equation
- At any point along a channel, the total energy is given by:

$$z + \alpha \frac{V^2}{2g} + \frac{P}{\rho g}$$
- Where α is the energy coefficient

To understand critical flow, we need to start with the energy equation. Let's take a point somewhere along a channel. This is no different from the energy equation used in pipe flow. You'll notice there is an energy coefficient (alpha) in this form of the equation, which just accounts for real-world variations in velocity and is typically close to one.

Slide 10

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Energy Equation

- Consider a point at the surface, so $P = 0$ (atmospheric)
- z = the height of the bottom of the channel and D = depth of the channel, so the energy equation for a point at the surface is:

$$z + D + \alpha \frac{V^2}{2g}$$

In fact, this relationship holds at any depth (not just the surface).

Now let's consider a point on the surface, where we know the pressure is zero, or atmospheric.

Z is the height of the channel bed and D is the depth of the channel.

So, the energy equation is z plus D plus velocity head.

This equation holds at depths below the surface too, because as D decreases P increases in proportion.

Slide 11

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Specific energy

- Specific energy** is simply the total energy minus the elevation of the channel bed (z):

$$E = D + \alpha \frac{V^2}{2g}$$

- Now, as $V = Q/A$:

$$E = D + \alpha \frac{Q^2}{2gA^2} = D + \frac{Q^2}{2gA^2}$$

(assuming $\alpha = 1$)

Now we're going to introduce a term called specific energy, which is the energy relative to the channel bed. Specific energy is water depth plus velocity head, so it's the same as the energy equation except that we've neglected Z .

Using $V=Q/A$, we get an equation for specific energy based on water depth, flow rate and cross-sectional area.

You should note that the term A accounts for both the channel width and the depth of the flow, so depth really occurs twice in this equation.

Slide 12

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Uniform flow

- Uniform flow

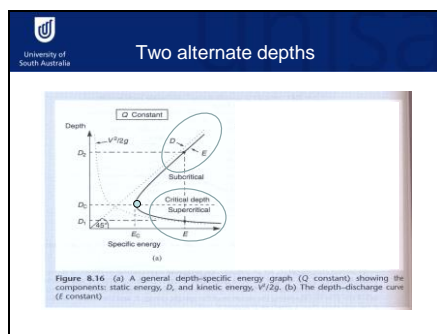
Unlike total energy, specific energy is the energy measured relative to the channel bed. In uniform flow, water depth and velocity remain constant and therefore, specific energy remains constant along the entire flow path.

Slide 13

A 4 minute video explaining the impact of depth using an excel spread sheet is provided – select play to begin.

Image source
http://farm5.staticflickr.com/4062/4288728056_076da425ab_z.jpg?zz=1

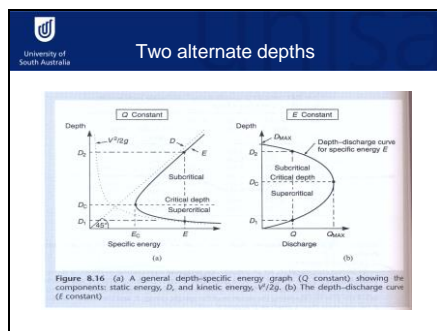
Slide 14



From the specific energy equation we can see there are two possible flow depths for each value of E. The figure here illustrates this point with a general depth-specific energy graph. In subcritical flow, specific energy is dominated by depth of water and in supercritical flow, it is dominated by velocity head. This graph illustrates that for a given value of specific energy, there's a shallow fast moving version and a deep slow moving version of the flow. The exception is critical flow, which is flow at the critical depth.

Image source- Les Hamill 2011, Understanding hydraulics.

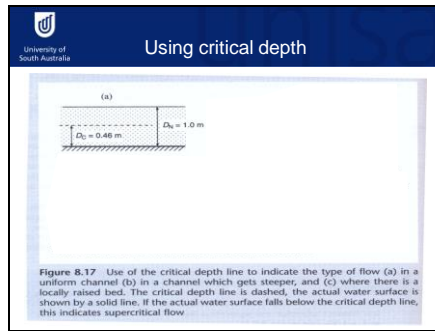
Slide 15



We can see the point illustrated in two different ways. The left side graph is for constant discharge, showing that if we take a line of constant E we get two alternate flow depths. The right side graph is for constant E, and again if we take a line through a constant flow rate Q, we get two alternate depths – one fast and shallow, one slow and deep.

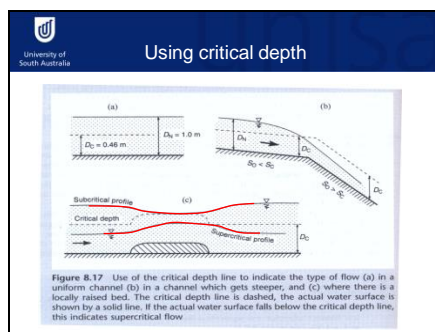
Image source- Les Hamill 2011, Understanding hydraulics.

Slide 16



If water is flowing higher than critical depth, it is subcritical. Channel which gets steeper, the flow is supercritical.

Slide 17



We denote critical flow depth D_c . If water is flowing at a depth greater than D_c , it is subcritical, like in the top left image here.

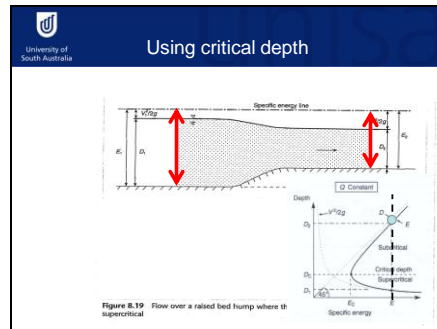
We can also define a critical slope S_c – when a channel has a bed slope shallower than S_c , the flow is subcritical but if it gets steeper it could induced supercritical flow. We'll look at critical slope a bit later.

The bottom image is quite interesting. It shows a bump in the channel bed, with two different outcomes depending on whether flow is supercritical or subcritical. In subcritical flow, the bump might reduce the flow depth, inducing a higher velocity. This can result in a localised depression in the water surface. On the other hand if flow before the bump is already supercritical, it can possibly flow up and over the bump in the bed slope.

The take-home message here is that depending on whether flow is subcritical or supercritical, there will be different results from perturbations like bumps or changes in bed slope.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 18



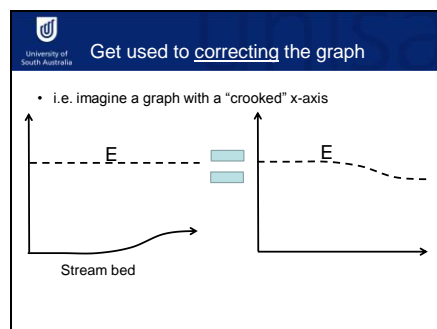
Let's look at these concepts a little more closely. Here we've got water at subcritical flow, flowing in a channel towards a raised bump. Upstream, the specific energy is mostly made up of the large depth plus a little bit of velocity head reflecting the slow flow. If we go back to that graph of depth-specific energy, the upstream flow might be around here.

When the water flows up over the bump, we raise the bed of the channel which means we have less specific energy.

So if we follow the curve we see we've now got a lower flow depth, in other words D_2 is smaller than D_1 – depending on how big the bump is, this could result in a significant depression in the the water surface above.

Image source- Les Hamill 2011, Understanding hydraulics.

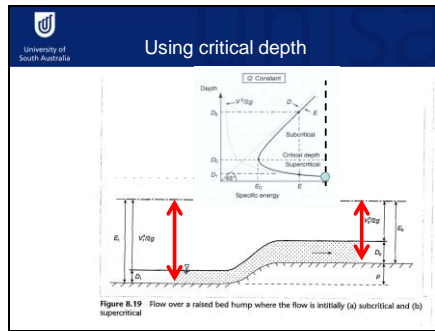
Slide 19



In the previous example the specific energy line was straight and horizontal like the graph here, but the stream bed changed because of the bump.

You need to remember that specific energy is measured relative to the channel bed, so the even though it looked like E was constant it was really changing.

Slide 20



Let's go back to our bump example but this time start with supercritical flow. Now the upstream specific energy comprises a little bit depth and a large amount of velocity head. The initial condition might be around here on the graph. When the flow hits the bump, just like the subcritical example there will be a reduction in specific energy. This means slightly deeper flow and less velocity head.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 21

- In subcritical flow, downstream conditions govern upstream effects
- In supercritical flow, disturbances cannot propagate upstream

The YouTube videos show experiments on subcritical, critical and supercritical flow.

One of the most important aspects of critical flow is that in subcritical flow conditions, disturbances introduced downstream cause changes to flow depth upstream. In supercritical flow, the water is moving too fast for disturbances to propagate back upstream. This has important implications in engineering design.

Slide 22

- At critical depth, E is minimised:
→ $dE/dD = 0$
- Derivations of equations 8.25 -8.27

Figure 8.21 An irregular channel with no specific shape and cross-sectional area, A , when the discharge is Q

Given the importance of subcritical and supercritical flow, we need equations to help us determine what flow conditions to expect in a channel. Earlier we used Froude number but this only holds in rectangular channels. The derivation of equations for critical flow conditions in irregular channels can be found in the textbook.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 23

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Calculating critical flow conditions

- At critical depth D_C the governing equation is:

$$\frac{Q^2 B_{SC}}{g A_C^3} = 1$$

These correspond to the surface width & cross-sectional area under critical flow conditions

One useful equation relates the water surface width B_{sc} to the flow rate Q and cross-sectional area of flow A_c . The subscript "C" denotes critical flow conditions and this relationship only holds under critical flow.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 24

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Calculating critical flow conditions

- Hydraulic mean critical depth D_{MC}

$$D_{MC} = \frac{A_C}{B_{SC}}$$

$$D_{MC} = \frac{Q^2}{g A_C^2}$$

May necessitate an iterative / trial-and-error solution if D_{MC} and A_C are both unknown

Lots of critical flow calculations need some sort of average value of critical depth. For an irregular shaped channel, we get the hydraulic mean critical depth, or DMC, by dividing the cross-sectional flow area A_C

By the surface flow width B_{SC} . This treats the channel like an equivalent rectangular section. Linking this equation with the one from the previous slide gives DMC in terms of flow depth and area. We might need to use trial and error if both DMC and A_C are unknown.

Slide 25

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Recall the Froude number:

$$F = \frac{V}{\sqrt{gD}}$$

- $F = 1$ for critical flow conditions.
- Substitute D_{MC} for D and $V=Q/A$ from the continuity equation:

$$\frac{\left(\frac{Q}{A_C}\right)}{\sqrt{g D_{MC}}} = 1 \quad \text{and as } D_{MC} = A_C / B_{SC} \quad \frac{\left(\frac{Q}{A_C}\right)}{\sqrt{g \frac{A_C}{B_{SC}}}} = 1 \Rightarrow Q = \sqrt{g \frac{A_C^3}{B_{SC}}}$$

Earlier on, we introduced the Froude number.

When $F = 1$, the flow condition is critical.

Substituting DMC and Q/A into the equation for the Froude number,

And the equation for DMC from the last slide,

we can derive the equation for the critical flow rate.

Slide 26

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Critical velocity

- Start with these:

$$V_C = \frac{Q}{A_C} \quad Q = \sqrt{g \frac{A_C^3}{B_{SC}}}$$

$$\Rightarrow V_C = \frac{\sqrt{g \frac{A_C^3}{B_{SC}}}}{A_C} = \sqrt{g \frac{A_C^3}{B_{SC} A_C^2}} = \sqrt{g \frac{A_C}{B_{SC}}}$$

If we want to know the velocity under critical flow conditions we can use the continuity equation and the equation for critical flow from the last slide.

Substituting for Q, we get an expression for critical velocity VC in terms of AC and BSC

Slide 27

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Critical velocity

- Recall that $\frac{A_C}{B_{SC}} = D_{MC}$
- So now: $V_C = \sqrt{g \frac{A_C}{B_{SC}}} = \sqrt{g D_{MC}}$
- i.e. $\frac{V_C}{\sqrt{g D_{MC}}} = 1$ This is just the Froude number, F = 1 !!! ©

Okay, so let's just check to make sure it all works, starting with our expression for DMC

So substitute into VC

Which gives us the Froude number equal to 1 in the critical condition.

Slide 28

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Example 8.11

- Q = 2.144 m³/s
- Is this critical flow?
- If critical, then: $\frac{Q^2 B_{SC}}{g A_C^3} = 1$

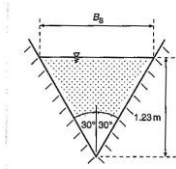


Figure 8.22

We can use these equations to test whether a particular flow rate is critical or not.

In this example we're given a flow of 2.144 m³/s.

Is it critical?

We can test it using this equation – if it comes out equal to 1, then flow is critical. You'll have to calculate BS and AC from the height and angles in the drawing. You can find the full workout procedure in the text book.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 29

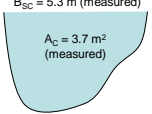
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Example 8.12

- Irregular cross-section
- Known to be at critical depth
- $Q = ???$

$B_{SC} = 5.3 \text{ m (measured)}$

$A_c = 3.7 \text{ m}^2 \text{ (measured)}$



$$Q = \sqrt{g \frac{A_c^3}{B_{SC}}}$$


Here's another example.

This time we've got an irregular-shaped channel and we know the water's flowing at critical depth.

We've got measurements of the channel cross-sectional area and surface water width.

We want to know flowrate Q .

Now all we need to do is find the equation that brings AC and BSC together with our unknown Q . Try this one.

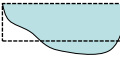
You can find the full workout procedure in the textbook.


Slide 30

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Example 8.13

- Irregular cross-section
- Mean hydraulic depth $D_M = 0.81 \text{ m}$
- Mean velocity $V = 2.97 \text{ ms}^{-1}$
- Subcritical or supercritical?
- What are the limitations of your conclusion?



$$F = \frac{V}{\sqrt{gD}}$$


Alright, now consider this one.

Another irregular shaped channel, this time we don't know whether it's subcritical or supercritical.

What we do know is the mean depth of flow, DM

And the mean velocity V .

To work out whether it's subcritical or supercritical we can use the Froude number.

Once you've worked out the Froude number, think about any limitations to the conclusions you make about the flow. Check the workout procedure in the textbook for more info.

Slide 31

Critical slope

- You should learn more about this in later courses (or feel free to read the rest of the chapter)
- For now, we'll just pluck the equation out of the book:

$$S_c = \frac{V_c^2 n^2}{R_c^{4/3}}$$

Critical slope
Manning's n
(a measure of surface roughness)

\
Critical hydraulic radius

The critical slope is the bed slope required to maintain critical conditions. Any steeper and flow's going to accelerate to supercritical, shallow and fast conditions. If bed slope is less than critical, flow starts to back up, producing deeper, slower subcritical conditions.

We won't cover the derivation this time round but you should learn more if you study hydraulics later.

Using the critical slope equation from the textbook you can see SC

depends on the critical velocity,

as well as a term called the critical hydraulic radius

and a measure of roughness called "Manning's n".

The Manning's n in open channel flow is very similar to the K factor in pipe losses. The rougher the channel surface, the greater the slope needs to be to maintain critical flow conditions.

Slide 32

Example 8.14

- What bed slope would maintain critical flow depth?
- Manning $n = 0.015 \text{ s/m}^{1/3}$

$$S_c = \frac{V_c^2 n^2}{R_c^{4/3}}$$

Important: Hydraulic radius $R_c = \frac{A_c}{P_c}$

"Wetted perimeter"

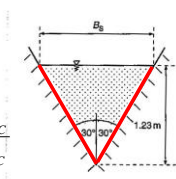


Figure 8.22

Here's an example of critical slope. It's the same channel as we looked at in Example 8.11, but now we're given a Manning's n value and asked what the bed slope would need to be to maintain critical flow.

You'll need to calculate the critical velocity if you didn't already in the earlier example.

An important bit of extra information is that RC is the hydraulic radius, which is found by dividing the area of flow by the wetted perimeter.

The wetted perimeter's the total perimeter in contact with the water.

The workout procedure of Example 8.22 is available in the text book.

Image source- Les Hamill 2011, Understanding hydraulics.

Slide 33



Flow transition from subcritical to supercritical

requires a velocity increase, which can be done by increasing bed slope or rapidly decreasing area, such as by introducing a bump in the flow.

This type of transition is relatively smooth.

Image source - http://s0.geograph.org.uk/photos/44/13/441399_cc75bac8.jpg

Slide 34



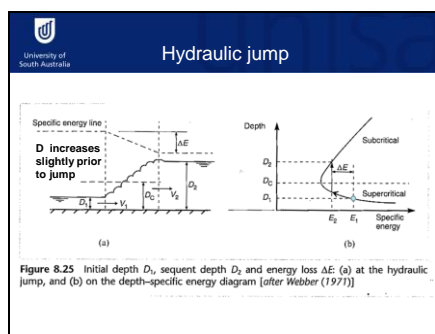
Transitioning from supercritical to subcritical gives rise to a phenomenon called a hydraulic jump.

This is the transition of flow from fast, shallow flow to deep, slow flow. A good example is water coming from a steep slope like the spillway of a dam, then transitioning back to deep, slow moving subcritical flow in the flatter stream.

These flow transitions are often very turbulent and there's a substantial loss of energy.

Image source - http://upload.wikimedia.org/wikipedia/commons/c/cd/South_Para_spillway.jpg

Slide 35

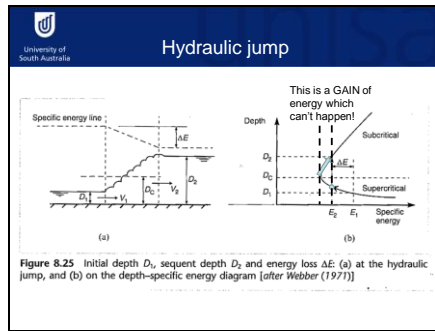


Let's use the depth-specific energy graph to help explain what happens in a hydraulic jump.

We start in supercritical conditions here. Depth increases slightly prior to jump, which results in the loss of energy.

The jump is what happens when we hop over to the subcritical position on the graph and you can think of the turbulence as being a result of temporarily departing from the depth-specific energy curve.

Slide 36



Okay, you might be wondering why the flow can't just smoothly follow the curve around like this.

The problem is that this part of the process involves moving from low to high energy

And without an external source of energy, the process can't simply gain energy

That's why the only way to transition from supercritical to subcritical flow involves the jump.

Slide 37

Dimensions of a hydraulic jump

Relationship between the flow heights before (D_1) and after (D_2)

$$D_1 = \frac{D_2}{2} \left(\sqrt{1 + 8F_2^2} - 1 \right)$$

$$D_2 = \frac{D_1}{2} \left(\sqrt{1 + 8F_1^2} - 1 \right)$$

Energy loss \rightarrow

$$\Delta E = \frac{(D_2 - D_1)^3}{4D_1 D_2}$$

There are two useful equations that we can use to relate the supercritical upstream flow depth to the subcritical depth downstream from the hydraulic jump.

The bottom equation here shows the energy loss (ΔE) due to the hydraulic jump.

Slide 38

Dimensions of a hydraulic jump

Relative height

$$\frac{(D_2 - D_1)}{E_1} = \frac{\sqrt{1 + 8F_1^2} - 3}{F_1^2 + 2}$$

Another term we might like to use is the relative height of hydraulic jump, as shown here.

Slide 39

F_1	L_j
<1.7	<i>Undular</i>
1.7	$4.0D_2$
2.0	$4.4D_2$
2.5	$4.8D_2$
3.0	$5.3D_2$
4.0	$5.8D_2$
5.0	$6.0D_2$
7.0	$6.2D_2$
14.0	$6.0D_2$
20.0	$5.5D_2$

Approximate length of the hydraulic jump (determined experimentally)


By their nature hydraulic jumps are turbulent and chaotic, but some people have gathered experimental data to help estimate the physical length of the jump. If the Froude number upstream (F_1) is less than 1.7, the flow's fairly close to critical and the jump won't be properly turbulent. But with larger Froude numbers we get decent supercritical conditions and we can estimate the jump length as a multiple of the downstream flow depth D_2 .

Slide 40

Example 8.18

- Channel 5.0 m wide
- $D = 0.65$ m, $Q = 19.0$ m³/s
- Channel goes horizontal

→ will a jump occur?
→ Determine ΔE , $D_2 - D_1$ and L_j



Here's an example from the textbook. The situation involves flow coming down a steep channel, like a spillway. It then goes out into a horizontal channel and we want to know whether the conditions are going to lead to a hydraulic jump forming. We're given some information about flow rate, depth and channel width which we can use to calculate velocity. First and foremost, we need to work out the Froude number to see if we've got proper supercritical flow. If we do, then it's likely that a jump's going to form as the flow transitions to slower-moving flow down the horizontal part of the channel. Then we've been asked to work out the energy loss and dimensions of the jump using the equations in the past few slides. Check the workout procedure in the text book if you need to.

Slide 41

Thank you

If you have any questions or desire further clarification please post a question or comment on the Discussion Forum.