ENR202 Mechanics of Materials Lecture 1B Slides and Notes

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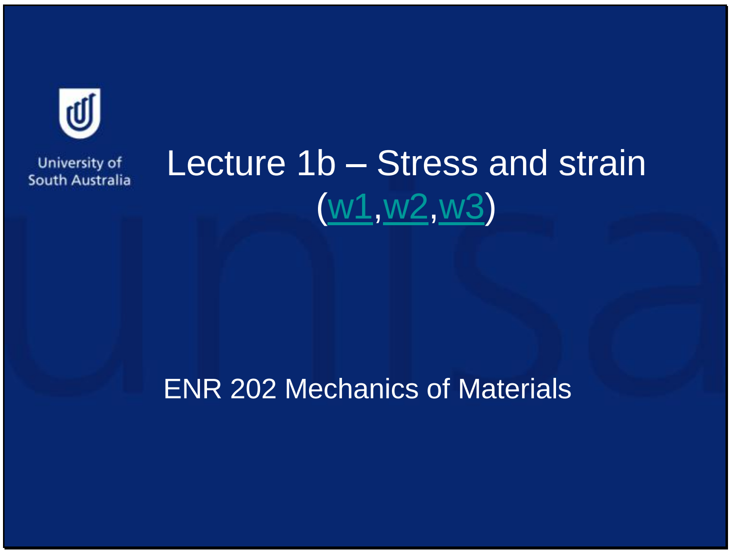
Slide 2



Welcome to Lecture Summary 1b for Mechanics of Materials. The most important concept to keep in mind for this course is safety principles. Before anything else, you must make sure your structure is safe. The second principle is serviceability. You must make sure that your structure will not grossly deform. Then you can consider economic design. The foundations for structural design are based on structural analysis and equilibrium equations. Equilibrium equations calculate all forces in all directions to be zero. For example, if you have a plane structure, all the forces in two directions must equal zero, and the moment of all forces taken at any point must together equal zero. This is what an equilibrium equation means. If you need to calculate reaction forces, you use a ‘’free body diagram’’. In a free body diagram, you remove support and replace the reaction forces. These are the basic principles of mechanics. We will also be looking at other basic concepts such as stress and strain. Stress is a safety issue, and strain is a serviceability issue.

Note that throughout all the lecture summaries for Mechanics of Materials, you will see live links, denoted by the letters W, P and V (for example, the three webpage links after the title in this slide). These links point to web pages, presentations and videos which will enhance your understanding of the content. You can pause the presentation at any time to access these links, and then go back to the presentation when you have finished looking at them.

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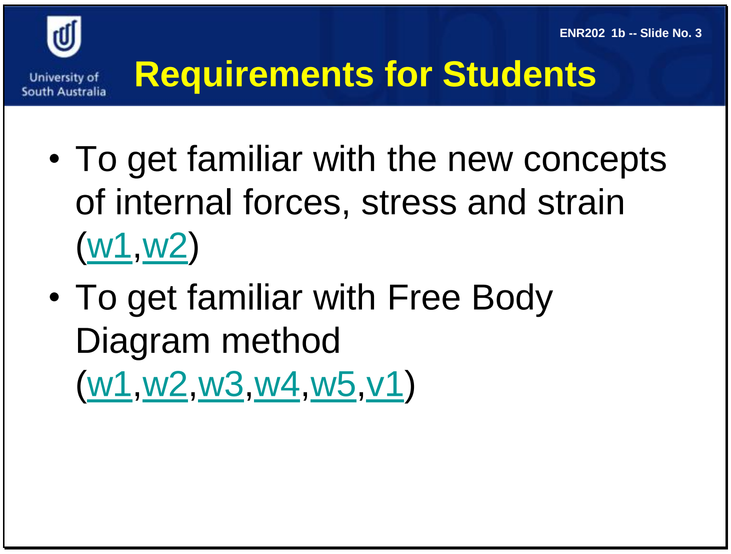

You will need to become familiar with the principles behind internal forces (and internal effect), stress and strain. We use the free body diagram method to calculate the internal forces. Click on the live links (the letters w and v) to access more detailed resources for each of these concepts.

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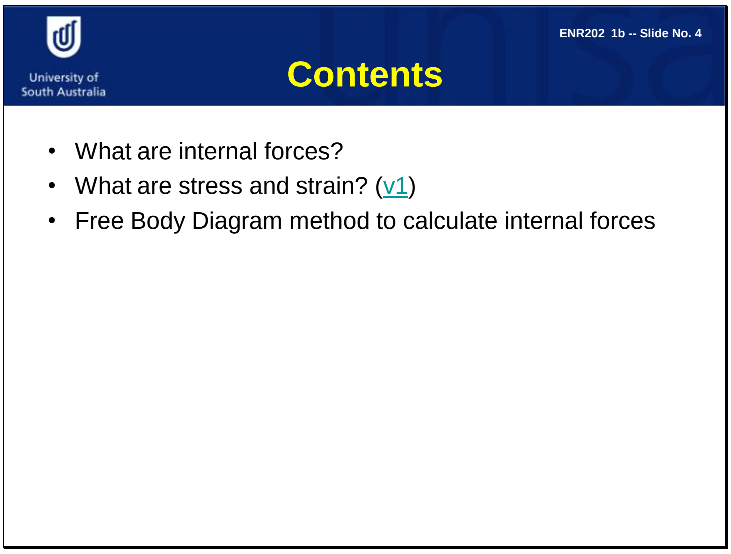

So in this lecture summary, we will look at what internal forces are, what stress and strain are, and how to use the Free Body Diagram method to calculate internal forces.

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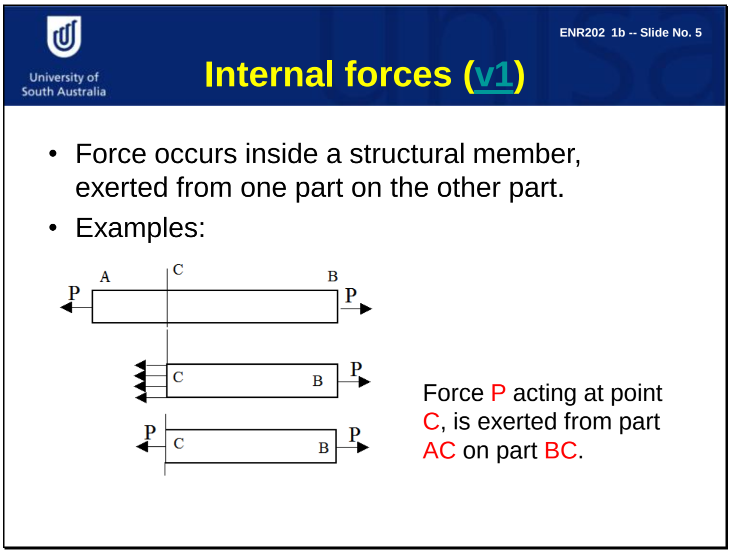

In this example, we have a bar subjected a force P acting at point A and another force in the equal and opposite direction at point B, as shown in the figure. I repeat one more time that the two forces are equal and are in opposite directions, so they create a balanced force system, and all parts of the bar will be in equilibrium.

We know that if the structure is in equilibrium, it means all part of the structure are in equilibrium. If you would like to calculate the internal forces at Point C, we cut the bar at cross section point C as shown in the figure. The structure is divided into 2 parts, and both parts are in equilibrium based on the equilibrium concept. One is part AC and the other is part BC. Now, let's consider the BC part. Imagine part AC is a support to part BC. The force coming from part AC is a reaction force of the AC support.

As we discussed earlier, in the Free-body diagram concept, we remove all supports from the object and replace them with reaction forces. So, we remove the part AC and replace it with a reaction force at point C. By using an equilibrium equation, all forces in a horizontal direction are equal to zero, so the reaction force at C is equal to P. This reaction force is an external force of part BC. However, this reaction force P at point C is an internal force of the whole bar AB.

So you can see that if you try to calculate the internal force, you have to use the equilibrium concept and free body diagram concept.

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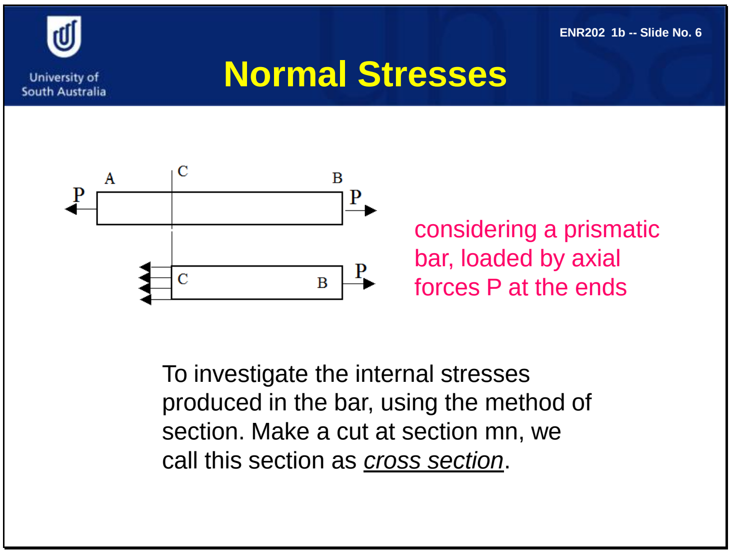

We now know the internal force acting at C. If we cut the section at C point that is perpendicular to axis of the bar, we say it is the C cross section. We call the intensity of the internal force over the cross section ‘stress’.

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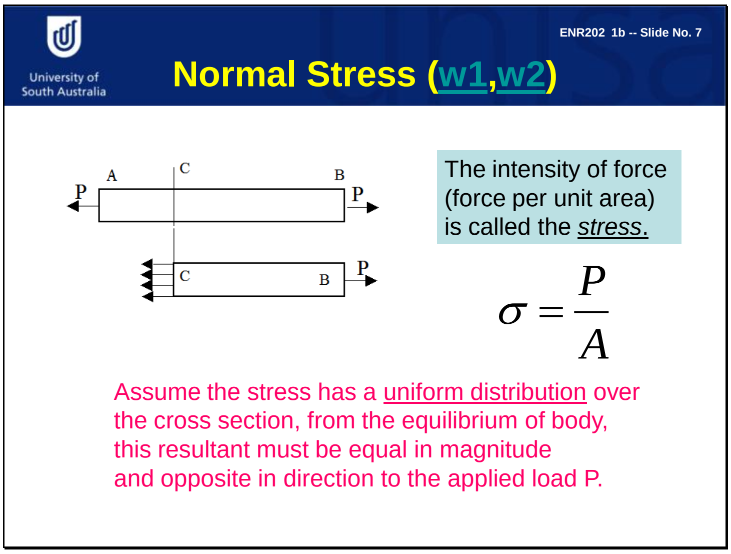

We use sigma to denote the stress. The intensity of the force means force per unit area. It is equal to P divided by A, because the total internal force acting at C is P. You use P divided by A to get the intensity of the force – that is, the stress.

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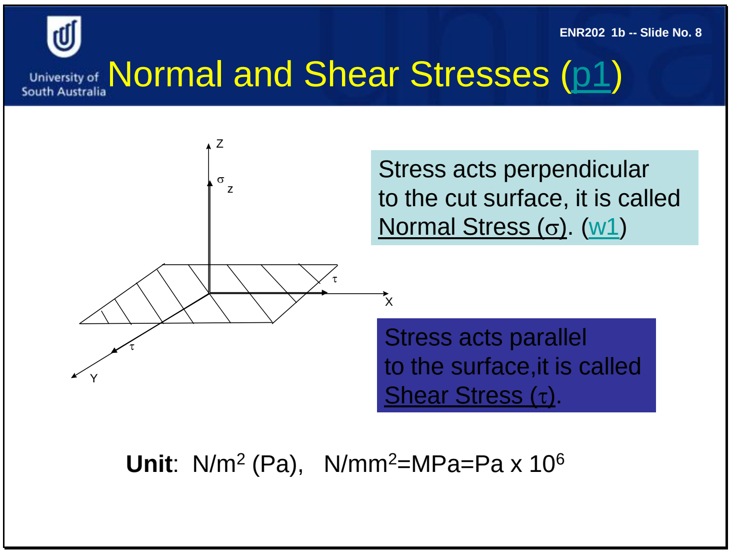

Suppose we cut the structure into two parts. Then, we consider the one of the cutting surfaces of the structure. We may have forces in different directions. The intensity of the force on that surface is called stress. Therefore, we may have stresses in three different directions. The component of stress in the Z direction is perpendicular to the cutting surface, called normal stress. We use sigma to denote normal stress. The component of stress in the X direction and the component of stress in the Y direction are parallel to the surface. We call these the shear stresses, and we use tou to denote shear stress. Therefore, we have two kinds of stresses, one is normal stress and the other one is shear stress. Once again, normal stress means stress acting perpendicular to the cutting surface, shear stress means stress acting parallel to the cutting surface.

For example if you pull a bar, you have tension stress, which is normal stress. This is different to shear stress, which is similar to frictions. The unit of the stress is force divided by area. So we consider the force in Newton and area in square meters. The stress unit is Newtons per square meter, which is also called Pascals. So we consider force in Newtons and area in square millimetres. The stress unit is N per square mm, also called Mega Pascals. Note the equation N/m2=10^6 N/mm2.

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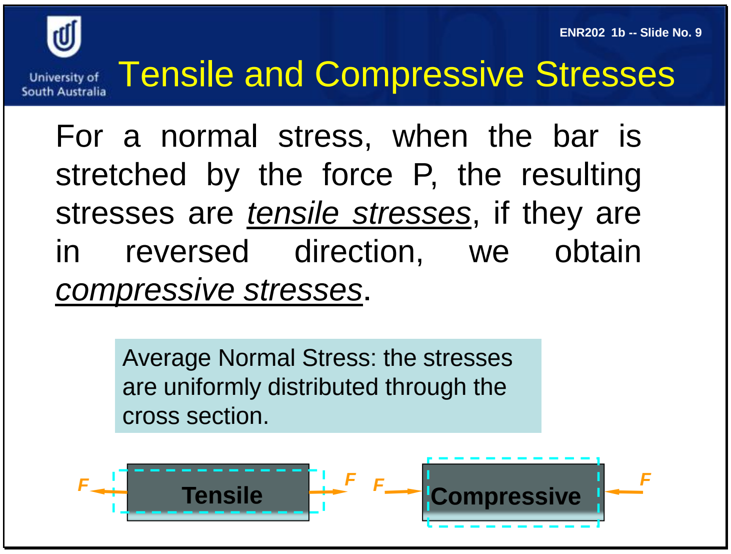

In normal stress, if a bar is stretched or expanded by force P, the resultant stress is tensile. The bar will get longer in length and thinner in cross section. If we apply forces in the reverse direction, we obtain compressive stress, and compressive stress will cause the bar get shorter in length but expand in cross section. Change in either length or cross section is called ***deformation***. Tensile force or compressive force act perpendicular to the cross section and the average normal stress is uniformly distributed throughout the cross section, so we average the force over the cross section area. So force divided by area equals average normal stress.

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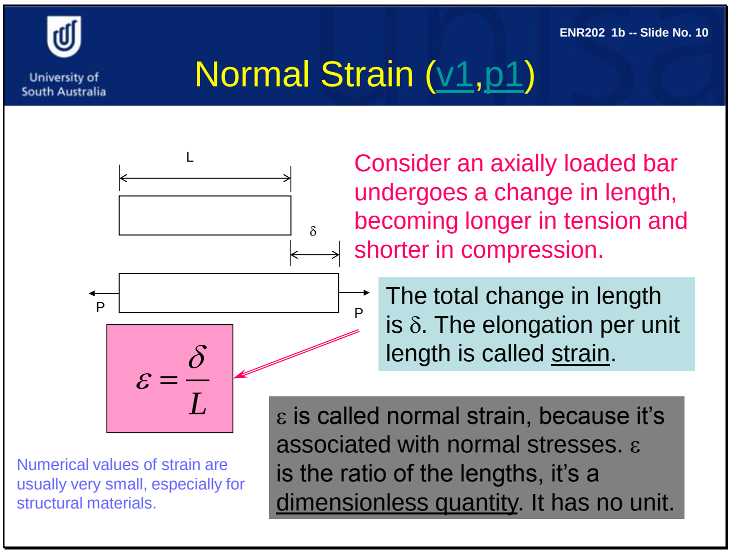

If we apply tensile force or compressive force on the bar, we can say that the bar is getting longer or shorter. The total change in length of whole bar called delta. Delta is the deformation of the whole bar. The elongation or contraction per unit length is called strain. We use the symbol epsilon to denote the strain. Epsilon is equal to delta divided by L, where L is the original length of the bar. The average deformation for the whole length is called normal strain. Because strain is the ratios of lengths, it doesn't have any unit. Strain is usually very small in real structures. For example, in steel structures, strain can be as small as 0.1%, so we cannot see that small deformation.

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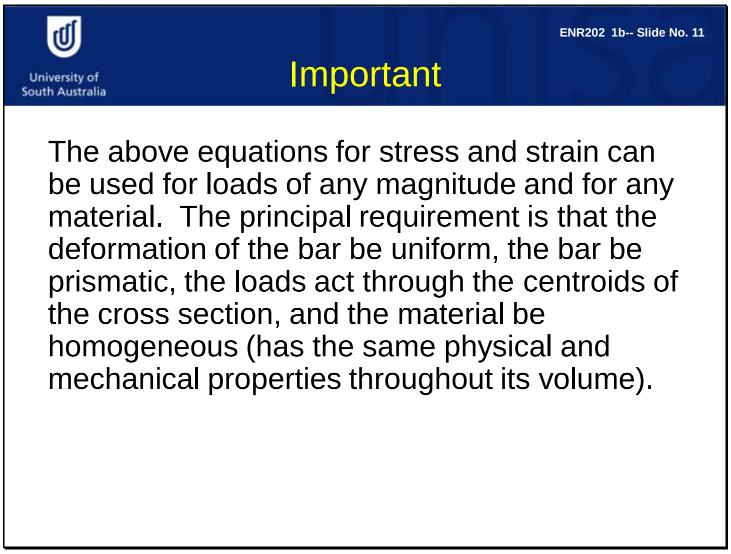

The equations for stress and strain can be used for loadings of any magnitude and any material. The principal requirement is that deformation of the bar is uniform. The bar should be straight and have a constant cross section throughout its length. The loadings act at centroid of the cross section. The material is also uniform.

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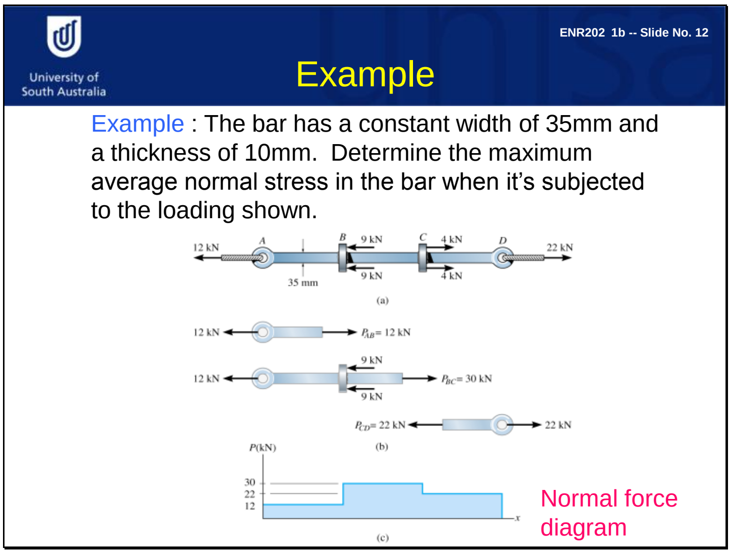

In this example, we have a bar with different parts. We have a load acting at point A of 12 Kilo Newtons, a load acting at point B which is altogether 18 Kilo Newtons, a load acting at point C which is altogether 8 Kilo Newtons, and a load acting at point D of 22 Kilo Newtons. The cross section of the bar is 35 millimeters in width and 10 millimeters in thickness. We want to calculate the maximum normal average stress in a bar.

We know that stress is equal to internal force divided by area. To calculate the maximum stress, we need to calculate the maximum internal forces.

(Note: If you have a non-uniform bar, that means different cross sectional areas at different points. If you want to work out maximum stress for different cross sectional areas, you need to consider the minimum cross sectional area too. Stress is denoted by sigma, which is equal to internal force divided by the area of the cross section. So, you have consider maximum internal force or minimum cross sectional area. However, in this example, the area of the cross section is uniform. That means that there is a uniform cross sectional area throughout length of the bar.)

Now let's calculate the maximum internal forces. There are different loads acting at points A, B, C, and D. So, we want to work out maximum internal forces in the AB, BC and CD parts of the structure. If we cut any cross section between B and A and use an equilibrium equation (that is, all forces in horizontal directions equal zero), we have internal force P_{AB} equal to 12 Kilo Newtons. That means that part AB has internal normal force of 12 Kilo Newtons.

Then we consider part BC, and cut at any point in that part. We have an external force of 12 Kilo Newtons acting at point A, and an external force altogether of 18 Kilo

Newtons acting at point B. We use the equilibrium equation and calculate the internal force P_{BC} to be equal to 30 Kilo Newtons.

Use same process for part CD. Cut at any cross section in CD, and consider the right part of the bar. The external force acting at D point is 22 Kilo Newtons. Apply the equilibrium equation. The internal force in CD is 22 Kilo Newtons.

Now we know that there is an internal force of 12 Kilo Newtons in part AB, 30 Kilo Newtons in part BC, and 22 Kilo Newtons in part CD. Our aim is to calculate maximum normal stress. Maximum normal stress occurs in part BC equal to internal force in part BC divided by the cross sectional area of BC, which is 35 millimeters multiplied by 15 millimeters.

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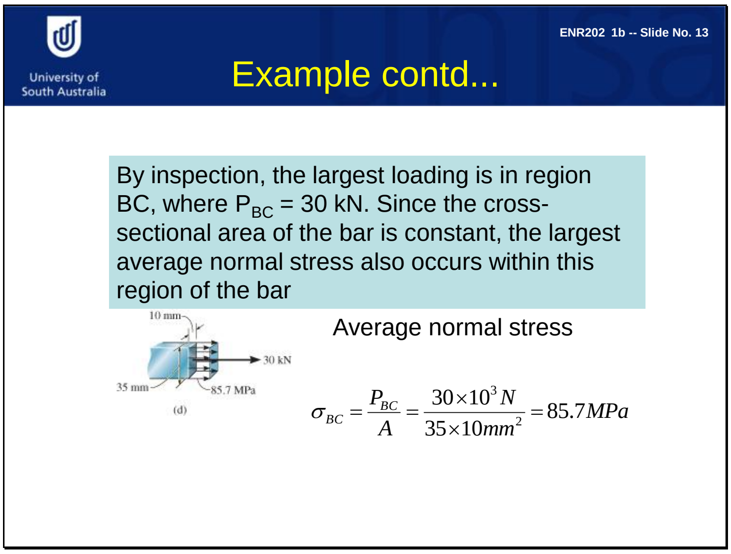

So, the maximum average normal stress is equal to 30 Kilo Newtons divided by the area of cross section. Convert force Kilo Newtons into Newtons. The area of the cross section is 35 mmms times 10 mms, which is equal to 350 square mms. That is 85.7 Newtons per square mm (also called Mega Pascals).

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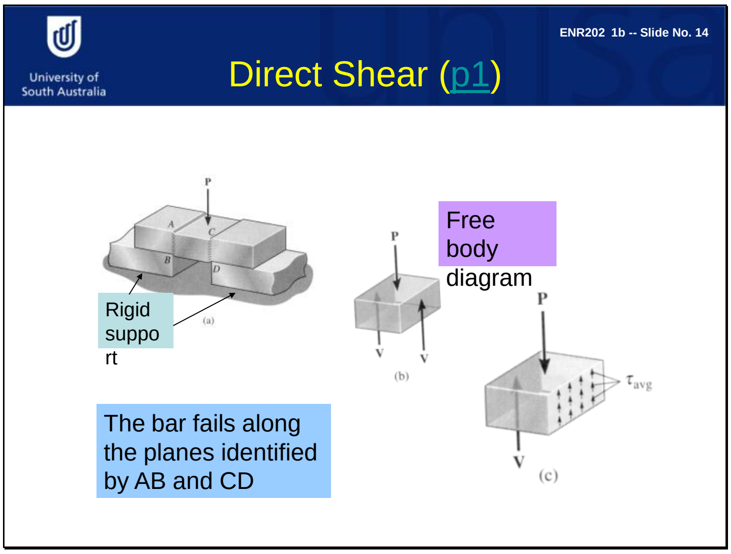

Let's look at shear stress. As an example, we have a short beam, supported by points A and C. There is a load P acting on the beam, if we cut the cross sections at points A and C as shown in the figure. If we analyse part AC, we have an external force P between points A and C. As I mentioned before, if the structure is in equilibrium, it means all parts in the structure are in equilibrium. Therefore, the AC part is in equilibrium. So, we must have upward direction balanced forces on cutting surfaces. These forces counterbalance force P, so we have two forces in cutting surfaces and the direction of these two forces is parallel to the cutting surface, so we denote with V for shear force. Shear force divided by the area of the cutting surface is called averaged shear stress.

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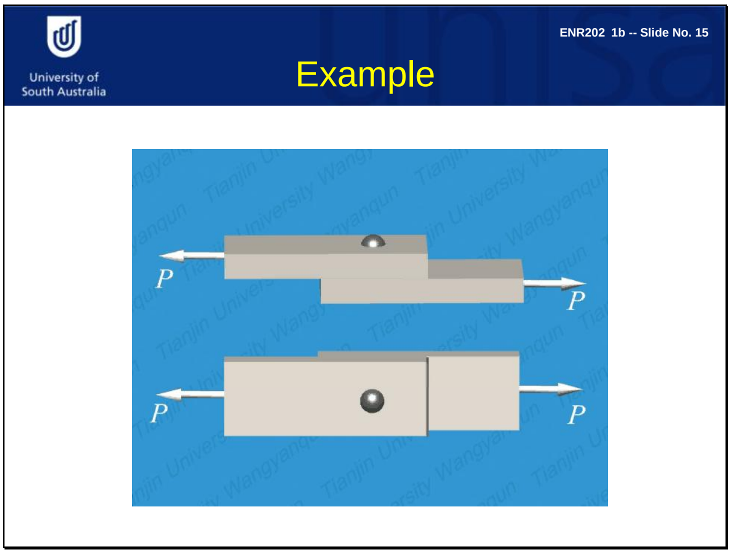

In this example, the two parts are connected by a bolt. If we have a connection bolted together, and an equal force P in opposite directions as shown in the figure, the bolt transfers the load P from one part to the other part. We can cut the cross section of bolt, and consider any one of the parts. The connection is in equilibrium, so any part in the connection is also in equilibrium. You must have forces at the cutting surfaces of the bolt cross section. This force at the cutting surface is equal to external force and parallel to the cutting surface of the bolt cross section.

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Shear Stress and Strain (v1,v2)

Shear stress - stresses act parallel or tangential to the surface. The average shear stress distributed over each sectioned area is defined by:

$$\tau_{avg} = \frac{V}{A}$$

τ_{avg} is in the same direction as V, also called as direct shear.

Shear is caused by the direct action of the applied load P. This type of shear often occurs in various types of simple connections that use blots, pins, welding materials, etc.

The Shear stress which is acting parallel to the cutting surface is equal to the shear force divided by the cutting surface area, called the average shear stress. It is denoted by the symbol ***tou*** acting in the same direction as shear force V.

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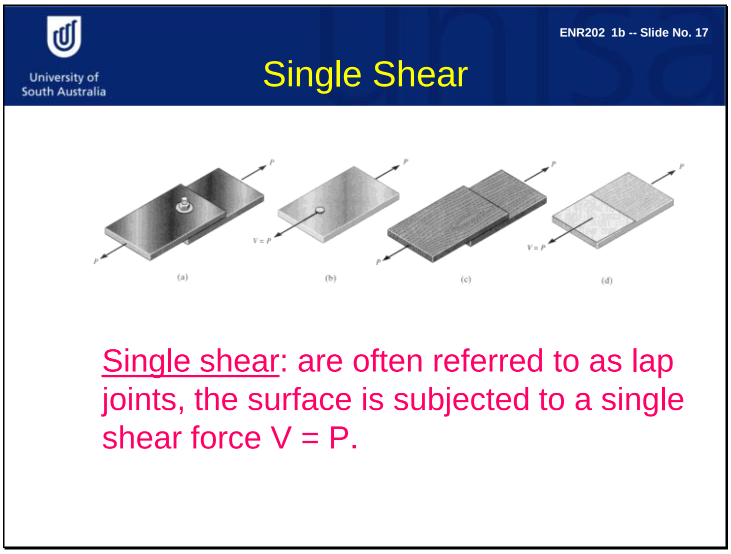

In this example, the connection of the bolt is resisted by the external force at only one shearing area. So we called this a single shear.

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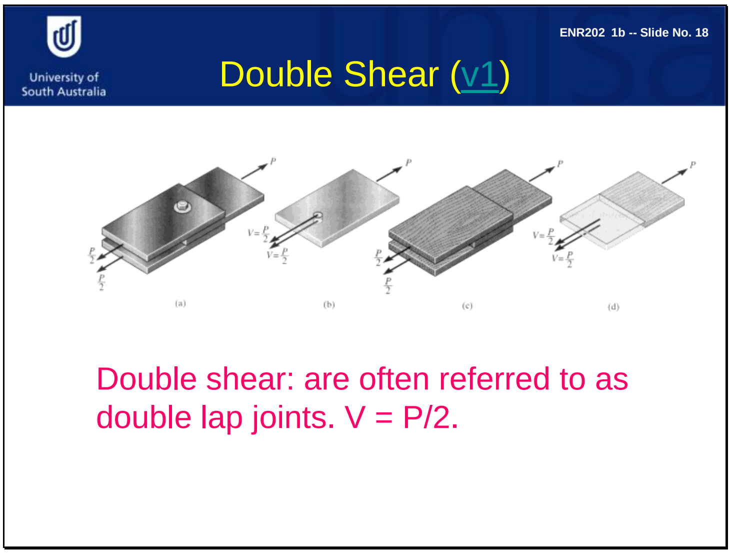

In this connection, the middle plate is subjected to force P, and the top and bottom plates are acting at half force P. We cut the bolt between the top and middle plates, and the bottom and middle plates cross section. Then, we analyse the middle plate with two shearing surfaces on the bolt. Total force is P shared by both shearing surfaces by half force P. So, this connection is a double shear. In figure C, the plates are joined by glue.

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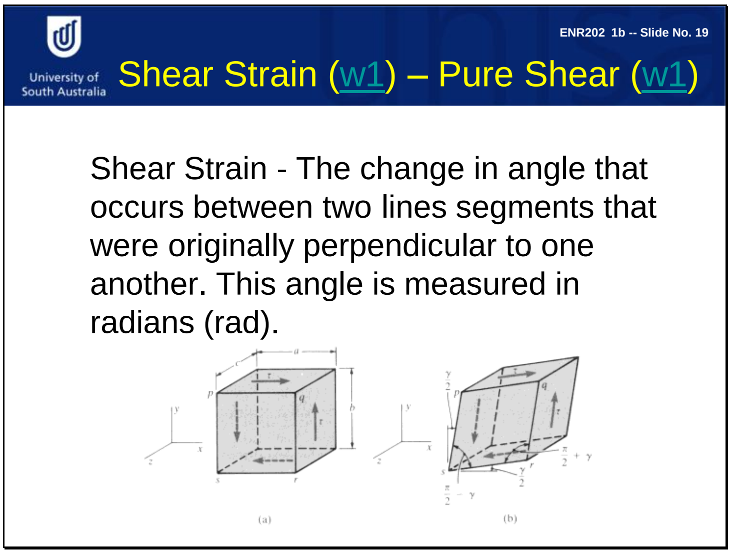

Shear strain occurs due to shear force. The shear force will cause the object to change in angle, resulting in shear strain. Shear strain is measured in radians.

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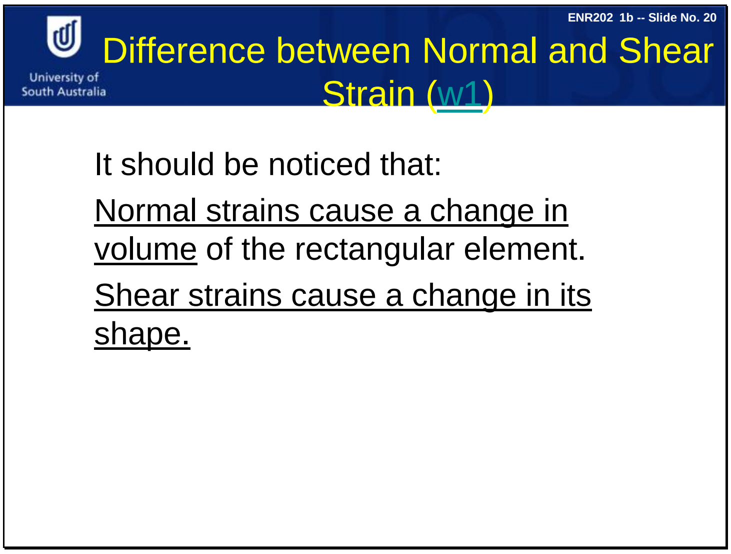

The difference between normal strain and shear strain is that normal strain causes a change in the volume of the object. For example, if we have tension, the bar gets longer and longer. However, shear strain only changes the shape, only changes the angle. There is no change in volume of the object due to shear strain.

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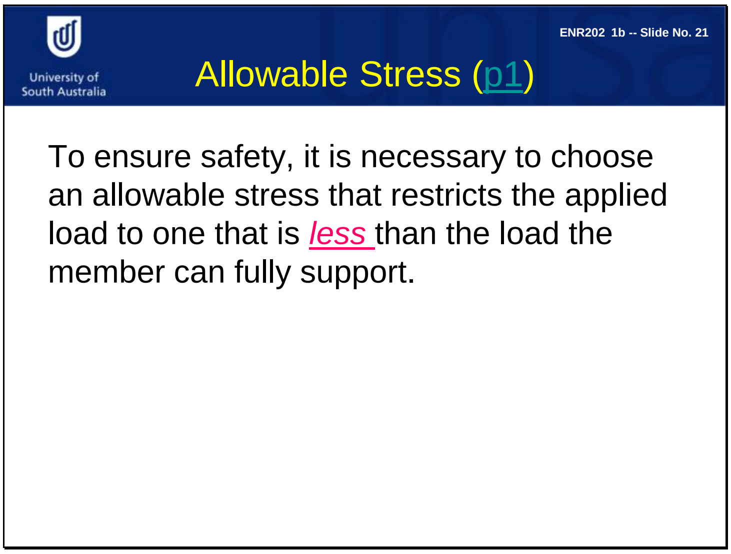

One of the most important concepts in this lecture is allowable stress. To make sure the structure is safe, we need to know the allowable stress. If you have a structural member, we can do an experimental test on this structural member, and work out ultimate load (that means the maximum load that the member can carry before it will break or damage). However, in reality, we cannot use ultimate loads, we just work out the safety load using allowable stress. The safety or allowable load is smaller than the structure member can carry.

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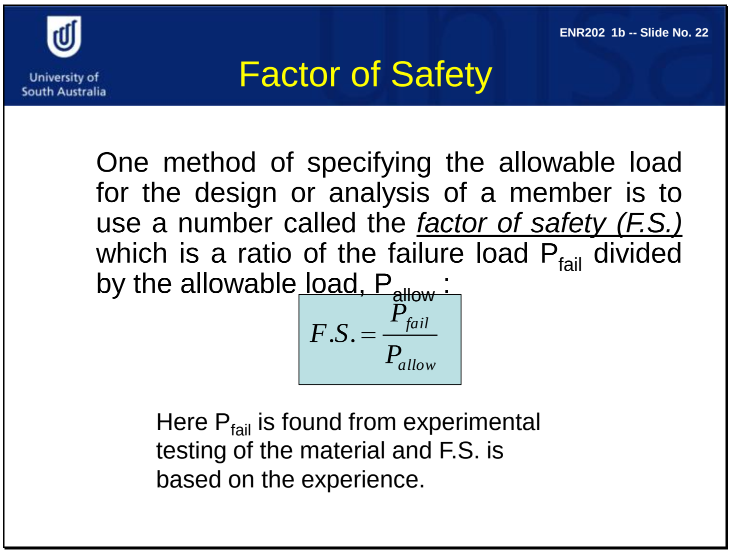

Similar to the allowable stress concept, we have the ***factor of safety*** concept. The factor of safety means the failure load divided by the allowable load. If we have a structure member, we can work out the failure load based on an experimental test. Considering the factor of safety of the structure, the allowable load is equal to the failure load divided by the safety factor. That means that in reality we use allowable load. This allowable load is smaller than the failure load, so we make sure the structure is safe.

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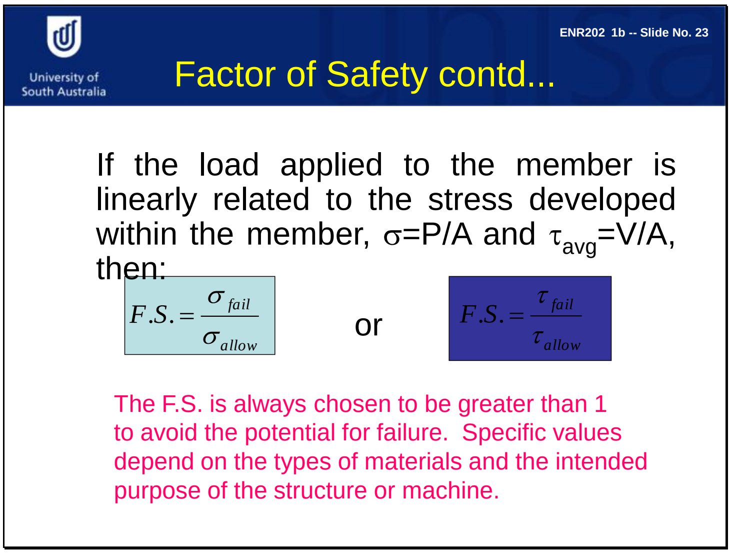

The factor of safety may also be defined based on maximum stress and allowable stress. That means that some kinds of material have failure stress or ultimate stress, which we know from an experimental test. We know that allowable stress is equal to failure stress divided by the factor of safety. Of course, the safety factor will be larger than one. We have another factor of safety related to shear stress, and this safety factor is equal to failure shear stress divided by allowable shear stress.

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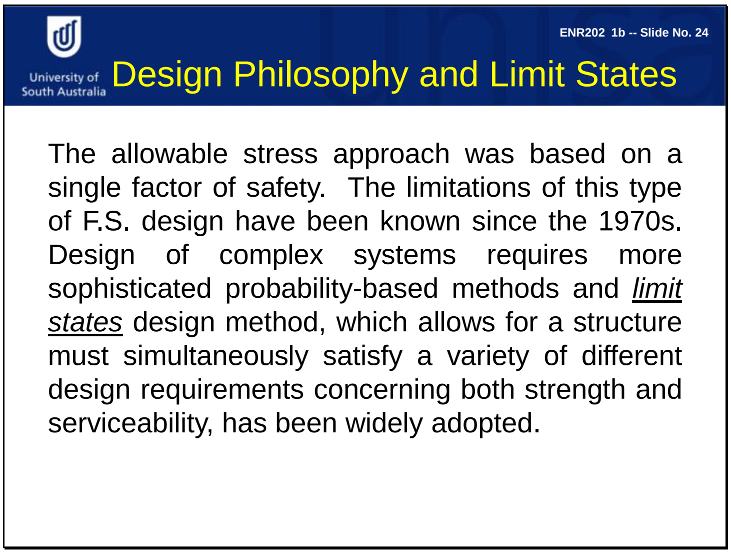

Design philosophy and limit states. In the 1970s, we used only one safety factor, the maximum loadings acting on the structure.
In the ***limit state design method***, we use multi safety factor. For example, we use different safety factors for materials based on test results (for example, steel maximum stress is 300 Mega Pascals, and we consider the one safety factor is 1.5, so we can use only 200 Mega Pascals for steel). Another safety factor for loadings. For example, if we consider dead load, we may use a safety factor equal to 1.2. If we consider the live load, we have some other safety factor equal to 1.5. So we use different kinds of safety factors for different things, for dead load, live load and materials. This is design philosophy.

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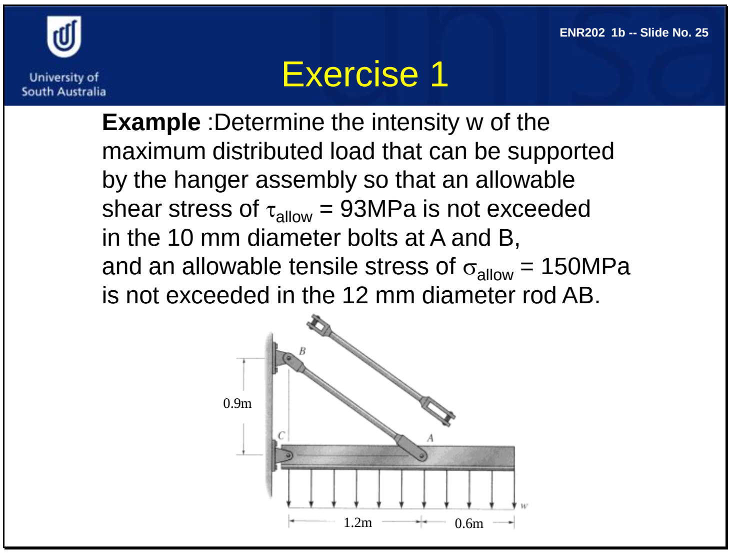

In this example, we have a beam and a hanger bolted at joint A. The bolt allowable shear stress is equal to 93 Mega Pascals. The bolt diameter is10 mms, and hangar allowable tensile stress is 150 Mega Pascals. The diameter of rod AB is 12 mms, and we need to calculate how much load can be applied on the beam. That means we just want to know the maximum uniform distributed load ‘w’. That is maximum load acting on the beam.

Pause this presentation now and try to work out this example. The solution is on the next two slides.

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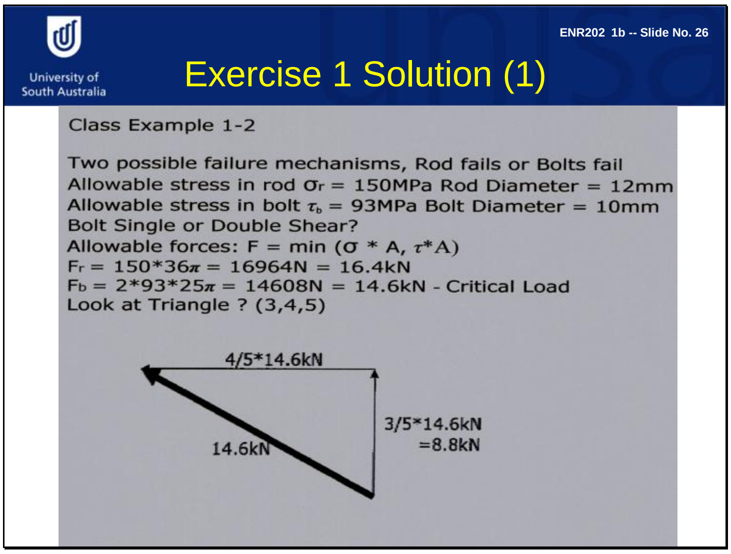

For the solution, you have to calculate the maximum load the rod can carry, the maximum load the bolt can carry.

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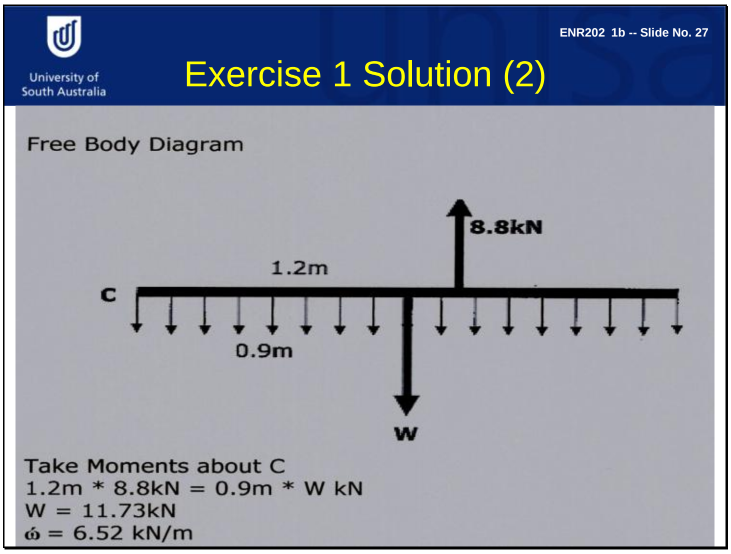

Now, consider the free body diagram of the beam. We have external loading w. Here, the length of the beam is 1.8 meters. The total uniform distributed load on the beam is equal to 1.8 times ‘w’. The vertical component from the rod is 8.8 Kilo Newtons (calculated in the previous slide). Therefore, we use the equilibrium equation that covers the moments about C: that is, the total load on the beam times the distance of 0.9 meters, because the total length of the beam 1.8 meters, and the resultant of the uniformly distributed load acting in the middle of the beam. The resultant load on the beam times 0.9 meters should be equal to 8.8 times the distance between points A and C, which is 1.2 meters. Finally we find the resultant load on the beam as equal to 11.73 Kilo Newtons. Therefore, the uniformly distributed load is equal to 11.73 Kilo Newtons divided by the total length of the beam (1.8 meters) which is 6.5 Kilo Newtons per meter, so this is the maximum load that can apply on the whole structure.

In this example, you must understand the calculation of the vertical component and horizontal component of the rod, and the maximum tension that the rod can carry, and the maximum shear force the bolt can transfer. So you calculate the maximum allowable tension in the rod, and allowable shear force on the bolt, and you choose minimum one as safe one can apply on the rod. Finally, calculate the maximum load that can apply to the whole structure.

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In this lecture summary, we have covered several basic concepts: internal force, stress, and strain, and how to use a free body diagram to work out internal forces. Thank you.