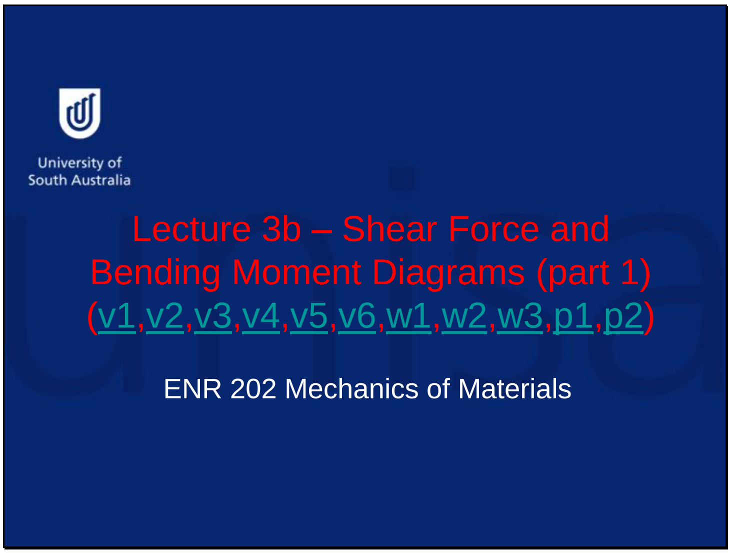
ENR202 Mechanics of Materials Lecture 3B Slides and Notes

Slide 1

ENR202 Mechanics of Materials Lecture 3B Slides and Notes

Slide 2



Hello, and welcome to Lecture Summary 3b. In past lecture summaries, we have learnt how to calculate reaction forces for a simply supported beam by using equilibrium equations and the concept of see saw and lever. In this lecture summary, we are going to learn about shear force diagrams (the abbreviation is SFD) and bending moment diagrams (the abbreviation is BMD). There are three parts to this. In the first two parts. we will study how to use the equilibrium method to draw SFDs and BMDs. In third part, we will look at using the graphical method to draw SFDs and BMDs.

Note that throughout all the lecture summaries for Mechanics of Materials, you will see live links, denoted by the letters W, P and V. These links point to web pages, presentations and videos which will enhance your understanding of the content. You can pause the presentation at any time to access these links, and then go back to the presentation when you have finished looking at them.

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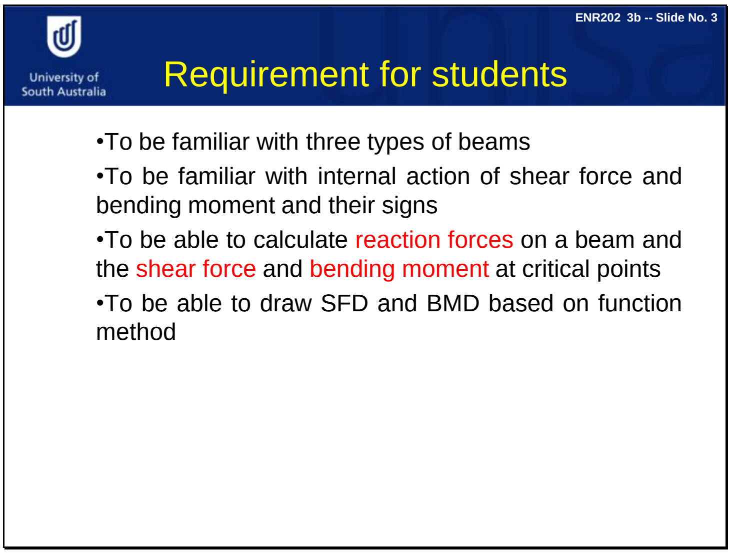

In this lecture, you are required to become familiar with three types of beams, with the internal action of shear force, and with bending moments. You will be able to calculate reaction forces on a beam and shear force, and bending moment at critical points inside the beam. You will also be able to draw shear force diagrams and bending moment diagrams based on the function method.

We have already introduced how to calculate reaction forces on a beam. Today, we will study how to calculate shear force and bending moment in the entire length of the beam.

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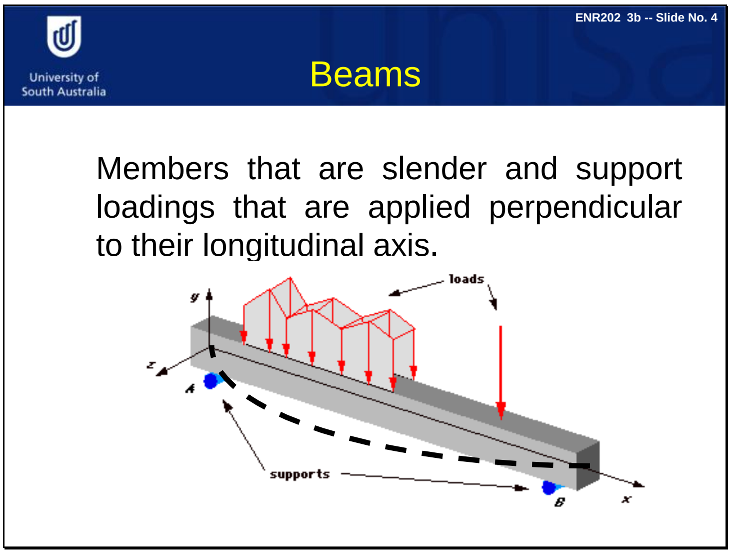

In fact, a beam is a straight member restrained by supports which carry lateral loads acting perpendicular to longitudinal axis of the beam. The beam is a very common structural member in civil engineering. A beam acts as a support for the load coming from the roof or slab of a building . If loads are applied on the beam, then the beam will bend, and this will cause deformation.

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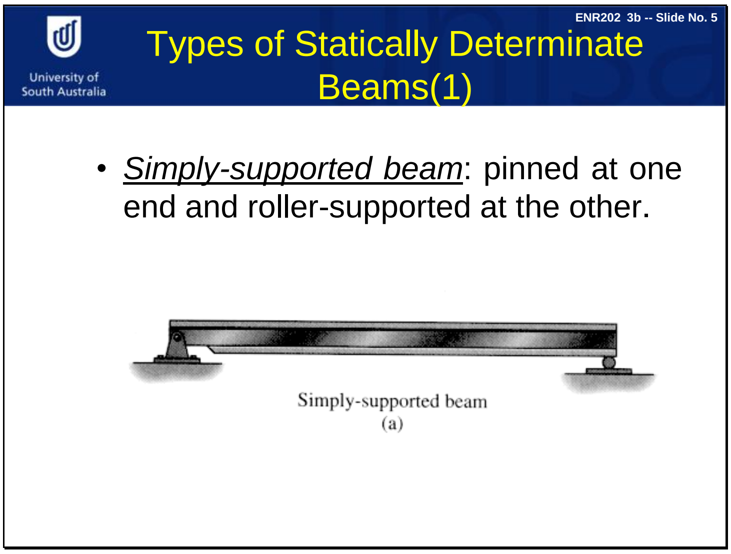

We have different kinds of beams, depending on the type of supports. For example, we can have one end pinned support and the other end a roller. This beam is called a simply supported beam. This beam can resist reaction forces in a horizontal and vertical direction at the pinned end, and can resist a vertical reaction force at the roller end. However, this beam cannot resist moment at both supports.

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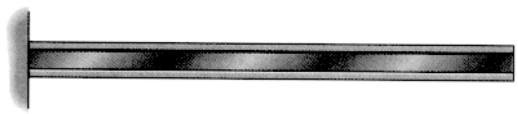
Types of Statically Determinate Beams(2)

- *Cantilevered beam*: fixed at one end and free at the other.



Cantilevered beam
(b)

The second type of beam is a cantilevered beam, in which one end is fixed and the other end is free, as shown in the figure. In the fixed end, we can have reaction forces in a horizontal and vertical direction, and also moment. So, you can see that this type of beam is totally different from a simply supported beam. We don’t have any support on the other end, so there are no reaction forces at that free end.

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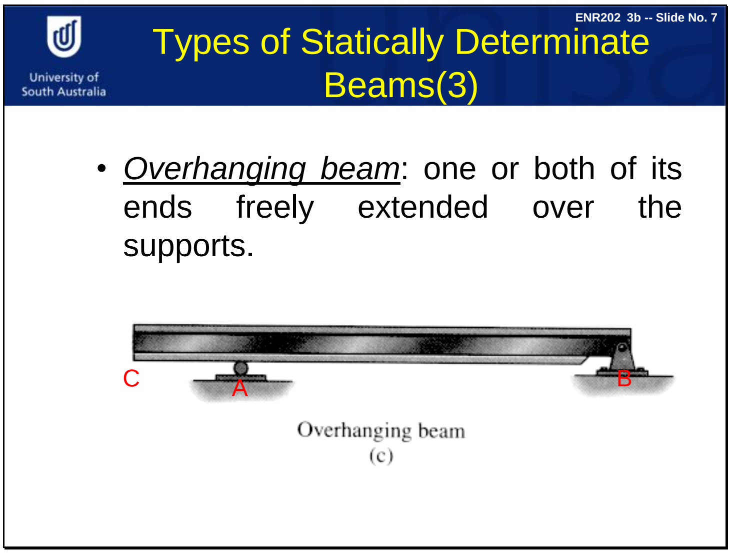

The third type of beam is an overhanging beam. This beam is a combination of a simply supported beam and a cantilevered beam. Point B is a pinned support , Point A is a roller support and Point C is a free end. Part AC is an overhang portion. Part AC is similar to a cantilever. Part AB is similar to a simply supported beam. Therefore, this overhang beam is a combination of a simply supported and a cantilevered beam.

You need to keep in mind that the beam is continued at point A, although there is no moment acting at point A. There is no disconnection of the beam at point A.

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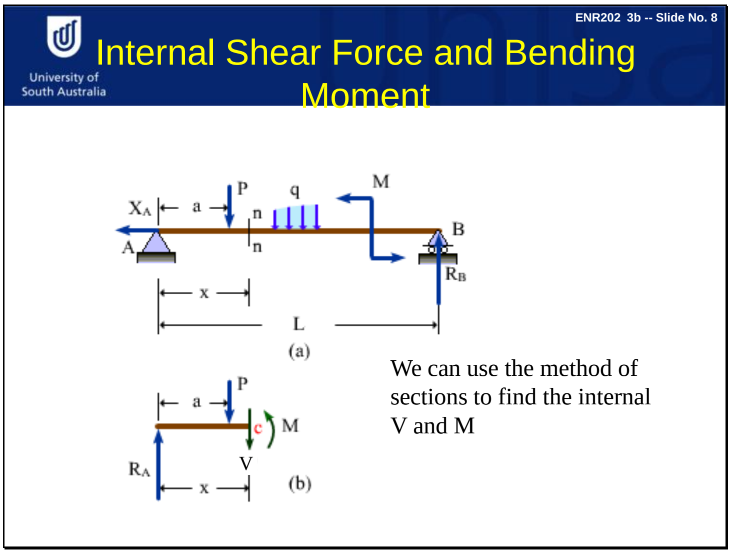

For all these kind of beams, we may have different lateral loads acting on the beam: distributed load ‘q’, concentrated load ‘P’, or concentrated moment ‘M” , as shown in the figure ‘a’.   Suppose you cut the beam at the n-n cross section, and draw the free body diagram of the left part of the beam. Then you have three internal actions at the n-n cross section. We have normal force (it is not shown in the figure because there is not horizontal loads acting on the beam), another force ‘V’, and bending moment ‘M’ in the beam segment, as shown in the figure ‘b’. The force ‘V’ is parallel to the cross section n-n, so we called this force ‘V’ shear force.

So there are two basic internal actions in the beam. One is shear force ‘V’ and other one is bending moment ‘M’’.

You need to calculate the shear force ‘V” and the bending moment ‘M’, to design a beam at that cross section n-n.   We have already looked at how to calculate reaction forces.

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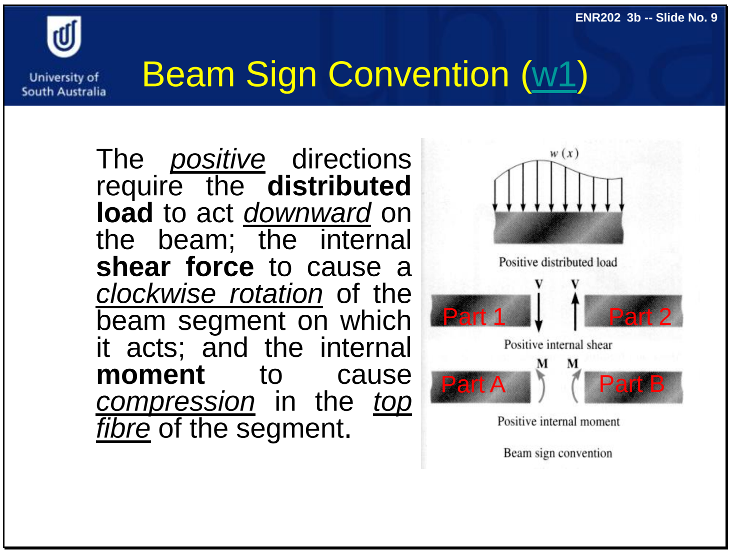

It is important to know beam signs conventions – that is, the signs of shear force and bending moment. It is very important in Shear Force Diagrams and Bending Moment Diagrams. Look at Part 1: if the shear force acting in a downward direction rotates Part 1 in a clockwise direction, and if the shear force acting in Part 2 in an upward direction rotates Part 2 in a clockwise direction, this shear force is positive. If bending moment causes tension in the bottom part of Part A and Part B of the beam segments, and causes compression in the top of part A and Part B of the beam segments, we call this bending moment positive: if the reverse is true, we call the bending moment negative.

The principle here is the same as we explained when talking about Axial loaded members. We consider the internal force affect, not the direction. If we cut the beam at any cross section, we know that shear force occurs at the cutting surface. If you consider the left part of the beam segment, the shear force is in an upward direction. If you consider the right part of the beam segment, the shear force is in a downward direction. However, both shear forces are the same, because both are acting at the same cutting surface. Both are positive shear forces – in this case, we can't say that up is positive and down is negative based on direction. Remember, we have to consider the force effect, not the direction of the force.

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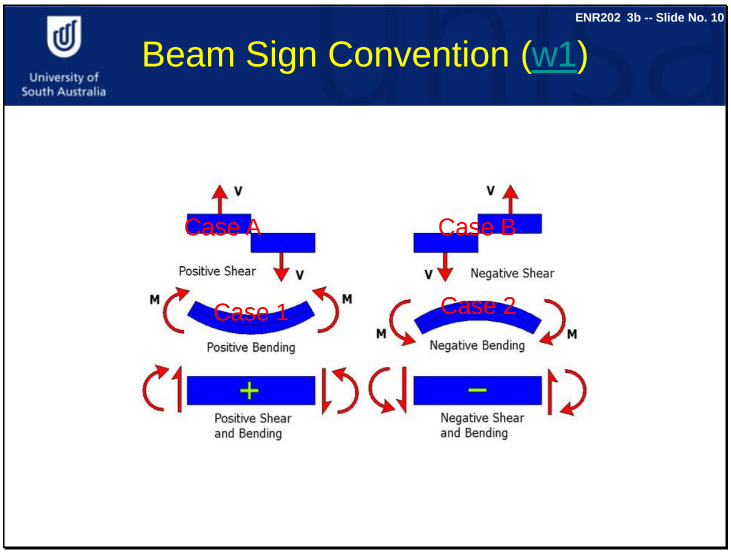

In Case A: Once you cut the beam, you need to know if the shear force is positive or negative. If the left beam segment goes up and the right beam segment goes down, then this type of shear force called positive shear force.

In Case B: If the left beam segment goes down and the right beam segment goes up, then this type of shear force is called negative shear force.

In Case 1: if we have bending moment ‘M’ acting as shown in the figure, then the top part of the beam segment is in compression and the bottom part of the beam segment is in tension. So this type of bending moment is called positive bending moment.

In Case 2: if we have bending moment ‘M’ acting as shown in the figure, then the top part of the beam segment is in tension and the bottom part of the beam segment is in compression. So this type of bending moment is called negative bending moment.

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General procedure to calculate shear force and bending moment in beams

- Calculate reaction forces
- Draw free body diagram with cutting at the calculating point. For cutting at continuous points, there are three internal actions: normal force(usually =0), shear force and bending moment.
- Calculate shear force and bending moment at important points based on equilibrium equations

The procedure for analysis is as follows:

Step 1, calculate reaction forces at the supports. This part may be very easy, but it is very important, because if you calculate the reaction forces incorrectly, your shear force and bending moment values will also definitely be wrong. So, the reaction force calculations must be correct.

Step 2, once you have calculated the reaction forces, you can draw the free body diagram of one of the beam segments after cutting at any cross section of the beam. As you know, we have internal actions at the cutting cross section.

Step 3, once you have drawn the free body diagram, then you calculate the shear force functions and bending moment functions by using equilibrium equations.

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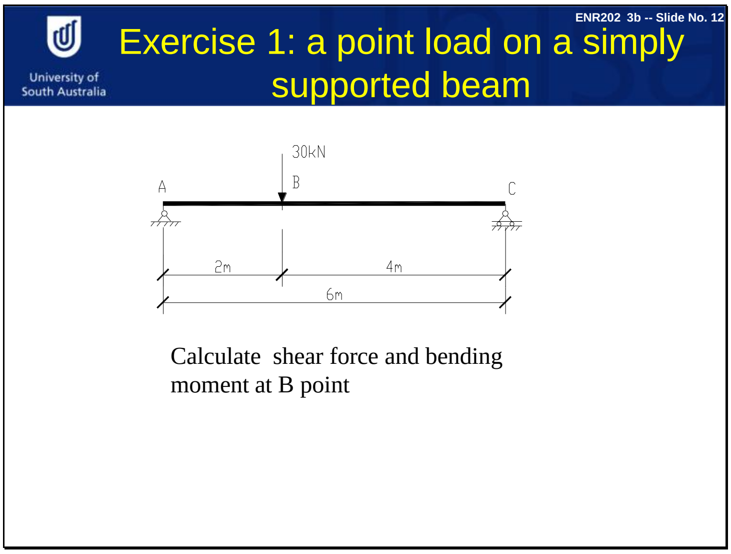

Now, pause this lecture summary, and try to do this exercise. A simply supported beam is subjected to a concentrated load (or point load) of 30 kilo Newtons at Point B (that is, at a distance 2 meters from point A) as shown in the figure. The total length of the beam is 6 meters. Calculate shear force and bending moment at Point B. (The solution is on the next three slides.)

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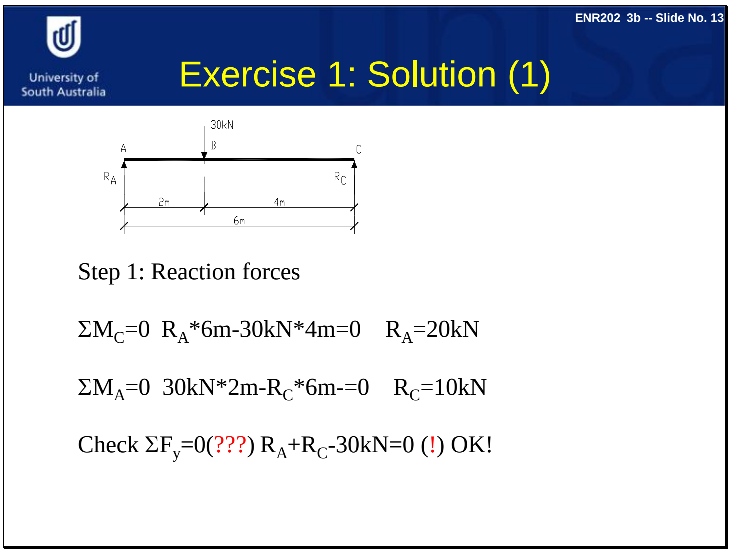

This is the solution to the exercise.

First, calculate the reaction forces for a simply support beam (go to Lecture Summary 3A if you are not sure how to do this.) The sum of the moments about Point C is equal to zero. We have two forces moments about Point C. One is the reaction force RA and other one is the point load of 30 Kilo Newtons. The moment of Reaction force RA about point C is equal to RA times the distance of 6 meters, and is in a clockwise direction, and the moment of point load about Point C is equal to 30 Kilo Newtons multiplied by a distance 4 meters, and is anti-clockwise. So we can work out the reaction force at A support is 20 Kilo Newtons.

Now, the sum of the moments about Point A is equal to zero. We have two forces moments about Point A. One is the reaction force Rc and the other is point load of 30 Kilo Newtons. The moment of Reaction force Rc about Point A equals Rc times the distance 6 meters, in an anti-clockwise direction. The moment of point load about Point C is equal to 30 Kilo Newtons multiplied by a distance of 2 meters, and is clockwise. So we can work out the reaction force at the c support is 10 Kilo Newtons.

We can check our results for the reaction forces at points A and c. We know that the equilibrium equation means that all forces in a vertical direction are equal to zero. So, the sum of the reaction forces at point A and point C is equal to a load of 30 Kilo Newtons. So, we know that our calculations of reaction forces are correct. This is the first step for calculating shear force diagrams and bending diagrams.

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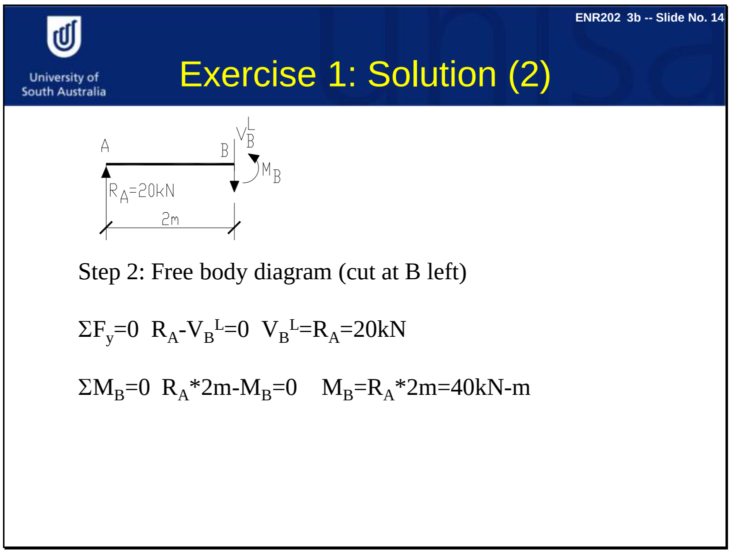

To continue this solution:

If you want to calculate the shear force and the bending moment at the left side of Point B, you just cut the beam there and consider the left part of the simply supported beam segment (as shown in the figure). Draw the free body diagram. In the free body diagram, the reaction force at support A is RA, and the left shear force at B is VB. VB, the shear force coming down at the cutting point, is a positive shear force because the left shear force at B rotates the beam segment in a clockwise direction. The moment at B at the cutting point is MB, and is in an anti-clockwise direction. It is a positive moment because the top part of the beam is in compression and the bottom part of the beam is in tension.

The next step is to apply equilibrium equations to the free body diagram. Let's look at the vertical forces. The reaction force at A minus the left shear force at B must equal zero. This means that the left shear force at B is 20 kilo Newtons. The next equilibrium equation is applied to the sum of all forces moments at Point B. We have 3 types of moments about Point B. One is the moment of reaction force RA about Point B, which is RA times 2 meters. Another one is the moment of the left shear force at point B, which is 'left VB' times zero, and the moment at Point B is MB. Altogether these moments equal zero. So we can calculate the moment at B as 40 Kilo Newton meters.

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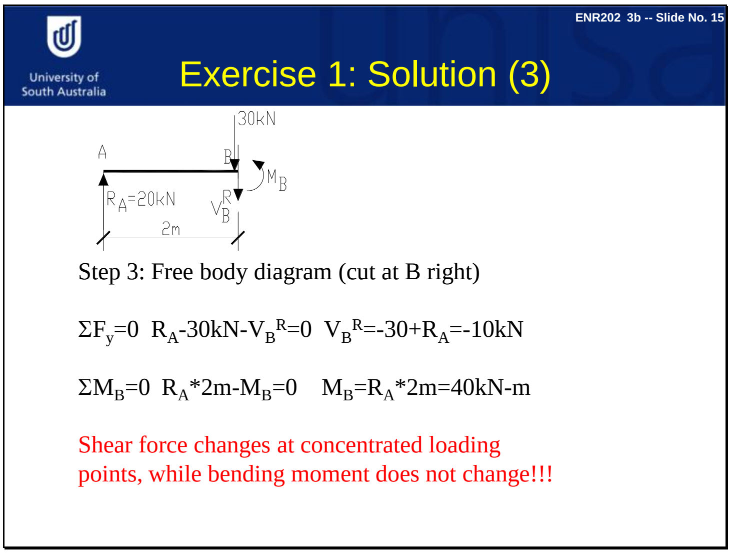

To continue this solution:

If you want to calculate shear force and bending moment at the right side of Point B, you just cut the beam there and consider the left part of the simply supported beam segment as shown in the figure. Draw the free body diagram. The free body diagram shows the reaction force at support A as RA, a point load of 30 kilo Newtons acting down. The right shear force at B is right VB, coming down at the cutting point, and is a positive shear force because the right shear force at B rotates the beam segment in a clockwise direction. The moment at B at the cutting point is MB, and is in an anti-clockwise direction. It is positive moment, because the top part of the beam is in compression and the bottom part of the beam is in tension.

The next step is to apply equilibrium equations to the free body diagram. All forces in a vertical direction must be equal to zero. That is, the reaction force at A minus the point load of 30 kilo newtons, the right shear force at B, must equal zero. Upward force can be positive or negative – it doesn't matter because you will get same answer; that is, the right shear force at B is minus 10 kilo Newtons. That means that we have a negative shear force at the right side of Point B.

The next equilibrium equation to work out is the sum of all forces moments at Point B. We have four types of moments about Point B. One is the moment of reaction force RA about point B, which is RA times 2 meters in a clockwise direction. The second one is the moment of right shear force at B about Point B, which is 'right VB' times zero. The third one is the moment of point load at Point B, which is point load times a distance of zero meters (because the point load is acting at Point B only). Finally, the moment at Point B is MB and is anti-clockwise. All together equal zero.

The clockwise direction can be positive or negative, again, it doesn't matter, because it will produce the same answer. The moment at Point B is 40 Kilo Newton meters.

You can compare the two results for the left side of Point B shear force and bending moment, and the right side of Point B shear force and bending moment. We have a different shear force, but the same bending moment. At point load, we have two shear force values just to the left side of the point load and just to the right side of the point load. The difference in these two values is equal to the value of the point load and is 30 kilo Newtons. However, you will get same bending moment at the point load irrespective of whether it is on the left or right side of the point load.

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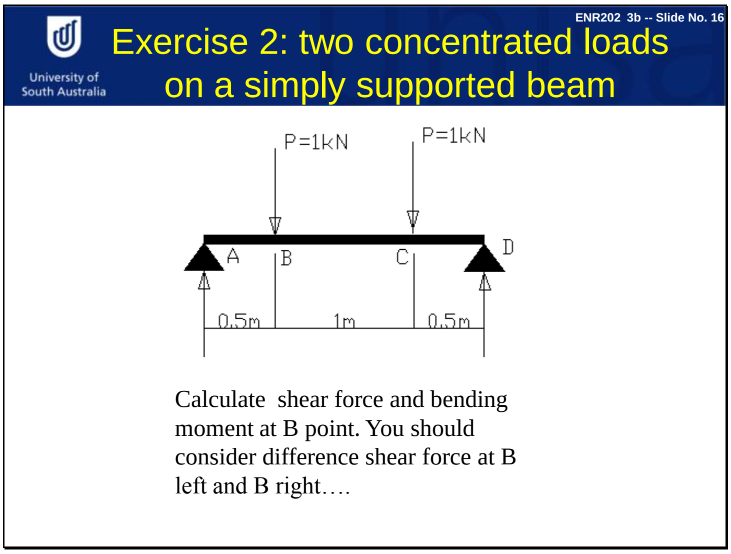

Now have a look at this second exercise. A simply supported beam subjected to two concentrated loads as shown in the figure. Both concentrated loads are with same values: that is, 1 kilo Newton. The total length of the beam is 2 meters.

Pause the presentation and calculate the shear force and bending moment at Point B. (You must cut the beam on the left side of Point B once and then on the right side of Point B, as explained in the previous example.)

The solution is in the following slide.

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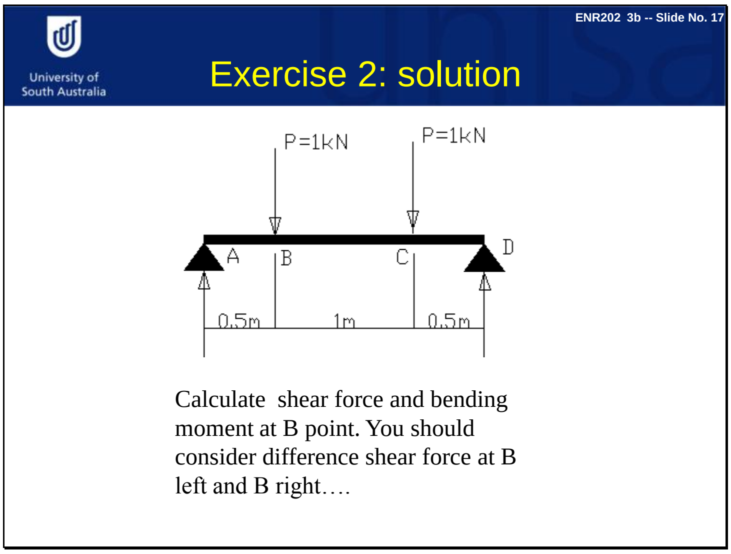

Exercise 2 solution.

The first step is to calculate the reaction forces. The reaction force at A point is 1 kilo Newton and reaction force at D point is 1 kilo Newton.
For the left side of the B point, shear force at B is 1 kilo newtons and bending moment is positive bending moment 0.5 kilo Newton meters.
For the right side of the B point, shear force at B is zero kilo newtons and bending moment is positive bending moment 0.5 kilo Newton meters.

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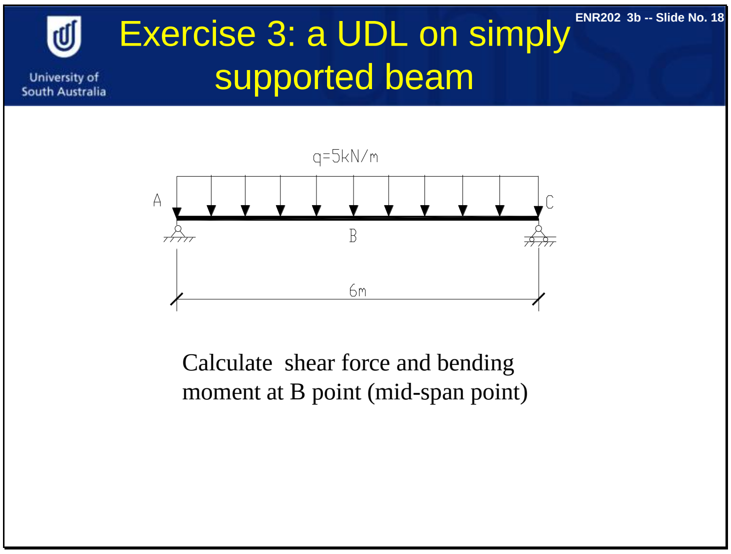

Pause the lecture summary here and try this exercise. A simply supported beam is subjected to a uniformly distributed load of 5 kilo Newtons per meter. The span is 6 meters. Calculate the shear force and bending moment at Point B (that is, the mid-span point of the beam). The solution is on the following two slides.

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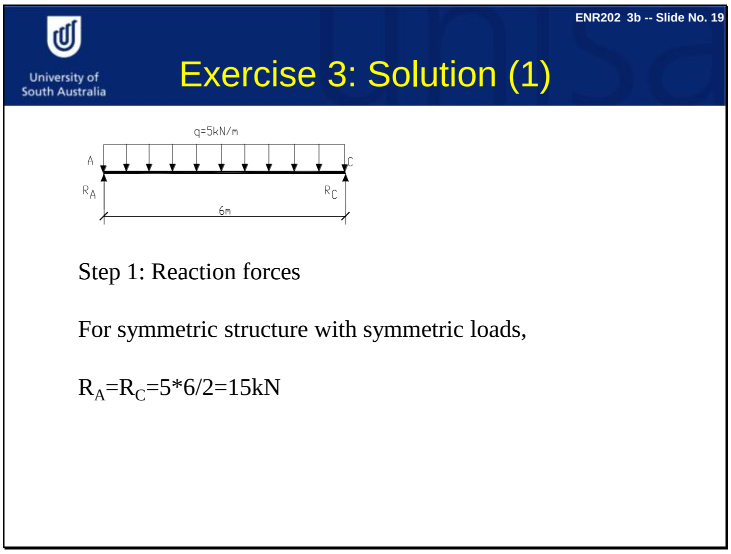

First, calculate the reaction forces at both supports. The beam is subjected to a symmetrical load. Therefore, we will get symmetric reaction forces which are the same and are equal to half of the uniform distributed load. The reaction force of the UDL is 5 kilo newtons per meter times six meters, which is equal to 30 kilo newtons. This acts on the centre of the UDL (that is, point B). The reaction force at A and the reaction force at C are equal to half of the 30 kilo newtons; that is, 15 kilo newtons.

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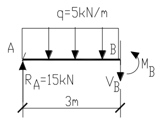
Exercise 3: Solution (2)



Step 2: Free body diagram (cut at B)

$\Sigma F_y=0 \;\; R_A\text{-}5*3\text{-}V_B=0 \;\; V_B=R_A\text{-}15=0\text{kN}$

$\Sigma M_B=0 \;\; R_A*3\text{m-}5*3*1.5\text{-}M_B=0$
$M_B=R_A*3\text{m-}15*1.5=22.5\text{kN-m}$

The solution to exercise 3.

To calculate the shear force and bending moment at Point B (the midspan of the beam), cut the beam at B and consider the left part of the simply supported beam segment as shown in the figure. Draw the free body diagram. The free body diagram shows the reaction force at support A as RA. The shear force at B is VB, and comes down at the cutting point. It is a positive shear force, because the left shear force at B rotates the beam segment in a clockwise direction. The moment at the cutting point Bi s MB, and is in an anti-clockwise direction. It is a positive moment, because the top part of the beam is in compression and the bottom part of the beam is in tension. The uniformly distributed load includes only the section from A to B of the beam segment. Note that you need to show the internal actions of shear force and bending moments which are positive only. The signs are coming from internal force effects, but not from direction.

The next step is to apply equilibrium equations to the free body diagram. First, apply an equilibrium equation to all forces in a vertical direction. The reaction force at A minus the shear force at B minus the resultant of the uniform distributed load (5 kilo newtons per meter times 3 meters which is 15 kilo newtons) is equal to zero. The shear force at B is equal to zero kilo Newtons.

The next equilibrium equation to apply is the sum of all forces moment about B. We have 3 types of moments about B. The first is the moment of reaction force RA, which is RA times 3 meters in a clockwise direction. The second is the moment of shear force at B, which is ‘VB’ times zero. The moment at B is MB in an anti-clockwise direction. The resultant UDL is 15 kilo newtons times the distance 1.5 meters (the centre of the distance between A and B), because the resultant UDL acts at the centre of the UDL from A to B. It is in an anti-clockwise direction. All together equal zero. So the moment at B is equal to 22.5 Kilo Newton meters.

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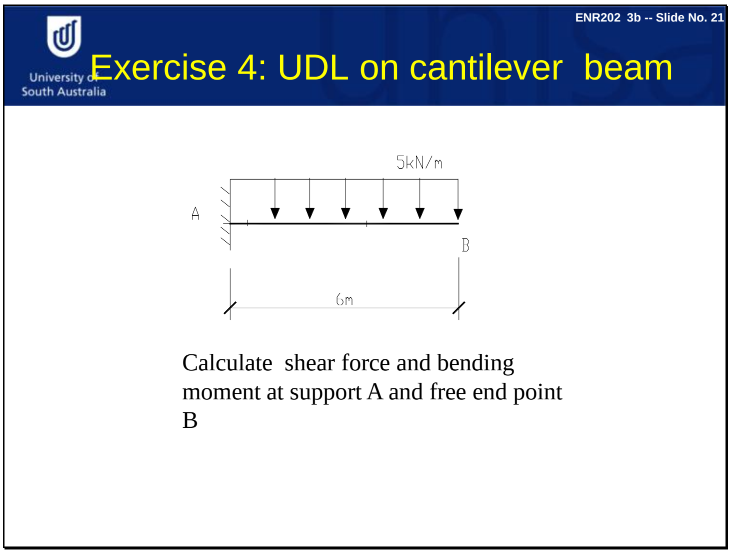

Now let's look at a cantilever beam. A cantilever beam is subjected to a uniformly distributed load of 5 kilo Newtons per meter. In the cantilever beam, one end is fixed and the other end is free. At Point A is a fixed support. Point B has no support.

Now pause the presentation, and calculate the shear force and bending moment at support A and at free end point B. The solution is on the following slide.

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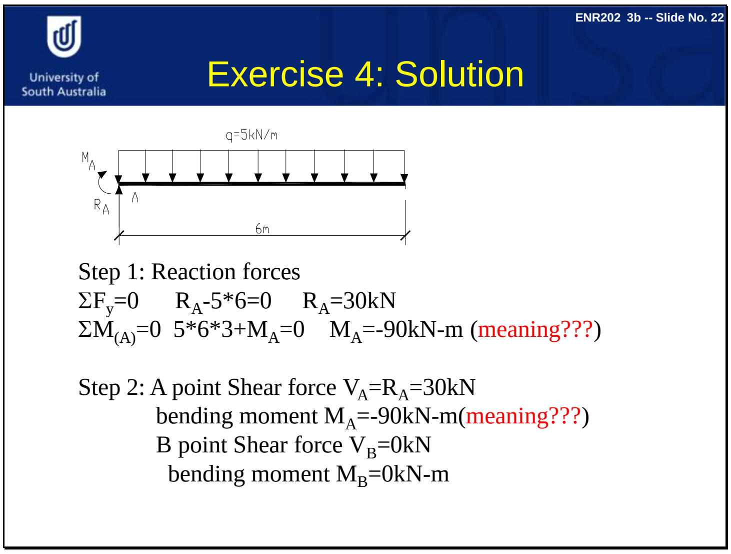

Solution to Exercise 3.

First, calculate the reaction forces. We have only one support (the fixed support at point A). The fixed support can resist three types of reactions: a horizontal reaction, a vertical reaction (RA), and a moment (MA).

We don’t have any horizontal loads in this beam, so the horizontal reaction force is zero. All forces in a vertical direction equal zero. The reaction force RA minus the resultant UDL (5 kilo Newtons per meter times 6 meters) is equal to zero. So, the reaction force RA equals 30 kilo Newtons. The sum of the force moment at Point A is zero. The moment of reaction force RA about A point is RA times zero distance. The moment of UDL about A is the resultant of UDL, which is 30 kilo newtons times 3 meters because the resultant UDL acts at the centre of the UDL. Moment MA is in a clockwise direction because the beam segment is on the right side for moment MA. Then we will get moment at Point A of minus 90 kilo Newton meters. This is a negative bending moment, which means that the top part of the beam is in tension and the bottom part of the beam is in compression.

Now let’s look at the shear force and bending moment. At A, we have a reaction force RA and a reaction moment MA. This 30 kilo Newtons reaction force is the shear force at A, because RA is acting parallel to cross section A.

What about the sign of the shear force? This reaction RA is acting upwards, and the beam segment is on the right side, so it rotates in a clockwise direction, so this shear force is positive. The reaction moment MA is negative. So, the moment at A is also negative, and the reaction moment MA is minus 90 kilo Newton meters. A cantilever beam is special: the top part of the beam will be in tension and the bottom part of the beam is in compression. In a simply supported beam, the top part of the beam will be in compression and the bottom part in tension.

Point B is the free end. We don't have any reaction forces in a vertical direction, a horizontal direction or moment. So, we don't have any shear force and bending moment at B.

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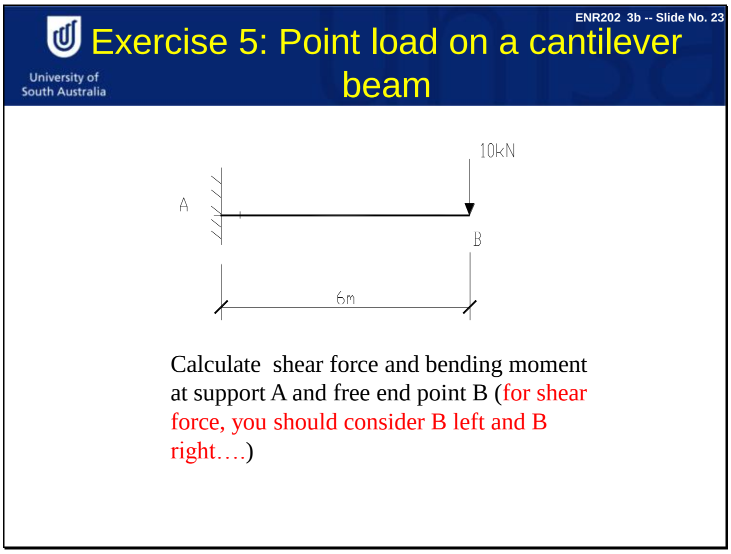

Now let's look at a point load exercise. A cantilever beam is subjected to a point load of 5 kilo Newtons. Point A is a fixed support and there is no support at Point B.

Pause this presentation, and try to calculate the shear force and the bending moment for the left side of Point B. The solution is on the next slide.

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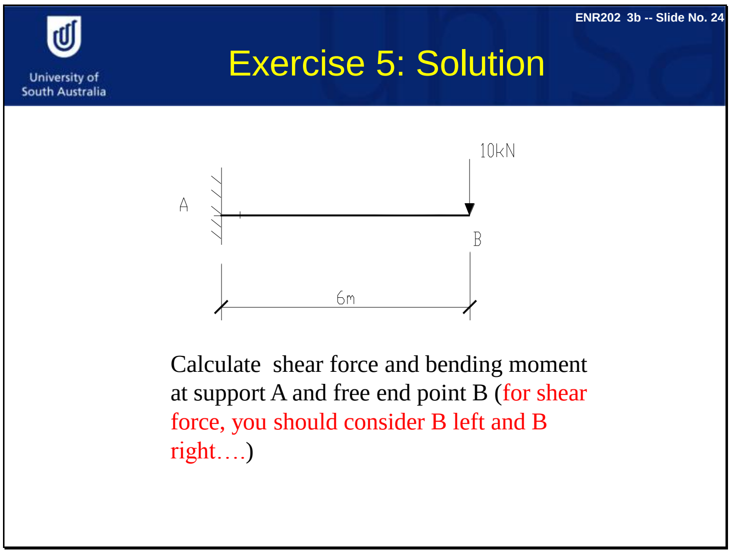

The solution for Exercise 5.

First calculate the reaction forces. We have only one support, the fixed support at point A. As you know, a fixed support can resist three types of reactions.

Draw the free body diagram and show the reactions. One is a horizontal reaction, the second is vertical reaction RA, and moment is MA.

We don't have any horizontal loads in this beam, so the horizontal reaction force is zero. All forces in a vertical direction equal zero. The reaction force RA minus the point load (10 kilo Newtons) is equal to zero, so the reaction force RA is 10 kilo Newtons. Now consider the moments about A point. The moment of reaction force RA about A is RA times zero distance. the moment of point load about A is the point load 10 kilo Newtons times the distance between A and B (10 kilo newtons times 6 meters). The moment MA is assumed in a clockwise direction. The sum of force moments at Point A is zero. So we will get that the moment MA that is minus 60 kilo Newton meters. This is a negative bending moment, which means that the top part of the beam is in tension and the bottom part of the beam is in compression.

Now lets look at the shear force and bending moment. At Point A, we have a reaction force RA and a reaction moment MA. This reaction force is zero, but the shear force at A is 10 kilo Newtons, because RA is acting at parallel to cross section A. What about the sign of the shear force? This reaction RA is acting upwards and the beam segment is on the right side, so it rotates in a clockwise direction, thus this shear force is positive. The reaction moment MA is negative, so the moment at A is also negative, and the reaction moment MA is minus 60 kilo Newton meter.

If you want to calculate the shear force and bending moment on the left side of Point B, you have to cut the beam on the left side of B and consider the left side of the beam and show the internal reactions of the shear force and bending moment on the cutting cross section. You can calculate that the shear force on the left side of Point B is 10kN and the bending moment on the left side of B is zero. Make sure that you are familiar with all the process involved in solving the questions in these exercises.

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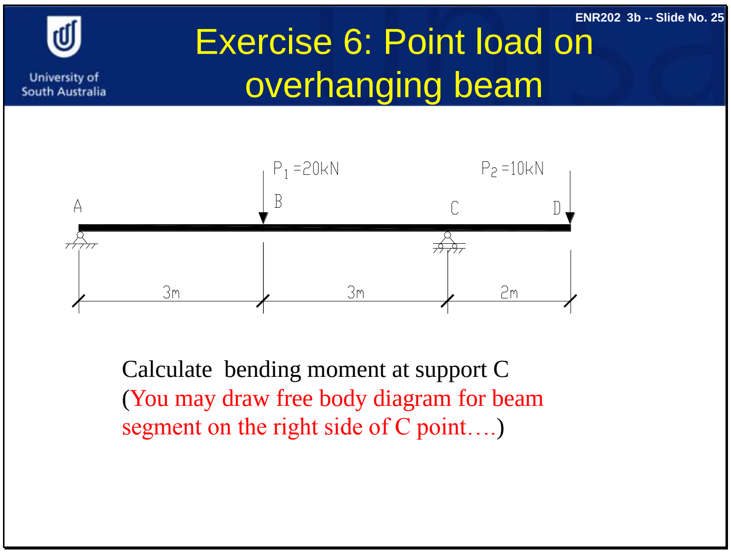

One last exercise. An overhang beam is subjected to a concentrated load of 10 kilo Newtons at Point D and another concentrated load of 20 kilo Newtons at B point. There is a pinned support at A, and a roller support at C, as shown in the figure.

Pause this presentation and calculate the bending moment at support C. The solution is on the following slide.

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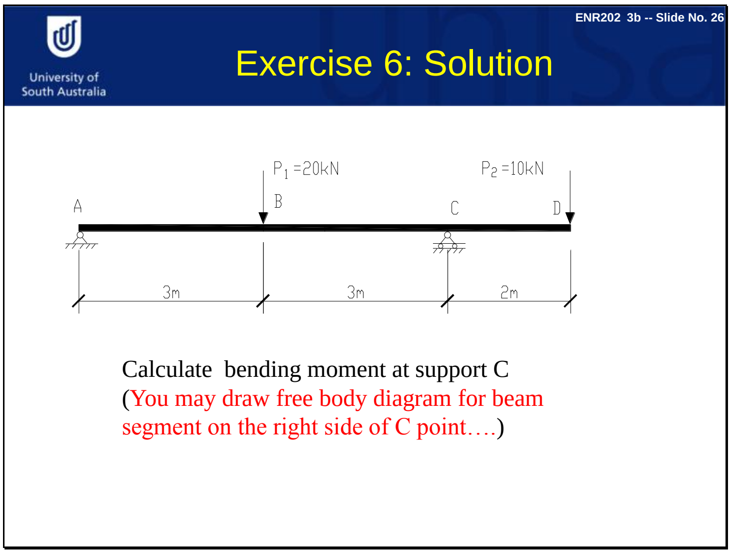

The solution for Exercise 6. Cut at the right side of Point C, and show the positive internal actions of the shear force and the bending moment at the cutting cross section on the right side of C. Then apply equilibrium equations and calculate the bending moment on the right side of C. The answer is minus 20 kilo Newtons meters.

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University of South Australia

Shear Force and Bending Moment Diagrams

The variation of shear force **V** and bending moment **M** over the length of a beam provides information necessary for the design analysis of the beam. The maximum magnitude of the bending moment is usually the primary consideration in the design and its value and position should be determined.

If we calculate all shear forces and bending moments along the length of a beam, we can draw shear force and bending moment diagrams for that beam. If you know the variation of shear force and bending moment over the length of the beam, you can calculate the maximum magnitude of bending moment and where it occurs. This value is useful in the design of the beam. You also need to calculate the position of maximum bending moment (where it occurs), the sign of bending moment, and the magnitude.

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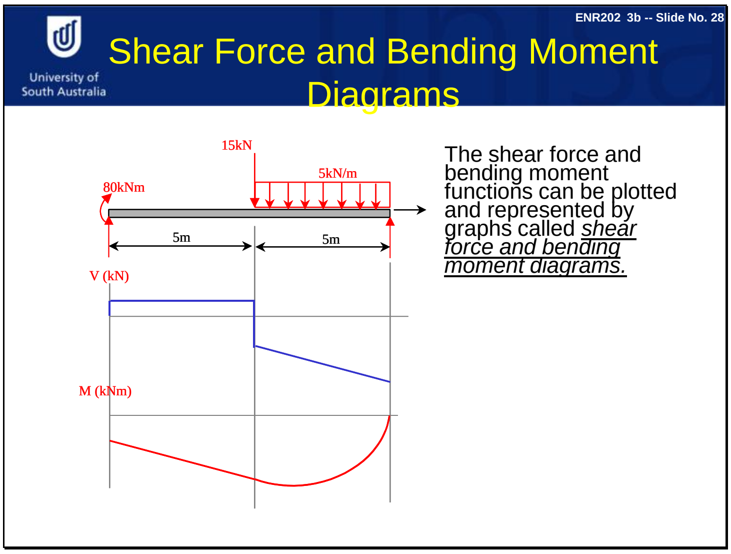

If you know the shear force function and bending moment function over the length of the beam, you can draw the shear force diagram and bending moment diagram. Later on in this unit, we will cover how to find the functions of Shear Force and Bending Moment over the length of the beam to draw SFDs and BMDs.

In this shear force diagram and bending moment diagram, you can easily find out at what point the maximum shear force and bending moment is. You can find the SF and BM at all points in the length of the beam. This is why it is so important to draw SFDs and BMDs along the length of the beam.

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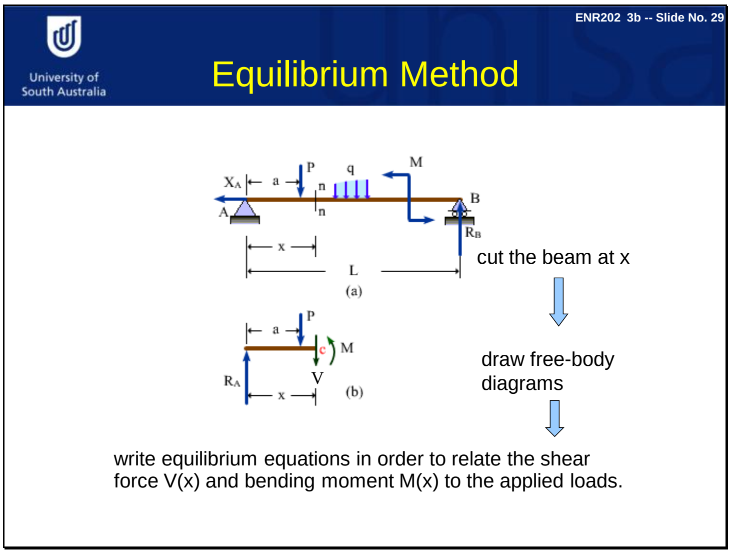

We have two methods of drawing SFDs and BMDs. If you use the equilibrium method of ascertaining the functions of SF and BM along the length the beam, you need to follow three steps.

The first step is to cut the beam at any cross section along the length of the beam (for example, a cut at n-n cross section, at a distance of ‘x’ from the left end as shown in the figure ‘a’).

The second step is to draw a free-body diagram for the left part of the beam or the right part of the beam (for example, we consider the left part of the beam as shown in the figure ‘b’). The third step is to use equilibrium equations to find the shear force function and the bending moment functions at x distance from left end.

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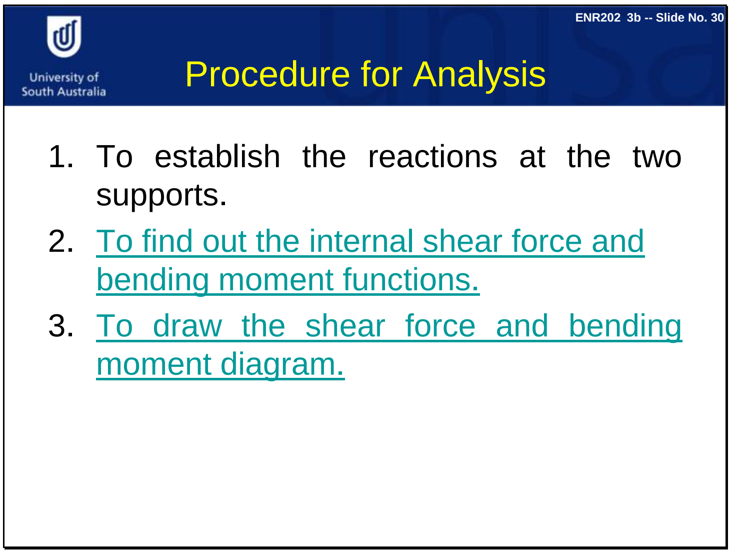

This is the procedure for the equilibrium method. First, calculate reaction forces at the supports. Second, find the internal shear force and bending moment functions along the length of the beam. Third, draw the shear force diagram and bending moment diagram.

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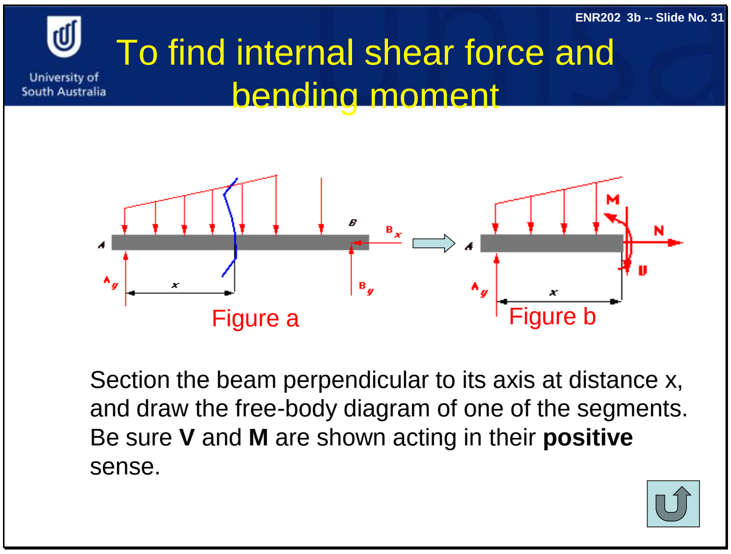

For a beam subjected to vertical loads, cut at distance ‘x’ from the left end, as shown in the figure ‘a’, and draw the free body diagram for the left part of the beam, as shown in the figure ‘b’. You can show positive internal forces as normal force ‘N’, shear force as ‘V’ and bending moment as ‘M’, as shown in the figure ‘b’. We can then write the equilibrium equations for that free body diagram, and draw SFD and BMD.

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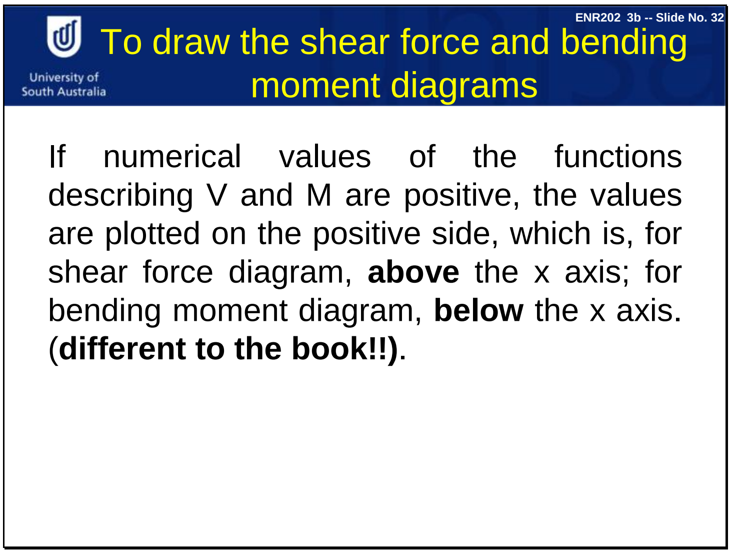

If the numerical values of the functions describing V and M are positive, the values are plotted on the positive side, which is, for shear force diagram, is above the x axis; for bending moment diagram, it is below the x axis. (different to the book!!).

The reason for this is that in civil engineering, we traditionally draw a bending moment diagram at the tension side. In positive bending moment, the bottom is in tension and the top is in compression, which means that tension will occur at the bottom part of the beam. Therefore, we draw the BMD below the x-axis.

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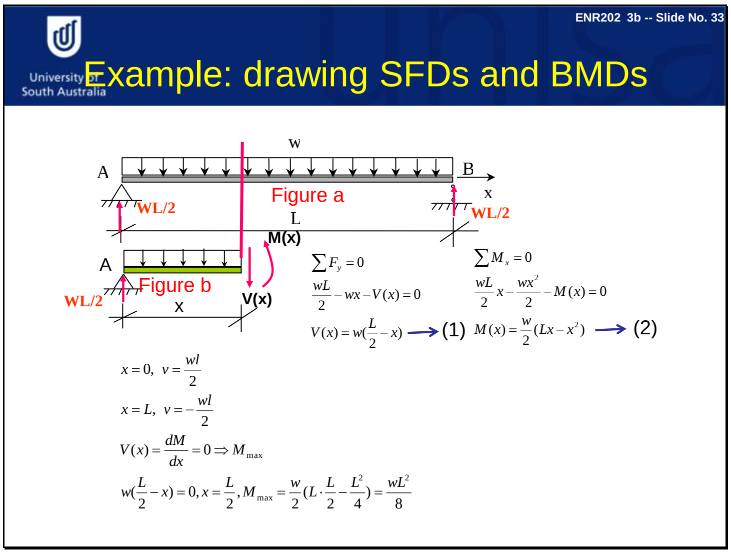

Here is an example of how to draw an SFD and a BMD.

A simply supported beam is subjected to a uniform distributed load (UDL) ‘w’ throughout the length of the beam as shown in the figure ‘a’. How would you draw the SFD and BMD for this simply supported beam?

We know the first step, which is to calculate the reaction forces at the supports. You can calculate reaction forces at A and B. This is a symmetrical beam which is subjected to symmetrical loading. The resultant of the UDL is equal to ‘w’ times the length of the beam. Therefore, the reaction forces at A and B are equal, and the value is UDL ‘w’ times the length of the beam ‘L’, divided by 2.

The next step is to cut the beam at a distance ‘x’ from the left support. We have two internal force actions at the cutting surface. One is shear force and the other is bending moment. Now draw the free body diagram of the left side of the beam as shown in the figure ‘b’. As you know, in the free body diagram, you remove supports and replace reaction forces, and show loads on the structure and internal forces of shear force ‘v(x)’ and bending moment ‘M(x)’ (as shown in the figure ‘b’).

V(x) and M(x) represent positive shear and positive bending moments for the left side of beam, as shown in the Figure ‘b’. The reason is that this shear force ‘v(x)’ is acting downward and rotates the beam segment in a clockwise direction. Due to the bending moment ‘M(x)’, the top part of beam is in compression and the bottom part of the beam is in tension. You can write the equilibrium equations for the left part of the beam. The first equilibrium equation is applied to all forces in a vertical direction. We have three forces. One is a reaction force RA (‘w’ times L divided by 2), the second is the resultant UDL (‘w’ times ‘x’). The internal shear force at the cutting cross section is ‘v(x). Finally, we work out that the first equation for shear force ‘V(x) is equal to w times L divided by 2 minus x, as equation 1 in terms of x that means linear equation.

The second equilibrium equation is that the sum of all force moments about ‘x’ is equal to zero. We have four moments about Point ‘x’. The first moment is a moment of reaction force about ‘x’ point, which is RA times the distance ‘x’ in a clockwise direction. The second moment is the moment of resultant UDL ‘wx’ times x divided by 2, because the resultant UDL occurs at half the length of the beam in an anti-clockwise direction. The third moment is the moment of M(x) which is anti-clockwise. The fourth moment is the moment of ‘v(x)’ about x which is equal to ‘v(x)’ times zero distance. This means that we get the moment function ‘M(x) as shown in the equation 2 in terms square ‘x’, which means a quadratic equation.

Now to find out where the maximum bending moment occurs. You know the basic concept of differentiation. If we differentiate the bending moment function with respect to ‘x’ being equal to zero, then we can get the x equal to half the length. That means that the maximum bending moment occurs at the middle-span point of the beam with a shear force of zero.

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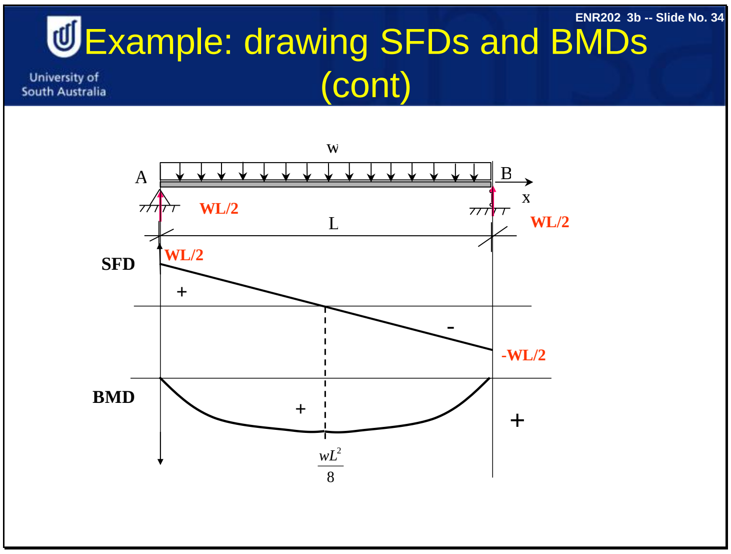

Therefore, the shear force function ‘v(x)’ of equation 1 is a linear equation. So, it is a straight line in the SFD. In this equation, if you substitute ‘x’ equal to zero, you will get a shear force of half the ‘w’ times ‘L’, and if you substitute ‘x’ equal to ‘L’, you will get a shear force of minus half the ‘w’ times ‘L’. This is a shear force diagram based on shear force function. We have zero shear force in the middle.

In the bending moment diagram, in the bending moment function of equation 2, we have a quadratic equation, which means the bending moment diagram is a parabola. If you substitute ‘x’ equal to zero into equation 2, the bending moment function is zero. If you substitute ‘x’ equal to L in equation 2, the bending moment function is also zero. If you substitute ‘x’ equal to half the length in equation 2, the bending moment function is w times square L divided by 8 as shown in the bending moment diagram.

Finally, the maximum shear force occurs at supports and the maximum bending moment is at the middle of the beam. As we have mentioned, in the SFD, above the x-axis is positive, and in the BMD, below the x-axis is positive.

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Thank you for your attention.