

ENR202 Mechanics of Materials Lecture 2A Slides and Notes

Slide 1



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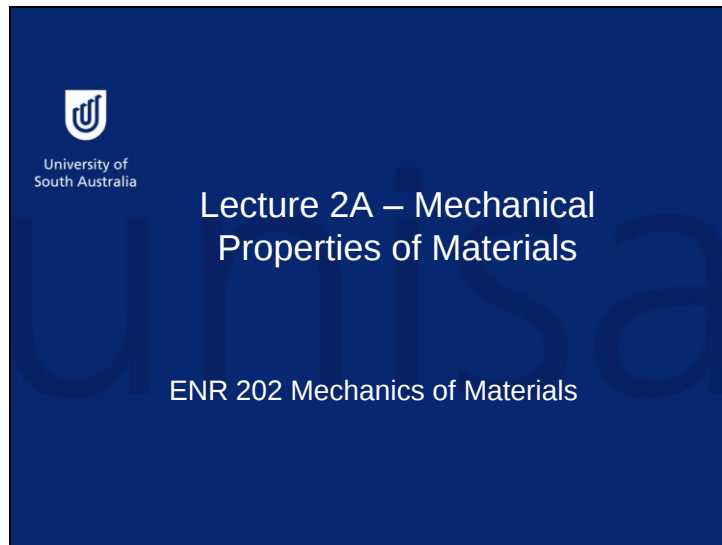
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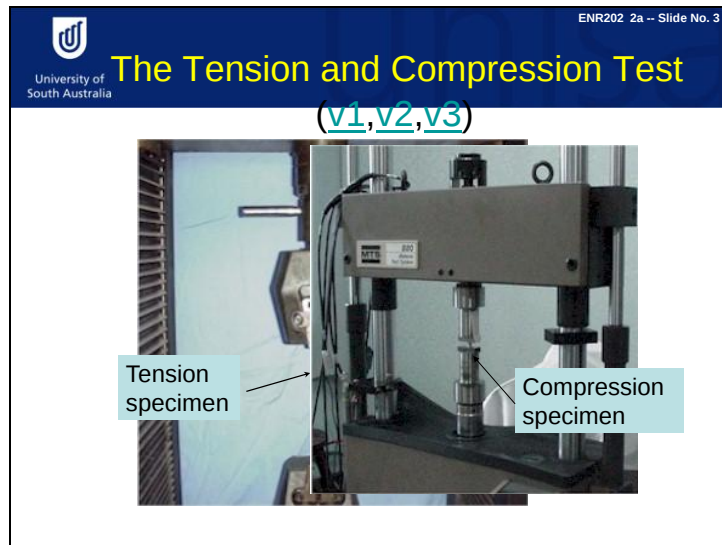
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Slide 2



Welcome to Lecture Summary 2A for Mechanics of Materials. In this Lecture Summary, we will start looking at the properties of materials.

Note that throughout all the lecture summaries for Mechanics of Materials, you will see live links, denoted by the letters W, P and V. These links point to web pages, presentations and videos which will enhance your understanding of the content. You can pause the presentation at any time to access these links, and then go back to the presentation when you have finished looking at them.



Because we use different materials for structural design, you have to know the mechanical properties of these materials. In mechanical properties, we need to know behaviour of materials in compression and tension. That is why we have to do compression and tension tests. In a tension test, the two ends of the specimen are fixed on the machine. The bottom end cannot move, the top end is connected with a load cell which can move up or down. If the load cell moves up, that means the specimen is getting longer. Thus we have both elongation and tension in this specimen, we can work out the relationship between forces and displacements/deformations in this specimen. If we move the load cell down, that means we will compress the specimen, we are doing compression tests.

Click on the live links (the letters v) to access more detailed resources for each of these concepts.

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Slide 4



The slide features a dark blue header with the University of South Australia logo on the left and the text "ENR202 2a -- Slide No. 4" on the right. The main title "Tensile test online" is centered in yellow. Below the title, a green URL is displayed: <http://www.youtube.com/watch?v=W5A8gU37wGg>.

Here you will find a video of the tensile test. In this tensile test, we apply tension on this specimen, which gets longer and longer. However, some areas are getting smaller and smaller. There is a reduction in the area of cross section, finally it breaks.

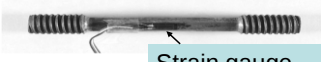


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Stress-Strain Diagrams

- The stress and strain of a specimen at various magnitudes of the load could be determined via tension/compression test
- A diagram can then be plotted which is called a stress-strain diagram
- A stress-strain diagram is characteristic of the material and conveys important information: the mechanical properties and type of behaviour.



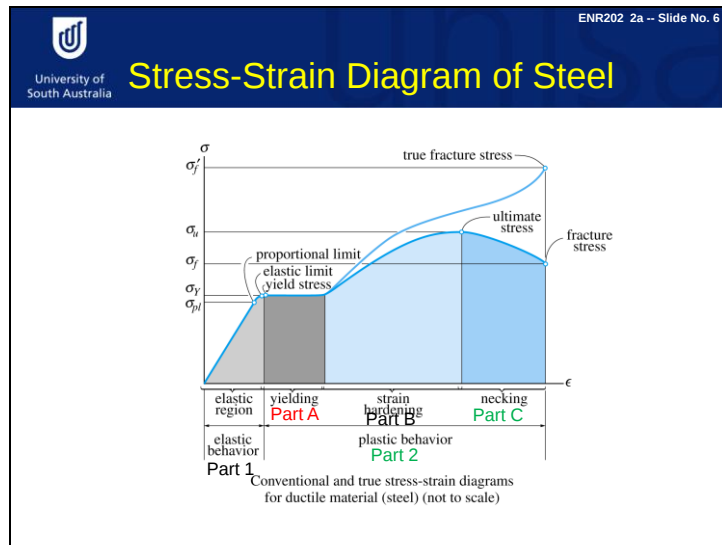
Strain gauge

From the tests, we can work out the relationship between load applied on the specimen and deformation (elongation or contraction) of the specimen. In this specimen, we have loads on the specimen 'P', and also we have a total length of specimen 'L'. There may be some elongation deformation which is 'delta L'. Finally we get the relationship between delta L and load P, for this specific case.

If we want to get the material properties, sometimes we use P divided by Area of cross section 'A', and delta L divided by length of the specimen L, so we have stress denoted by σ equal to Load P over Area of the cross section, and strain denoted by ϵ equal to delta L over Length of the specimen. This means that from the test, we can finally get the relationship between stress and strain.

The stress strain relationship is a mechanical property of the material.

In tension or compression tests, sometimes we use strain gauge on the specimen, that means we don't have to calculate delta L divided by Length of the specimen, we can obtain strain (ϵ) directly by using strain gauge.



This is the stress-strain relationship of steel, and it is very important, because all engineers should have knowledge of steel. From the diagram, we can say there is a total curve of 2 parts. Part one is elastic behavior and part two is plastic behavior. The elastic region means that if we increase stress or increase load, we have larger deformations. However, if we decrease the stress then we have smaller strain, if we remove all the stress or unload to zero, then the strain will also be zero. That means that in this region, the deformation or strain can recover and come back to zero.

However, if we increase the load after elastic region, we come into the plastic region. Plastic strain or plastic deformation is permanent, it cannot come back. In the plastic region there are 3 parts. Part A is yielding, in fact it is a straight line, with an increase in strain but there is no increase in stress. Therefore, we have large deformation in this part without increase of our load. After the yielding, we come to the Part B region that is strain hardening region. During this region we can have increasing stress with increasing strain, and this peak point is ultimate stress. After this point, In Part C region that is necking, stress will decrease even though strain increases. We have large deformation but with smaller load. And finally it breaks at one point. This point is fracture stress.


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Elastic Behaviour

- The material back to the origin when the load is removed.
- The elastic limit – maximum stress the material can sustain without permanent deformation
- This course only deals with the material behaviour under the elastic region



In Stress-Strain diagram in previous slide, The first part of the stress-strain diagram is elastic behaviour. That means the material comes back to zero strain when the load is removed. At one important point is the elastic limit – if the stress exceeds that point, elastic will become plastic. Another important point here is ‘sigma proportional limit (PL)’ – this means that prior to this we have a linear relationship between stress and strain. In fact, proportional limit is very close to yield stress or elastic limit. Sometimes we consider them as one point. That means we use only yield stress as the proportional limit. During the elastic region, we have a linear relationship between stress and strain.



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Yielding

A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to *deform permanently*

The stress causes yielding - *Yield Stress* - σ_y

Deformation – *plastic deformation*



Now we come to yielding. If we slightly increase stress, we have a large deformation (plastic deformation), and this is permanent. Yield stress is similar to elastic limit.



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Strain Hardening

A further load applied, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress.

The maximum stress – *ultimate stress*, σ_u

The rise in the curve in this manner is called – *strain hardening*



Now we come to strain hardening. After yielding, we can increase the load, increasing stress and increasing strain, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress.



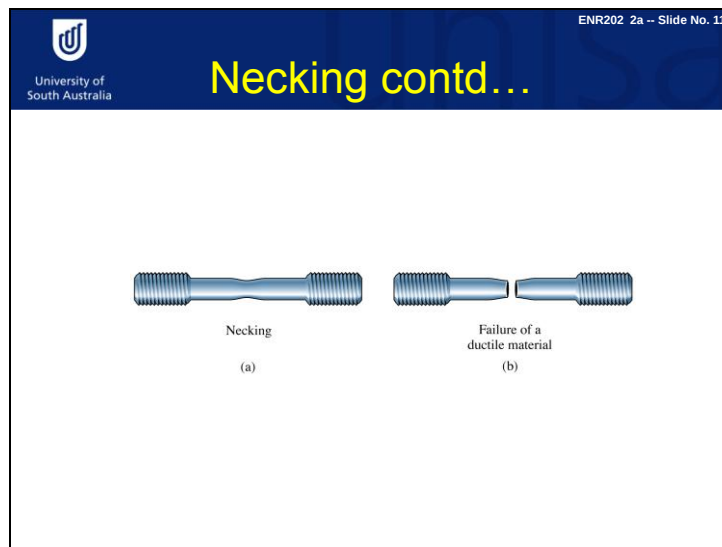
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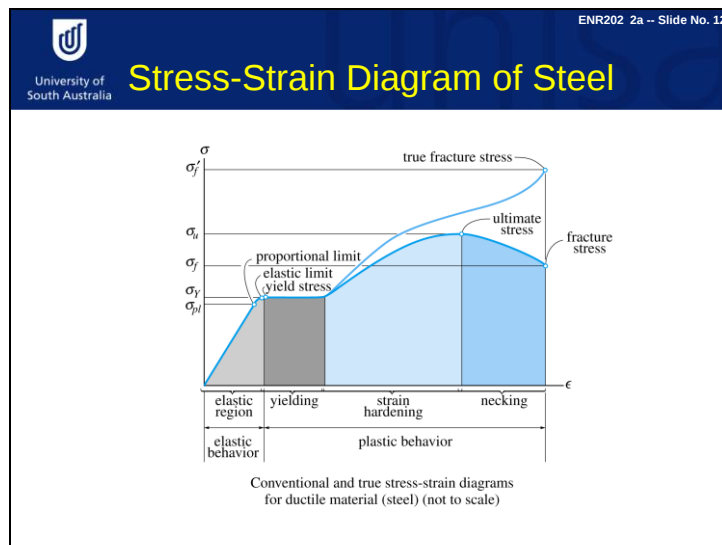
Necking

- The cross-sectional area begins to decrease in a localised region.
- A “neck” gradually tends to form in this region.
- The stress-strain diagram tends to curve downward until the specimen breaks at the fracture stress, σ_f .

Necking means the cross sectional area begins to decrease in the localized region, and a neck gradually forms in this region. The stress strain diagram tends to curve downwards until the specimen breaks, decreasing stress and decreasing loadings even though strain and deformation are increasing.



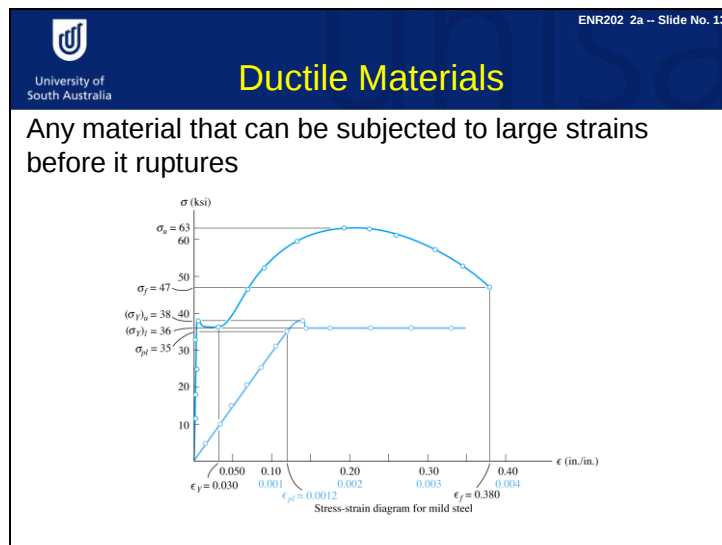
In figure a, you can see a reduction in the cross sectional area. With the reduction in cross section area, the loading should decrease, and finally it breaks at the necking point, as shown in figure b.



Looking at this diagram, the bottom curve is nominal stress-nominal strain. What does that mean? This is a stress-strain diagram, which we know from the test, and we have a load P and ΔL . If we try to work out the stress denoted by σ , it is equal to Load divided by the Area of the cross section. However, the cross section is changing, so what will we use for Area of the cross section?. If you use the original cross section area ' A ', we have the bottom curve of the stress-strain diagram. If we consider the necking part of the cross section area, there is a decrease in cross section area and you have the top curve of stress-strain diagram. This is fracture stress. The real fracture stress is the highest point.

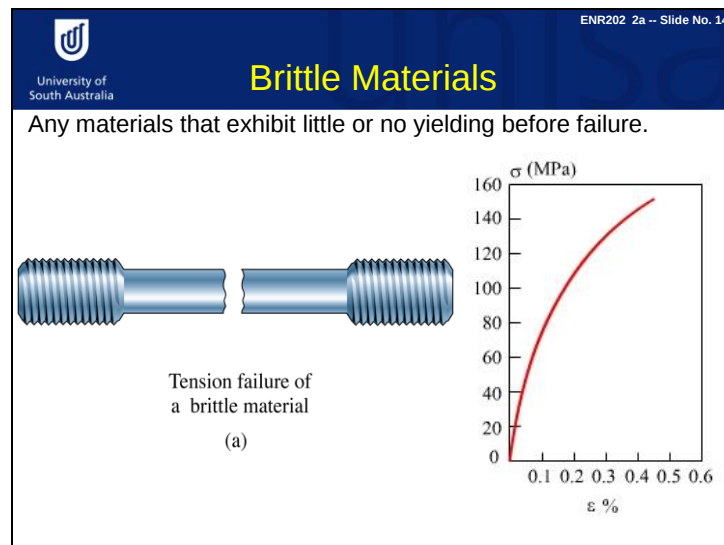
In fact, if we don't consider the non-linear behaviour that means plastic behaviour, we use only the linear part that is elastic behaviour, if you want to do research study, you may consider non-linear behaviour of different kind of material, consider the real stress-real strain diagram.

Just to clarify the notations of stress strain diagram: σ_{pl} is proportional limit stress, σ_y is yield stress, σ_u is ultimate stress, σ_f is fracture stress, σ_f' is true fracture stress. σ is stress and ϵ is strain.



In engineering applications, we have two kinds of materials. One is ductile material and the other is brittle material. Ductile material can have large deformations. For example, steel can have a maximum deformation of up to 40% of length of the specimen. We use ductile material because it can absorb energy. The area below the curve in this diagram is the energy the material can absorb. What does that mean? Let's look at an example. Earthquakes produce energy, and if we have material which can absorb large amounts of that energy, the material is safe during the earthquake. Because steel can have a elongation as high as 40%, it will perform very well in earthquake. Another important value is the elastic limit. The elastic strain should 0.1% to 0.3% for steel. If the strain is below this limit the material should be elastic, otherwise it might be plastic.

And the maximum yielding point could be said to be 3%, but in fact we cannot use 3% deformation in our design, 3% is too high. We usually allow a maximum yielding strain of less than 1%.



In brittle material, we have only small deformation, and the maximum strain might be 0.5%. It is very small compared to 40% for ductile material! Because the area under this curve is very small compared to ductile material, this material is not good for dynamic loading. If you have dynamic loading in a structure, you should use ductile material in your structure, but not brittle material.



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Hooke's Law

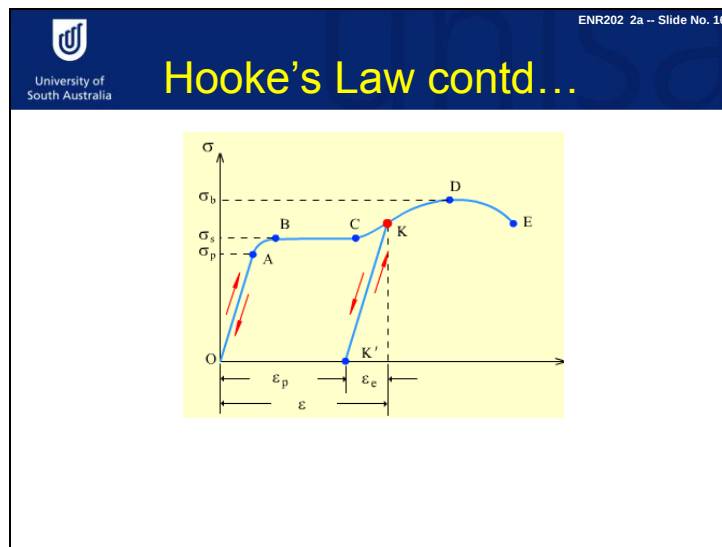
- The linear relationship between stress and strain can be expressed by:

$$\sigma = E\varepsilon$$

E is the constant of proportionality, which is called the modulus of elasticity or Young's Modulus

E represents the slope of the line, has same unit of stress. It indicates the stiffness of a material

Hooke's law is simple but very important. It represents the relationship between stress and strain. Stress is internal force, and strain is the deformation. Stress and strain are proportional to each other and have a linear relationship. For example, the slope of this line is E, which is constant of proportionality, and is called the modulus of elasticity or Young's modulus. Sigma equals E times epsilon. If you know the stress, you can use this formula to calculate epsilon that is the strain. If you have internal forces, from those forces you have stress, you can calculate the strain based on this formula, and then calculate the whole deformation.



Hooke's law is slope of the straight line. It is interesting that in the steel stress-strain diagram, in the elastic region, we have a linear relationship between stress and strain. However if we unload in the plastic region, the stress-strain relationship follows a parallel line, so you can still use Hooke's law.

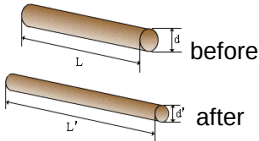


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Poisson's Ratio

- When a prismatic bar is loaded in tension, the axial elongation is accompanied by ***lateral contraction***



Longitudinal strain: $\epsilon = \frac{\Delta L}{L}$

Lateral strain: $\epsilon = \frac{\Delta d}{d}$

Poisson's ratio means that if you have deformation in axial direction (longitudinal direction), automatically you have deformation in lateral direction. For example, if you have a bar and apply tension, it gets longer and longer. We have elongation in an axial direction or longitudinal direction; however, we have lateral deformation as well. We have lateral strain and diameter change. The longitudinal strain is ΔL over Length of the specimen, and the lateral strain is Δd over diameter of the cross section.



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Poisson's Ratio contd...

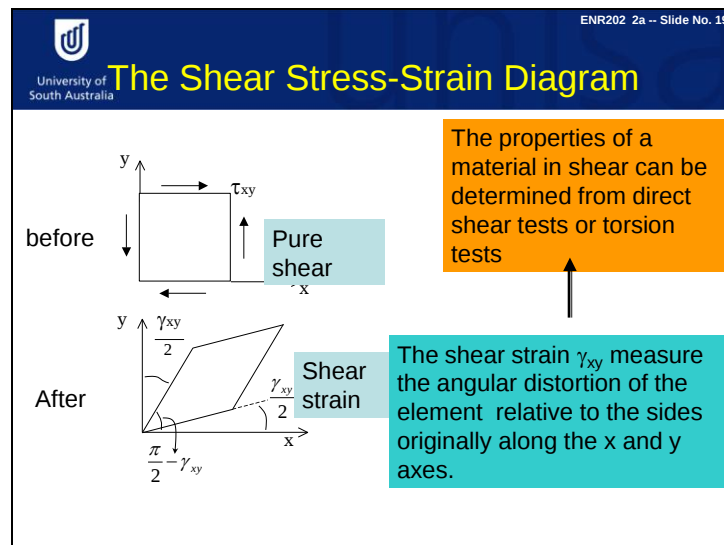
It has found out that the lateral strain is proportional to the axial strain in the **linear elastic range**, provided the material is both homogeneous and isotropic

The ratio of the strain in the lateral direction to the strain in the axial direction is known as **Poisson's ratio**.

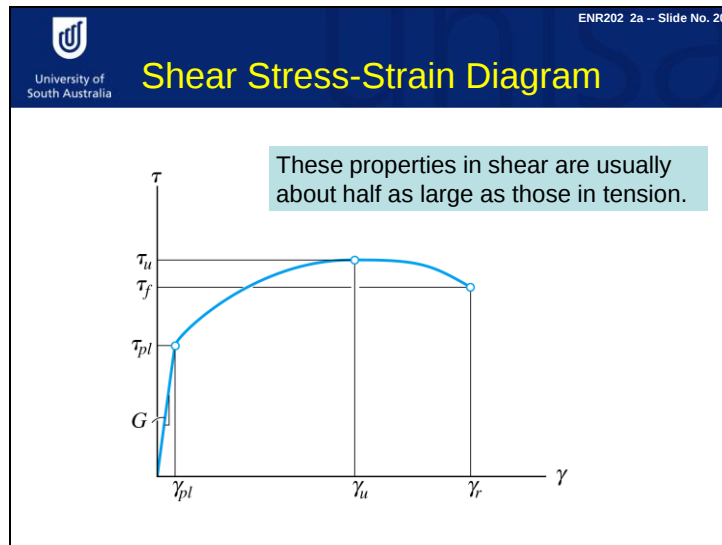
$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

Poisson's ratio has a positive value for most materials. It is dimensionless and $0 \leq \nu \leq \frac{1}{2}$

Lateral strain and longitudinal strain have a relationship like this: negative lateral strain divided by longitudinal strain is equal to Poisson's ratio. For certain materials, we have certain values. Poisson's ratio is dimensionless and the value is between 0 and 0.5, so during the test if you get a result larger than 0.5, you must have done something wrong! The range must be between 0 and 0.5.



The mechanical properties of a shear stress-shear strain diagram can be worked out from a direct shear test or from a torsion test. The shear strain γ_{xy} measures the angular distortion of the element relative to the sides of the element as shown in the figure.



As we have stated, we have an elastic region and a plastic region. In the elastic region, we have a linear relationship. The symbol τ denotes shear stress, and γ denotes shear strain. The slope of this line is called ' G '.



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Hooke's Law in Shear

- The linear relationship between shear stress and strain can be expressed by:

$$\tau = G\gamma$$

G is the constant of proportionality, which is called the shear modulus of elasticity or Modulus of rigidity

G represents the slope of the line, has same unit of stress.

$$G = \frac{E}{2(1+\nu)}$$

G is used for this formula: shear stress is denoted by τ and is equal to G times shear strain, which is denoted by γ (this is also Hooke's law, but it is in shear.) G is the shear modulus of elasticity or modulus of rigidity. Modulus of rigidity G is equal to Young's modulus E divided by 2 times (1+ μ). μ is Poisson's ratio. μ is between 0 and 0.5. so G is less than 50% of E. So the relationship between E and G is that G is less than half of E.

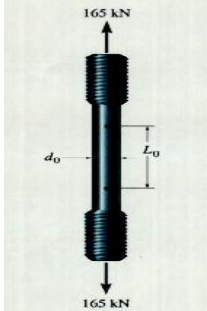


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Exercise 1

An aluminium specimen shown in the figure has a diameter of $d_0=25\text{mm}$ and a gauge length of $L_0=250\text{mm}$. If a force of 165kN elongates the gauge length 1.20mm , determine the modulus of elasticity. Also, determine the contraction of the diameter by this force. $G_{\text{al}} = 26\text{GPa}$ and $\sigma_Y = 440\text{MPa}$.



In this example, the rod is made from aluminium and the original diameter ‘ d_0 ’ is 25mm. The gauge length ‘ L_0 ’ means the distance between these two measuring points as shown in the figure, and is 250mm. If a force of 165 Kilo Newtons is applied, the gauge deformation is 1.2 mms. The modulus of rigidity of aluminium equals 26 Gega Pascals, and σ_Y means yield stress equals 440 Mega Pascals.

Now pause this presentation and try to determine the modulus of elasticity, and also the contraction of diameter of the rod. That means that you need to determine the change in diameter. The solution is on the next slide.



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Exercise 1 Solution

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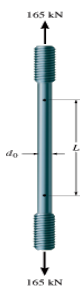


Fig. 3-26

SOLUTION

Modulus of Elasticity. The average normal stress in the specimen is

$$\sigma = \frac{P}{A} = \frac{165(10^3) \text{ N}}{(\pi/4)(0.025 \text{ m})^2} = 336.1 \text{ MPa}$$

and the average normal strain is

$$\epsilon = \frac{\delta}{L} = \frac{1.20 \text{ mm}}{250 \text{ mm}} = 0.00480 \text{ mm/mm}$$

Since $\sigma < \sigma_Y = 440 \text{ MPa}$, the material behaves elastically. The modulus of elasticity is

$$E_{al} = \frac{\sigma}{\epsilon} = \frac{336.1(10^6) \text{ Pa}}{0.00480} = 70.0 \text{ GPa} \quad \text{Ans.}$$

Contraction of Diameter. First we will determine Poisson's ratio for the material using Eq. 3-11.

$$G = \frac{E}{2(1 + \nu)}$$

$$26 \text{ GPa} = \frac{70.0 \text{ GPa}}{2(1 + \nu)}$$

$$\nu = 0.346$$

Since $\epsilon_{long} = 0.00480 \text{ mm/mm}$, then by Eq. 3-9,

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

$$0.346 = -\frac{\epsilon_{lat}}{0.00480 \text{ mm/mm}}$$

$$\epsilon_{lat} = -0.00166 \text{ mm/mm}$$

The contraction of the diameter is therefore

$$\delta' = (0.00166)(25 \text{ mm}) = 0.0415 \text{ mm} \quad \text{Ans.}$$

We know force acting on the bar, the original cross sectional area of the member, deformation after application of the force, and the Original length of the bar.

Therefore, First we can calculate the stress equal to Force divided by cross sectional area of the bar. The force is 165 Kilo Newtons and converted into Newtons and then calculate cross sectional area of the member, equal to Pi times square diameter divided by 4. Finally we can calculate the Stress, denoted by sigma, equal to 336.1 Mega Pascals or Newton per square millimeter.

Second you need calculate the strain denoted by epsilon equal to change in length divided by original length. Change in length or deformation equal to 1.2 millimeters and original length of the bar is 250 millimeters. Dividing these two lengths gives us a strain of 0.00480. As you can see from this calculation, we are dividing millimetres by millimetres so these units cancel each other out, reinforcing our statement that there are no units for strain

From Hooke's law. Young's modulus of the Aluminium is equal to stress divided by strain. Finally we can calculate the Young's modulus of the Aluminium as 70 Gega Pascals.

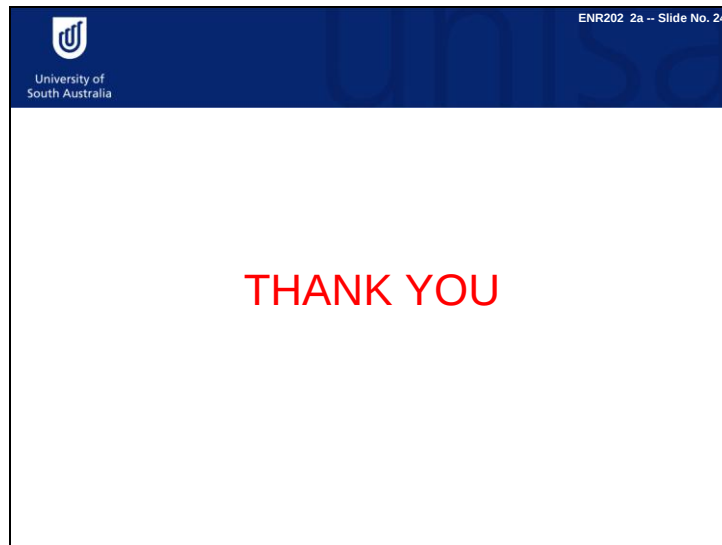
We know Modulus of Rigidity of Aluminium G_{al} , is 26 Gega Pascals. We just calculated Young's modulus E_{al} , equal to 70 Gega Pascals. We know the relationship between E and G that is G equal to E divided by 2 times $(1 + \mu)$. From this. We can calculate Poisson's ration (μ) as 0.346. We know the definition of Poisson's ratio is equal to negative lateral strain divided by longitudinal strain. We already calculated the longitudinal strain as change length divided by original length, so we can calculate lateral strain. The lateral strain is equal to negative 0.00166. We

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know the definition of lateral strain is equal to change in diameter divided by original diameter. We know the original diameter is equal to 25 millimeters. Therefore we can calculate the change in diameter equal to 0.0415 millimeters. This value is negative meaning diameter contracts after the application of the tensile force of 165 Kilo Newtons.

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In this lecture summary, we learned the concepts of stress strain diagram of steel for Normal force and shear force, the behaviour of ductile material and brittle material, Poisson's ratio, Hooke's law for normal stress and shear stress.

As always, I'd like to thank you for your attention, and wish you all the best with the remainder of this course.

Please continue to the next lecture at your leisure.