ENR202 Mechanics of Materials Lecture 2B Slides and Notes

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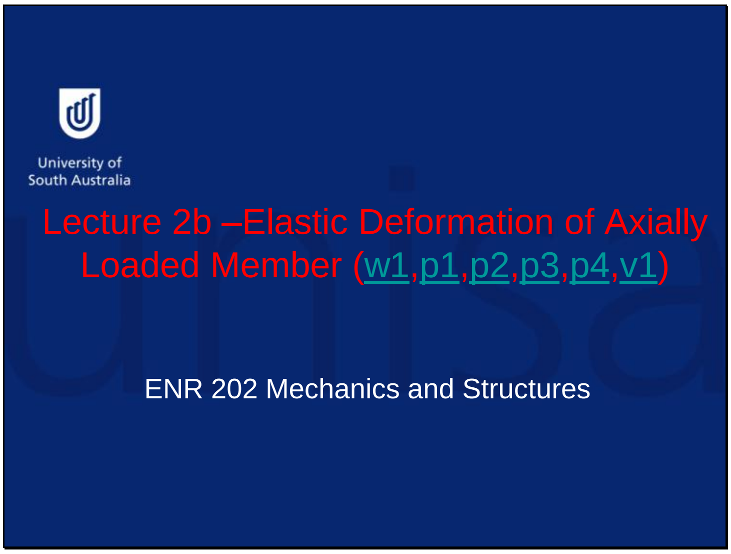

Welcome to Lecture Summary 2B, in which we will discuss elastic deformation of axially loaded members.

Note that throughout all the lecture summaries for Mechanics of Materials, you will see live links, denoted by the letters W, P and V. These links point to web pages, presentations and videos which will enhance your understanding of the content. You can pause the presentation at any time to access these links, and then go back to the presentation when you have finished looking at them.

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Requirements for students

- To understand definition of "axially loaded members"
- To get familiar with free body diagram method
- To be able calculate normal force, normal stress and axial deformation based on normal strain method.

Today, we will look at axially loaded members.

If we apply load on a structure, we will get internal actions in the structure. There are four types of loads: axial force, shear force, bending moment and torsion. In this lecture summary, we will introduce axial force in an axially loaded member. We have already studied some concepts of axial force, but now we will be extending our knowledge on axially loaded members. In future lectures, we will look at the other internal actions in a structural component: shear force, bending moment and torsion.

By the end of this lecture, you should understand what axially loaded members are, and you should be familiar with free body diagrams, because you will use these free body diagrams to analyse axial forces. You should also be able to calculate normal force, normal stress, normal strain and total axial deformation.

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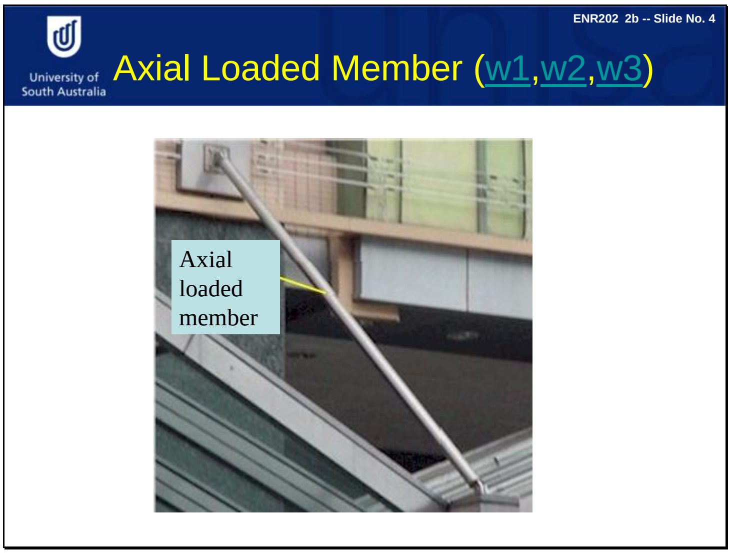

Let’s look at an example: this axial loaded member supports the roof. An axially loaded member may be in compression or in tension.

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Columns transfer loads from the roof to the foundation. Here, the columns are subjected to compression force. We also have some other structures, such as a truss structure. Truss members only carry axial load.

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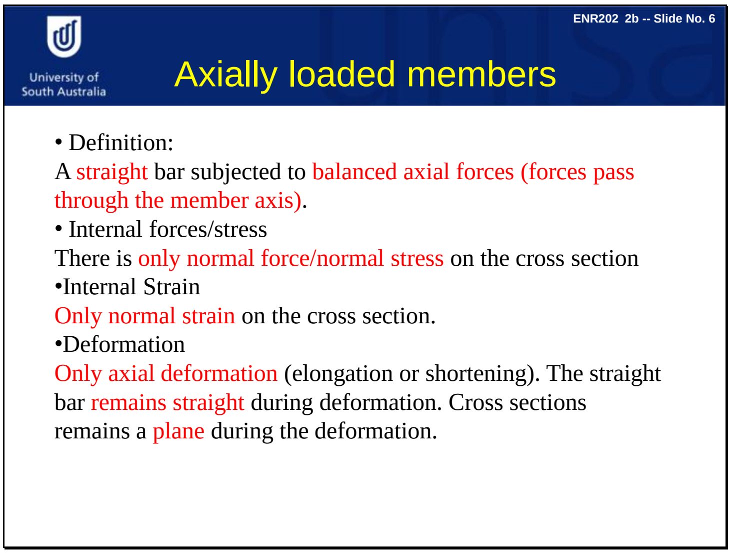

An axially loaded member is defined as a straight bar subjected to balanced axial forces, which means that the forces pass through the longitudinal axis or member axis. So what happens if bar is curved? If we have a curved bar, even though the bar subjected to axial force, we will get bending moment inside the bar. We will look at this in more detail in future lectures. If force acts in the direction of the longitudinal axis, but does not pass through this axis, what will happen? The bar will rotate about the longitudinal axis, which produces bending moment. Therefore, axial force passes through member axis and in the same direction as this axis. The bar must be a straight bar subjected to balanced axial force which may be compression or tension.

Due to axial load, we get internal forces in the axially loaded member. In this case, the internal force is just normal force, because it is perpendicular to the cross section. The normal forces cause normal stress on the cross section of the member. You will have elongation if axial force is in tension, or contraction if the axial force is in compression. These are called deformation of the bar. This deformation causes strain in the member. (Remember that we have studied Hooke's law which focuses on the relationship between stress and strain.)

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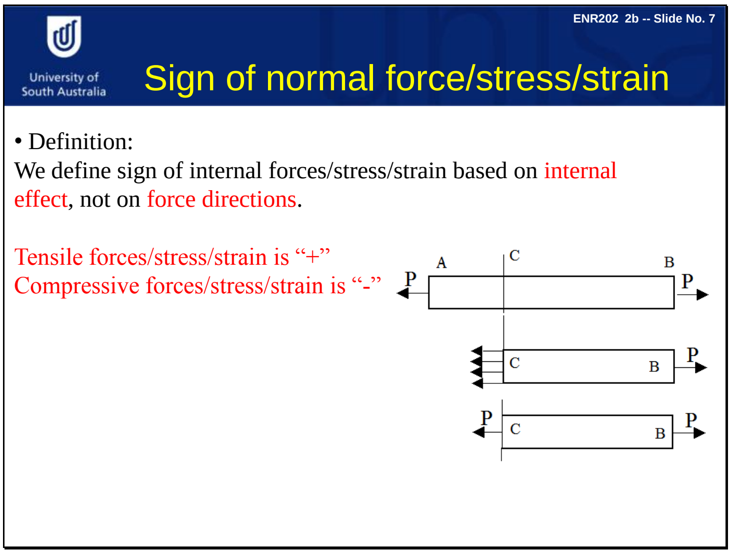

The sign of normal force, stress or strain is based on force effect. We don't consider the direction of the force. We only consider the force effect in mechanics of materials. For example, normal force may be compression or tension.

Let's look at an example. Suppose that we have a straight bar subjected to tension, and we cut the bar at point C. If we consider the right part (BC) of the bar, the internal force at C is in a left direction. However, if we consider the left part of the bar (AC), the internal force at C is to the right. So, the internal force at C is in both the left and right direction. We cannot define the sign of the internal force by direction. We cannot say the left direction is positive and the right direction is negative. We are considering the effect of the force, not the direction of the force, when we define the sign. So, remember that tensile force stress or strain is positive and compression force stress or strain is negative.

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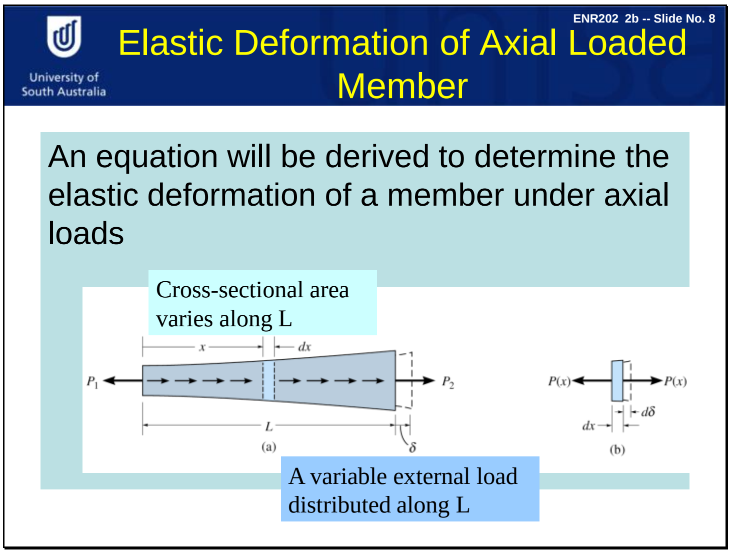

As you know, strain, denoted by epsilon, is equal to the change in length ‘delta’, divided by original length as shown in the figure. The average strain is equal to the total deformation divided by the total length. However, if you have a variable axial loaded member which does not have a uniform cross sectional area throughout the length of member, you will get non-uniform stress. So, you will get non-uniform strain, and non-uniform deformation along the length of the bar. Here, we consider an infinitesimal length of the member ‘dx’ as shown in the figure. The length of this segment is dx. The total deformation of this segment is ‘d delta’.

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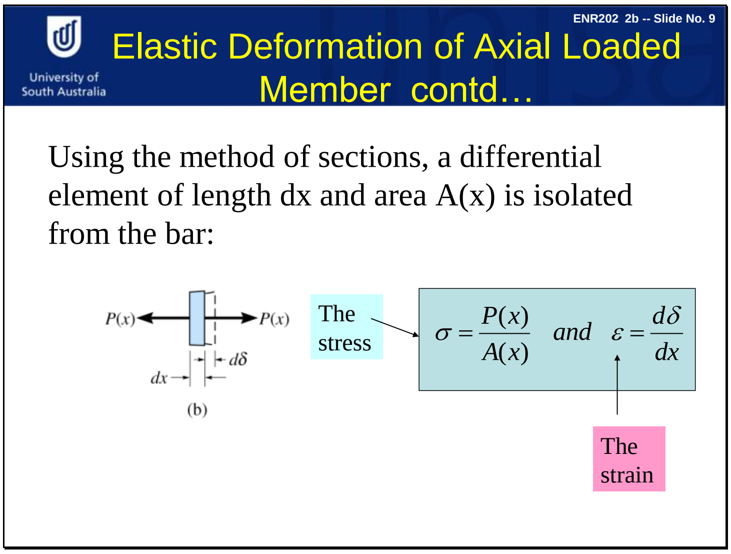

The definition of strain is the change in length 'd delta' divided by the length of the segment 'dx'. This, in fact, is a differential equation for strain changing along the x-direction. The stress denoted by sigma and is equal to the normal force at x, which is P(x), divided by the area of the cross section at x, which is A(x).

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Elastic Deformation of Axial Loaded Member contd…

If the materials are linear and elastic, apply Hooke's law (v1):

$$\frac{P(x)}{A(x)} = E\left(\frac{d\delta}{dx}\right) \Rightarrow d\delta = \frac{P(x)dx}{A(x)E}$$

For the entire length L of the bar:

The end displacement $\Rightarrow$ $$\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

Based on Hooke's law, Young modulus is defined as stress divided by strain. If you substitute the stress and strain at x into this equation, you will get the strain at the x point: top left side and the top equation. Then we will get the total deformation 'delta' for the length of the bar, which will be an integral as shown in the equation. This is the general formula to calculate the total deformation of an axially loaded member if the bar is subjected to variable force along the length.

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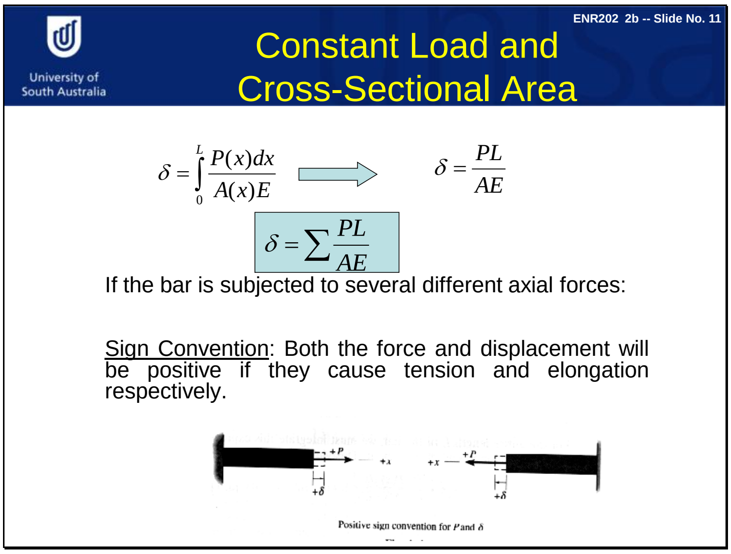

Now we can get a simplified formula for some simple cases.

For example, the axial force ‘P’ and the area of the cross section are uniform along the length of the bar. The integral finally changes to a simple formula: that is, axial force ‘P’ times ‘L’ divided by ‘A’ times ‘E’. We don’t need to do the integration. The condition for this formula is that the axial force ‘P’, and the area of the cross section ‘A’, and the materials do not change along the length of the member ‘A’. Note that if you use the general formula with integration, there is no need to follow these conditions. However, if you use simple formula, you must follow the conditions.

If we have several axial loads acting on one member, we have to separate the structural member into several segments in such a way that the condition is valid: the axial force ‘P’ and the area of the cross section ‘A’ and the material does not change within each member segment. Finally, we can get the total deformation ‘delta’ as equal to sigma ‘P’ times ‘L’ divided by ‘A’ times ‘E’.

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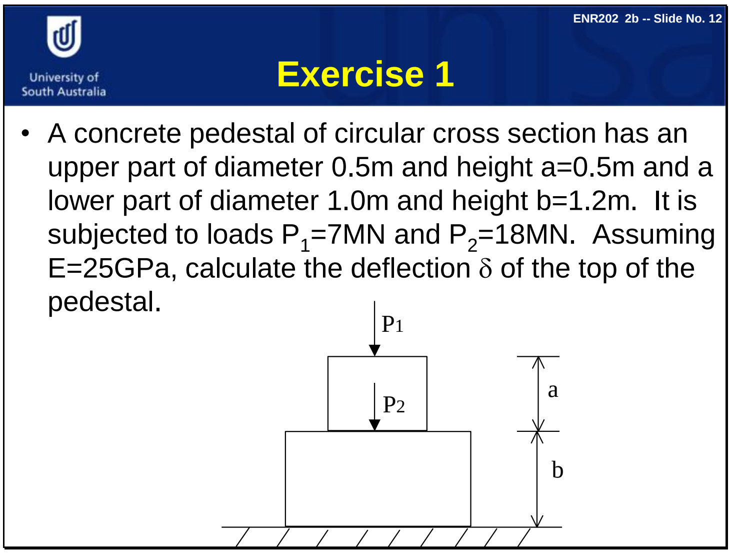

In first exercise, we have a concrete pedestal with a circular cross section. The top part of diameter is 0.5 meters and the height of the top part ‘a’ is 0.5 meters. The diameter of the lower part is 1 meter and the height of the bottom part ‘b’ is 1.2 meters. The load P1 is 7 Mega Newtons acting on the top of the top part of the pedestal, as shown in the figure. There is a second load P2, which is 18 Mega Newtons, acting between the bottom and top part, as you can see in the figure. We already know that the Young’s modulus of concrete is 25 Giga Pascals.

We need to calculate the total deformation of the concrete pedestal. If you think that you can work this out yourself, pause the presentation and give it a try. If you would like to see the solution, it is on the next two slides.

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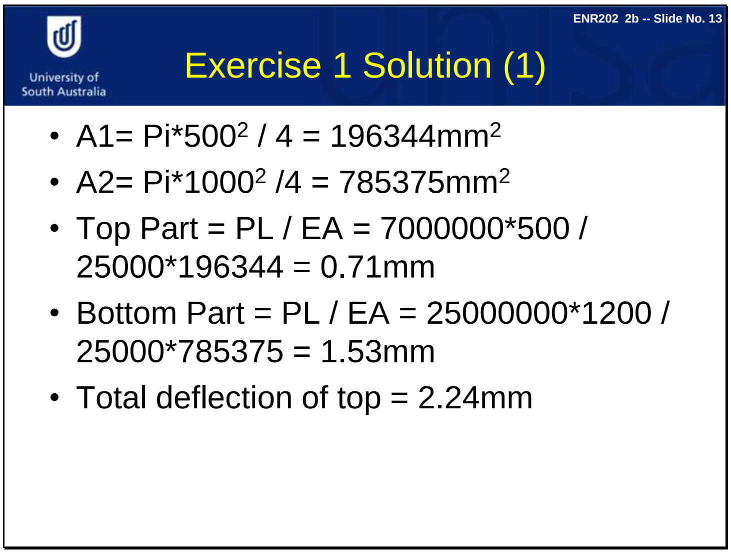

This is the solution to Exercise 1.

In fact, we have two parts here. The first part is the top part and the second part is the bottom part. The two parts have different cross sectional areas and different normal force. You should use free body diagrams to calculate the normal forces in each part.

If you draw the free body diagram for the top part, you can see that there is external force P1, the normal force in the top part, which is equal to 7 Mega Newtons. If you draw the free body diagram for bottom part, you can see that there are two external loads, P1 and P2. So, the normal force in the bottom part should equal to P1+ P2, which is 7 Mega Newtons plus 18 Mega Newtons, which is 25 Mega Newtons. You can use a simple formula to separately calculate total deformation for the top part and bottom parts.

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Exercise 1 Solution (2)

- A1= Pi*500^2 / 4 = 196344mm^2
- A2= Pi*1000^2 /4 = 785375mm^2
- Top Part = PL / EA = 7000000*500 / 25000*196344 = 0.71mm
- Bottom Part = PL / EA = 25000000*1200 / 25000*785375 = 1.53mm
- Total deflection of top = 2.24mm

Now we can calculate the deformation of the top part. The area of the cross section of the top part is equal to 196 344 square mm. The normal force for the top part is 7 Mega newtons, and the length of the top part is 0.5 meters. The material is the same for both parts. Therefore, Young's Modulus is 25 Giga Pascals.

Make sure that you use consistent units in the formulas. Convert all force units into Newtons and all length units into mms. The area of the cross section is in square mm, Young's modulus is in Newtons per square mm.

The condition for the simple formula is that the axial force 'P' and the area of the cross section 'A' and the materials do not change along the length of the top part. From this, we get the deformation of top part as 0.71 mm.

Now, we can calculate the deformation of the bottom part. The area of the cross section of the top part is 785 375 square mm. The normal force for the bottom part is 25 Mega newtons. The length of the top part is 1.2 meters. The material is the same for both parts. Therefore, Young's modulus is 25 Giga Pascals. From this, we get the deformation of bottom part as 1.53 mms.

The total deformation of concrete pedestal is the sum of the deformation of the top part and the deformation of the bottom part, which is 2.24 mm.

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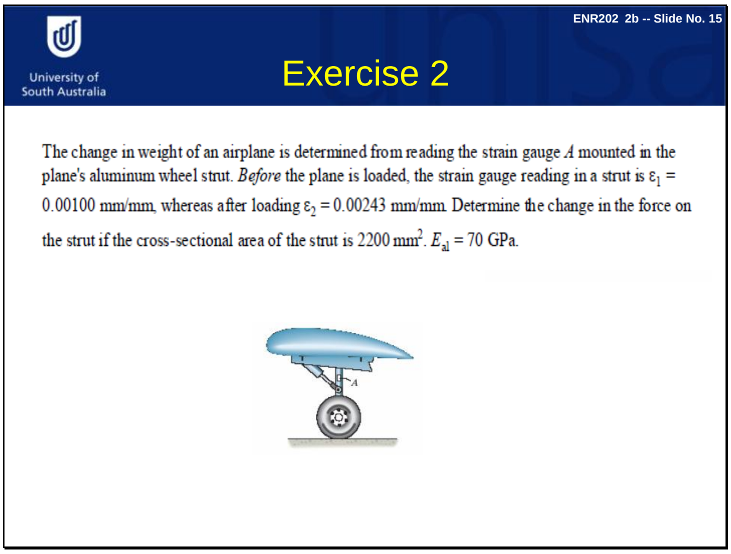

In this second exercise, we have a change in weight in an airplane, which is the difference between the weight before the plane is loaded and the weight after the plane is loaded. The change in weight is based on the reading from a strain gauge, ‘A’, which is mounted in the plane’s aluminium wheel strut. Before the plane is loaded, the strain gauge reading shows that epsilon 1 is equal to 0.00100 mm per mm. After the plane is loaded, the strain gauge reading shows that epsilon 2 is equal to 0.00243 mm per mm. The cross-sectional area of the strut is 2200 square mm. Young’s modulus of the aluminium wheel is 70 Giga Pascals.

You need to calculate the change in the weight on the strut (which means that really, you are calculating the weight of the passengers). Pause this presentation and try to solve this problem. The solution is on the next slide. Note that this example is based on Hooke’s law, and on the definition of stress and strain.

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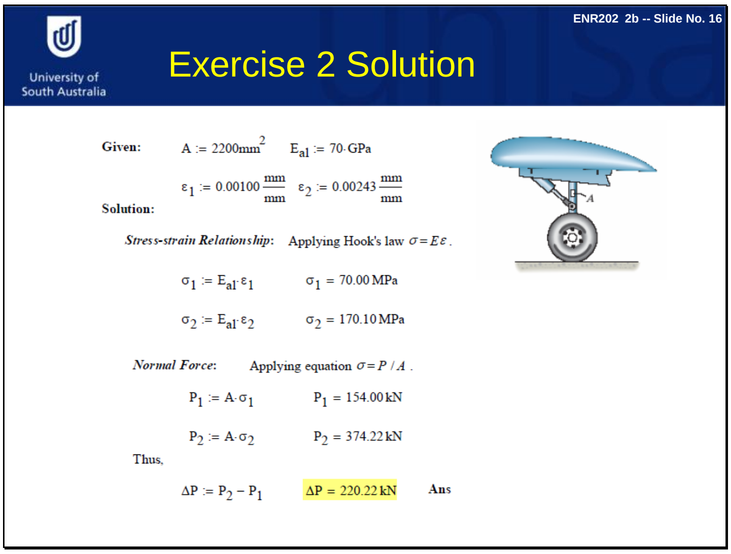

Here is the solution for the second exercise. You have been told that the area of the cross section is 2200 square mm. You have also been given Young's modulus, and the strain gauge readings before and after the plane is loaded.

By applying Hooke's Law, you can calculate the stress before and after the plane is loaded. The stress before the plane is loaded is equal to Young's modulus times the strain gauge reading, which is equal to 70 Mega Pascals or Newtons per square mm. The stress after the plane is loaded is equal to Young's modulus time the strain gauge reading, which is equal to 170.1 Mega Pascals.

We already know that normal stress is equal to normal force divided by the area of the cross section. Based on this rule, we can calculate the load P1 (before the plane is loaded) to be 154 Kilo Newtons. We can calculate the load P2 (after the plane is loaded) to be 374. 22 Kilo Newtons. Finally, we work out that delta P is equal to P2 minus P1, which is equal to 220.22 Kilo Newtons.

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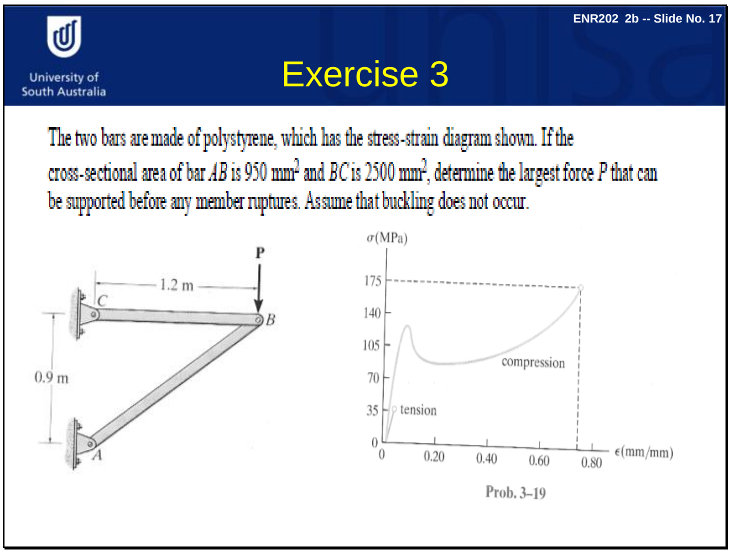

In this third exercise, a two bar truss made of polystyrene carries a load P acting in a downward direction at point B, as shown in the left figure. The cross sectional area of the inclined bar AB is 950 square mm and the cross sectional area of the horizontal bar BC is 2500 square mm. The length between A and C is 0.9 meters, and the length between C and B is 1.2 meters. This means that the length of the AB bar should be 1.5 meters and BC bar is 1.2 meters. The material is special, because it doesn't have the same strength in compression and tension. Therefore, we have different stress strain diagrams for compression and for tension, as shown in the figure. The maximum compressive stress is 175 Mega Pascals and the maximum tensile stress is only 35 Mega Pascals.

We have to check the safety of both bars AB and BC. You need to know which bar carries tension and which bar carries compression, based on load P acting in a downward direction at point B. Pause the presentation here and try to do this exercise.

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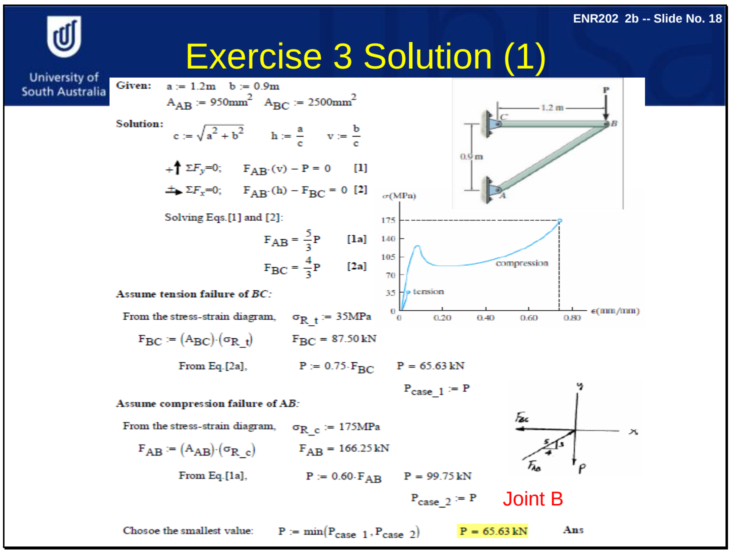

Here is a solution to the third example. We know that the distance between A and C (called ‘b’) is 0.9 meters, and the distance between B and C (called ‘a’) is 1.2 meters. Therefore, we can easily calculate the distance between A and B (called ‘c’). It is the square root of ‘a’ squared plus ‘b’ squared, which is equal to 1.5 meters. The horizontal component ‘h’ is equal to ‘a’ divided by ‘c’ and the vertical component ‘v’ is equal to ‘b’ divided by ‘c’.

Now, we have to calculate the internal forces in the member BC and the member AB. To do this, we draw the free body diagram of joint B. To draw the free body diagram, remove all supports and replace with reaction forces. So, for joint B, you remove members AB and BC and replace them by the reaction internal forces in the members. The force in member AB is FAB and the force in member BC is FBC . The load P is acting in a downward direction. The internal force in member AB acts in an upward direction, which means that the force in member AB is compression. If you resolve the FAB into a horizontal direction, this force acts in a right-ward direction. That means that FBC is acting in a leftward direction. Therefore, the force in member BC is tension.

As you know, if a structure is in equilibrium, that means that all structural members and all structural joints are in equilibrium. Therefore we can apply the equilibrium concept and use equilibrium equations at joint B. The first equilibrium equation states that all forces in a vertical direction are equal to zero. Therefore, the first equilibrium equation is that FAB times v minus P is equal to zero. The second equilibrium equation states that all forces in a horizontal direction are equal to zero. Therefore, the second equilibrium equation is that FAB times h minus FBC is equal to zero. So we have two equations and two unknowns FAB and FBC. We can calculate the force in member AB by working out that FAB is equal to 5 divided by 3 times P and the force in member BC is FBC equal to 4 divided by 3 times P.

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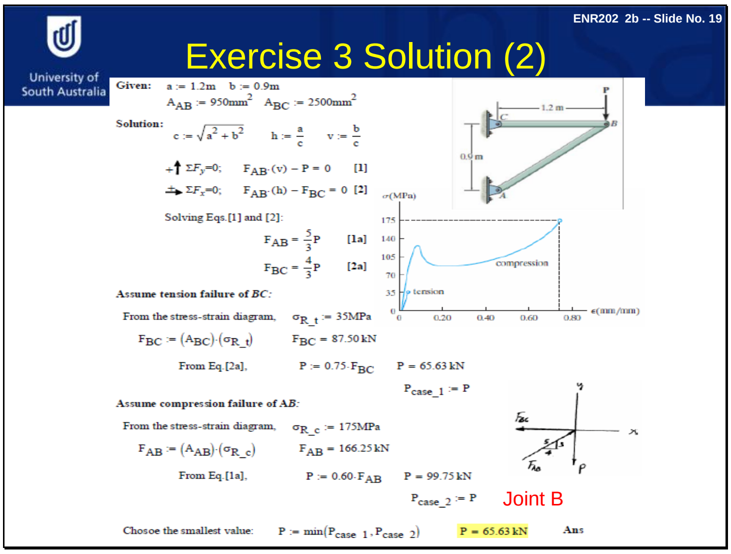

Here, there are two possibilities to get failure of the two bar truss. The first is compressive failure in member AB, and the second is tensile failure in member BC. Therefore, we need to check both possibilities to make sure that the structure is safe. We know the stress strain diagram of polystyrene in compression as well as tension.

Case 1: We know the force in member BC is in tension in terms of load P. Tensile failure stress from the stress strain diagram is 35 Mega Pascals. The area of the cross section of member BC is 2500 square mm. We already know that the definition of stress is force divided by area. Therefore, we can calculate load P as equal to 65.63 kilo Newtons.

Case 2: We know the force in member AB is in compression in terms of load P. Compressive failure stress from the stress strain diagram is 175 Mega Pascals. The area of the cross section of member AB is 950 square mm. Therefore, we can calculate load P as being equal to 99.75 kilo Newtons.

Therefore, if you apply 99.75 kilo Newtons, member AB is safe, but member BC will fail. If you apply 65.63 kilo Newtons, both members are safe. So a force of 65.63 kilo Newtons is the minimum load P for both cases.

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Thank you for your attention.