ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 1

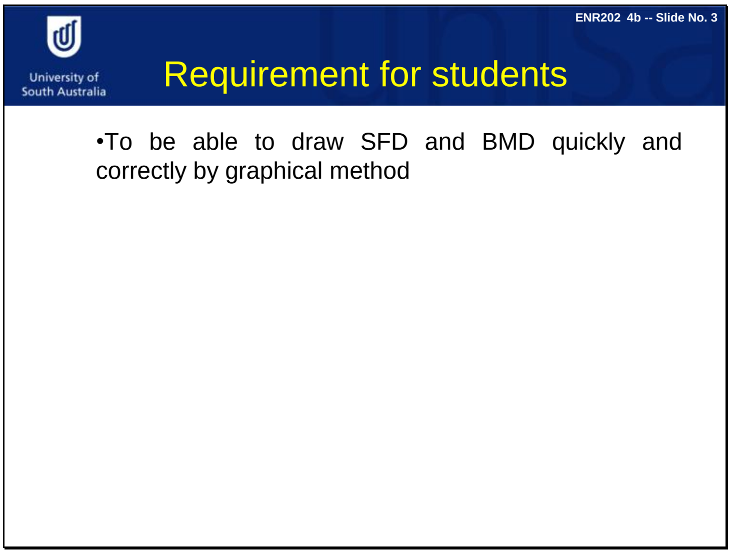
ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 2

Welcome to lecture summary 4b. In this lecture summary, we will continue looking at shear force diagrams and bending moment diagrams. In the last two lecture summaries, we learnt how to draw shear force diagrams and bending moment diagrams using the equilibrium method based on the principles of important points. In this presentation, we will learn a second method, the graphical method.

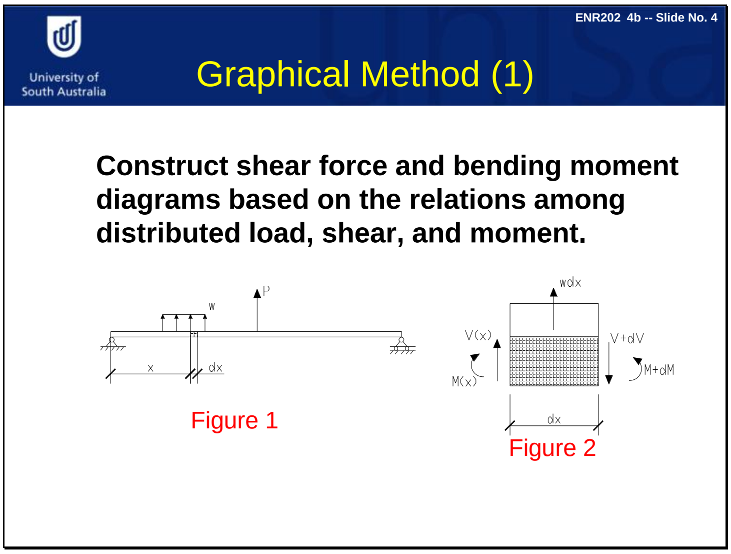
ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 3



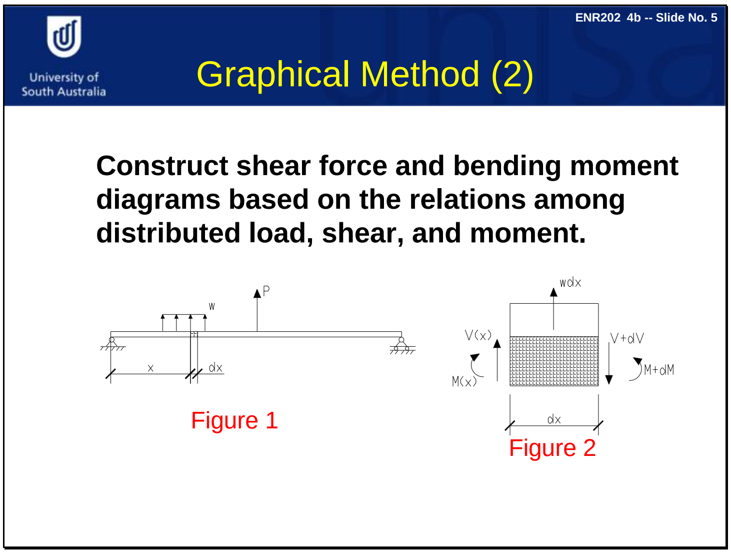
In this lecture summary, you will learn how to draw Shear Force Diagrams and Bending Moment Diagrams quickly and accurately using the graphical method.

Slide 4



The graphic method is based on the relationship between shear force and external loading and the relationship between bending moment and shear force. We can draw the shear force diagram and bending moment diagram based on these two relationships. The graphical method is an easier method to use.

Slide 5



Let's say that a beam is subjected to various types of lateral loads such as a uniform distributed load 'w' and point load 'P', as shown in Figure 1. We can analyse one small segment of the beam 'dx' at a distance of 'x' from the left support, as shown in Figure 1. In this small segment, we have two cutting cross sections.

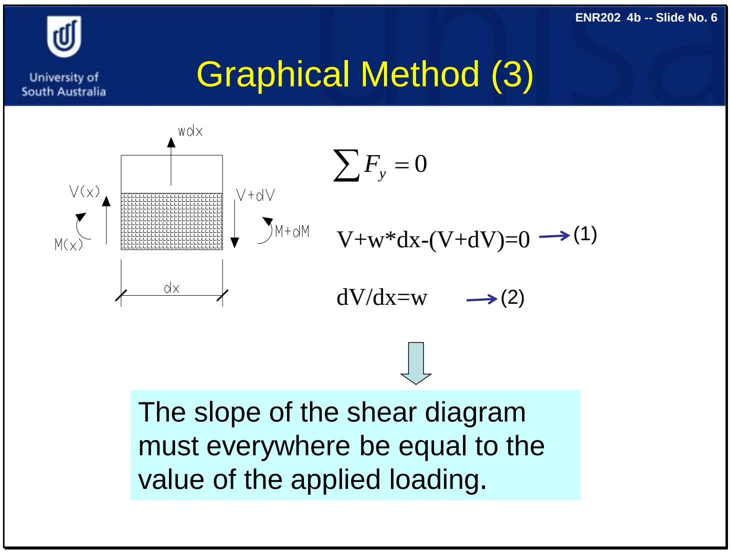
We have already discussed that we have two internal force actions at a cutting cross section. They are the shear force and the bending moment. We can see positive shear force and positive bending moment at the cutting cross section. (You should be familiar with the sign conventions for shear force and bending moment at a cutting cross section).

On the left cutting cross section, we have a positive shear force 'V' acting in an upward direction and rotating the beam segment in a clockwise direction, and a positive bending moment 'M', acting in a clockwise direction. So the top part of the beam is in tension and the bottom part of the beam is in compression, as shown in Figure 2.

On the right cutting cross section, we have a positive shear force 'V plus dV', which is acting in a downward direction and rotates the beam segment in a clockwise direction, and we have a positive bending moment 'M plus dM', which is acting in an anti-clockwise direction. So the top part of the beam is in compression and the bottom part of the beam is in tension, as shown in Figure 2.

The shear force and bending moment at the right side cross section is different from the shear force and bending moment at the left side cutting cross section. The free body diagram of this beam segment also includes the external loads acting on the beam segment, so we have distributed loadings acting on the beam segment as shown in Figure2.

Slide 6

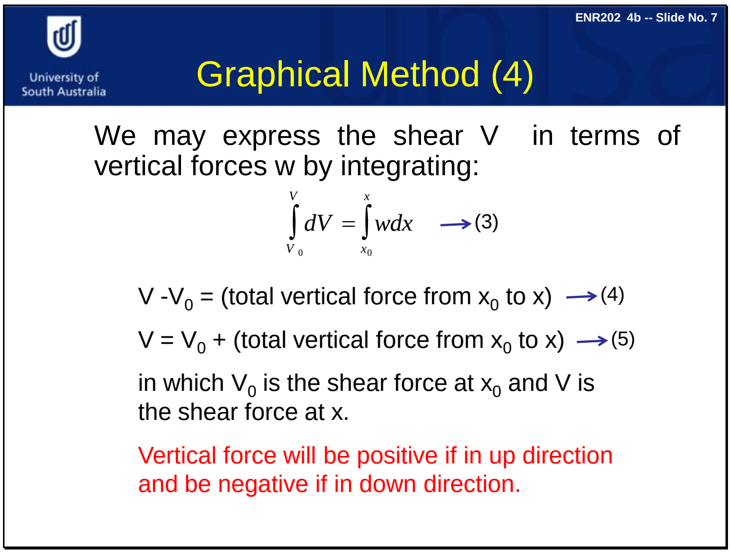


As you know, if the structure is in equilibrium, it means that all structural parts are in equilibrium. Therefore, the beam segment 'dx' is also in equilibrium.
In this figure, we can see a positive shear force and a positive bending moment at the left and right cutting cross sections, and the resultant uniform distributed load 'w times dx'. Now we can apply the equilibrium equation that is all forces in a vertical direction are equal to zero.

The shear force 'V(x)' is acting in an upward direction at the left cutting cross section. The shear force 'V plus dV' is acting in a downward direction at the right cutting cross section. The resultant uniform distributed load 'w times dx' is acting in an upward direction. So we get equation 1, that V plus 'w times dx' minus 'V plus dV' is equal to zero.

Now we can see the relationship between the shear force and external distributed load. It is 'w' equal to 'dV' divided by 'dx', which means the first derivative of the shear force is 'w'. Therefore, the slope of the shear force diagram must be equal everywhere to the value of the applied loading.

Slide 7



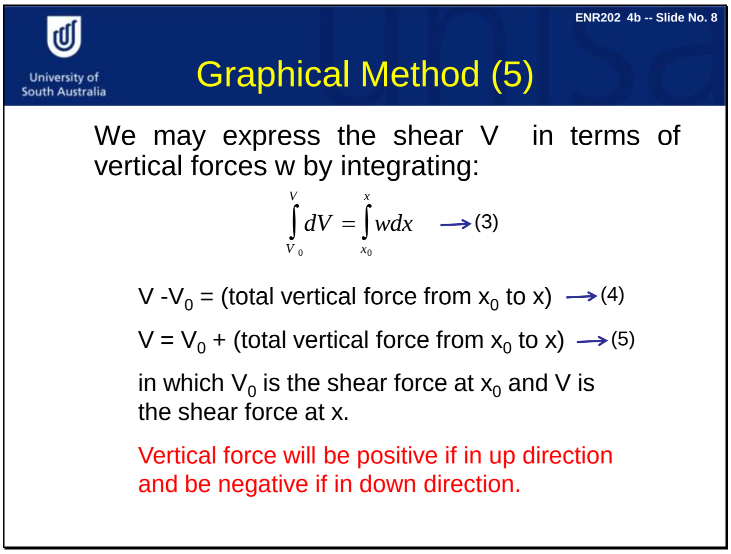
If you rearrange equation 2 and integrate both side of the equation, you will get equation 3, as shown in the slide. The limits of the integration on the left side of the equation is the shear force at the left side cutting cross section, which is ‘V0‘, to the shear force at the right side cutting cross section, which is ‘V’. The limits of the integration on the right side of equation is the distance of the left side cutting cross section from the left support, which is is ‘x0’, to the distance of the right side cutting cross section from the left support, which is ‘x’, as shown in Equation 3.

Once you integrate the left side of the integration, you will find out the shear force ‘V minus V0’, equals the right side of the integration is the total vertical force from distance ‘x0’ to distance ‘x’, as in Equation 4.

Note that the shear force at the left cutting cross section and at a distance of x0 is V0, and the shear force at the right cutting cross section and at a distance of x is V.

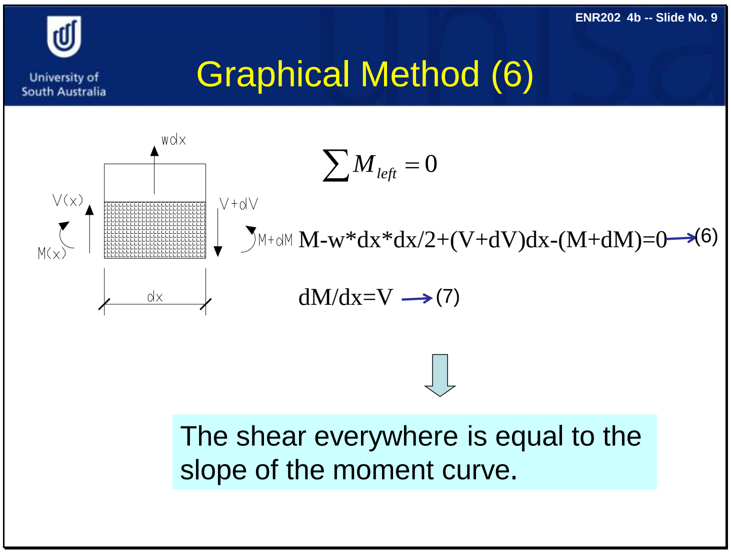
In the graphical method, we start by calculating the shear force from the left side of the beam. Once we know the shear force at the left side of the beam, then we can calculate the right side of the beam. The shear force at the right side of the beam ‘V’ is equal to the shear force at the left side of the beam ‘V0’ plus the total vertical force from x0 to x’, as in Equation 5. Note that the external load acting upwards has a positive shear force, so we use a positive sign, and the external load acting downwards has a negative shear force. (Make sure that you are familiar with sign conventions.)

Slide 8



We will start with the shear force at the left side cutting cross section, then calculate the shear force at the right side cross section. The shear force at the left side of the beam is ‘V0’, which is equal to the shear force at the right side of the beam ‘V’ minus the total vertical force from x0 to x’. If the external load is acting upwards, the shear force is negative, and if the external load is acting downwards, the shear force is positive. Because the external load is acting upwards, we use a negative sign.

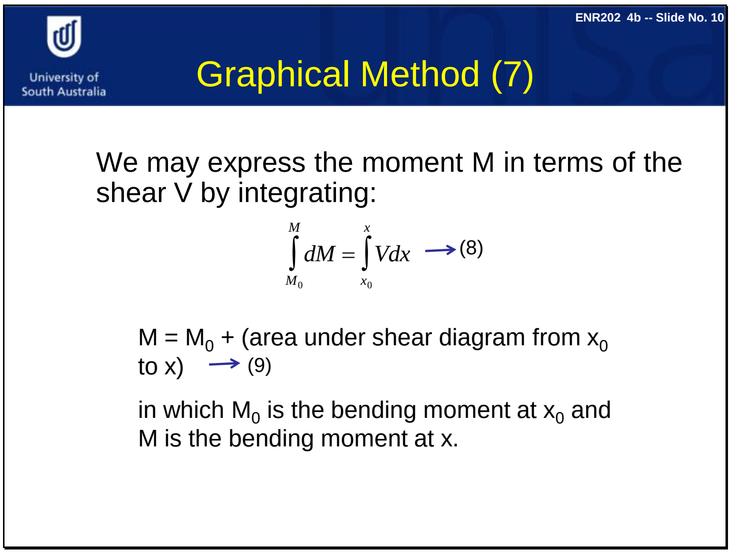
Slide 9



We know that the beam segment ‘dx’ is in equilibrium. In this figure, we can see positive shear force and positive bending moment at the left and right cutting cross sections, and the resultant uniform distributed load ‘w times dx’ . Now, we can apply the equilibrium equation which says that all force moments at the left cutting cross section point are equal to zero.

The moment of shear force ‘V(x)’ at the left cutting cross section is zero because ‘V(x)’ is acting at the left cross section. The moment of the shear force ‘V plus dV’ at the left cutting cross section is ‘V plus dV’ times ‘dx’ in a clockwise direction. The moment of the resultant uniform distributed load ‘w times dx’ times ‘dx divided by 2’ is in an anti-clockwise direction. The moment ‘M(x)’ acting at the left cutting cross section is in a clockwise direction. The moment ‘M plus dM’ acting at the right cutting cross section is in an anti-clockwise direction as in Equation 6. Simplify Equation 6, and ignore the term ‘square dx’ and you will get Equation 7. Equation 7 outlines the relationship between the bending moment and the shear force, and means that the first derivative of moment is shear force. Therefore, the shear everywhere is equal to the slope of the moment curve.

Slide 10

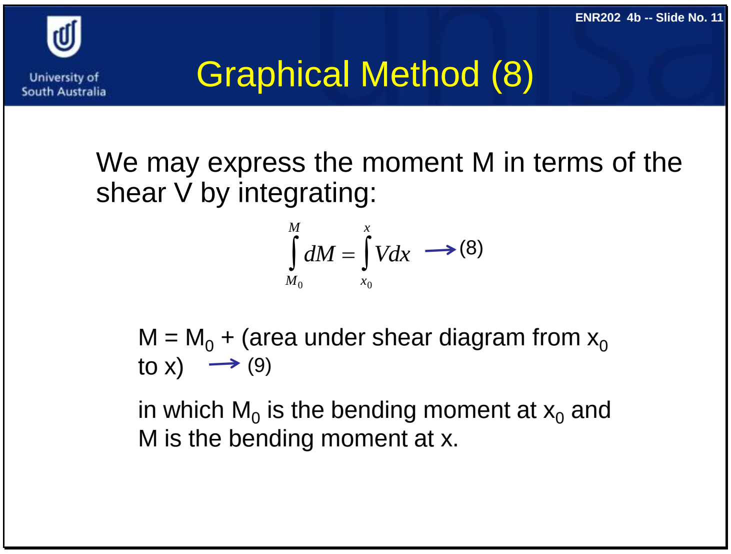


Now rearrange Equation 7 and integrate both side of the equation, to get Equation 8 as shown in the slide. The limits of the integration of the left side of the equation is the moment at the left side cutting cross section, which is ‘M0‘. The moment at the right side cutting cross section is ‘M’. The limits of the integration of the right side of the equation is the distance of the left side cutting cross section from the left support, which is ‘x0’ to the distance of the right side cutting cross section from the left support, which is ‘x’, as shown in Equation 8.

Once you integrate the left side of the integration, you get moment ‘M minus M0’, and the right side of the integration is the area under the shear force diagram from ‘x0’ to ‘x’, as in Equation 4.

Note that the moment at the left cutting cross section and at a distance of x0 is M0. The moment at the right cutting cross section and at a distance x is M.

Slide 11

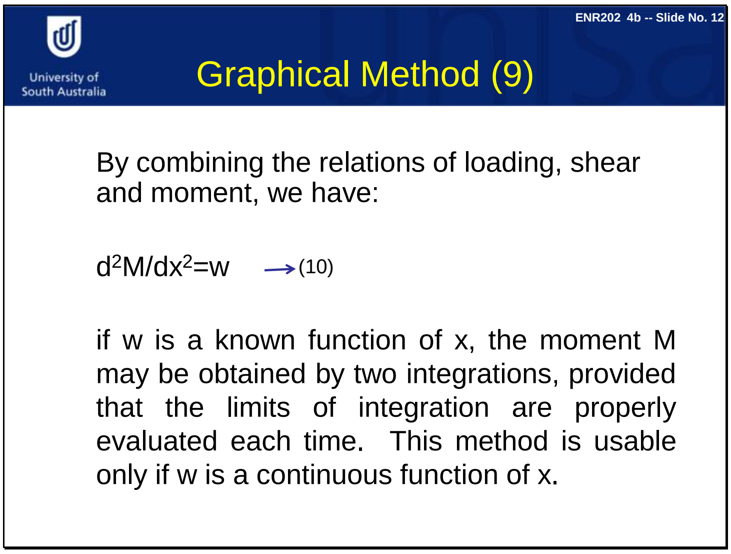


In the graphical method, we calculate the moment from the left end to the right end of the beam.

The moment at the right side of the beam 'M' is equal to the moment at the left side of the beam 'M0' plus the area under the shear force diagram from x0 to x', as in Equation 9.

If the area under the shear force diagram is positive, use a positive sign to calculate the moment at the right side cutting cross section. If the area under the shear force diagram is negative, use a negative sign to calculate the moment at the right side cutting cross section.

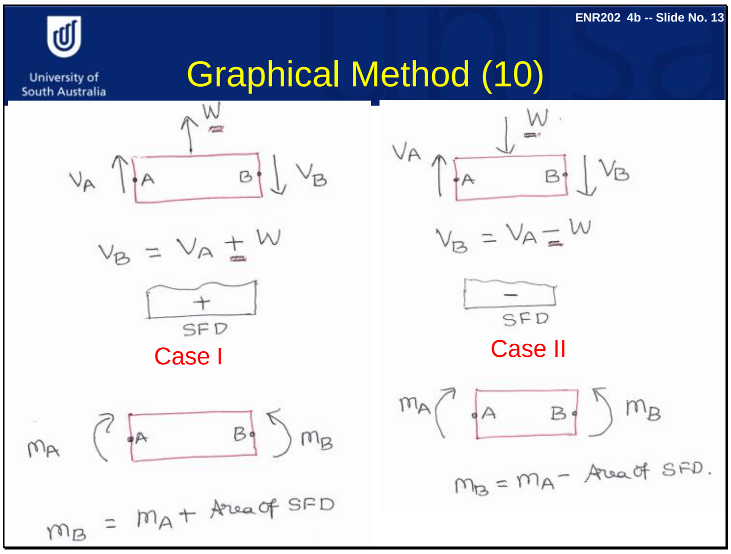
Slide 12



If you combine the relationship between shear force and external loading, and the relationship between bending moment and shear force, you get Equation 10. If external loading ‘w’ is a known function of x, then you can work out bending moment M by two integrations, provided that the limits of integration are properly evaluated each time. However, you can only use this method if the external load ‘w’ is a continuous function of x, which means a uniform distributed load, or triangle (linearly varied) distributed load.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 13



In this slide, we know shear force and bending moment at A, and we need to calculate shear force and bending moment at B. We present two cases.

Case I: In a beam, we consider a beam segment between point A (on the left side of the beam segment cross section) and point B (on the right side of the beam segment cross section) and we show a positive shear force and positive bending moment at points A and B, as shown in the figure. Here, we know the shear force at A and we assume the resultant external load is acting in an upward direction. We need to calculate the shear force at point B, which is equal to the shear force at A plus an external load acting on the beam segment.

Case II: In a beam, we consider a beam segment between points A and B as shown in the figure. Here, we know the shear force at point A and we assume that the resultant external load is acting in a downward direction. We need to calculate the shear force at point B, which is equal to shear force at A minus the external load acting on the beam segment.

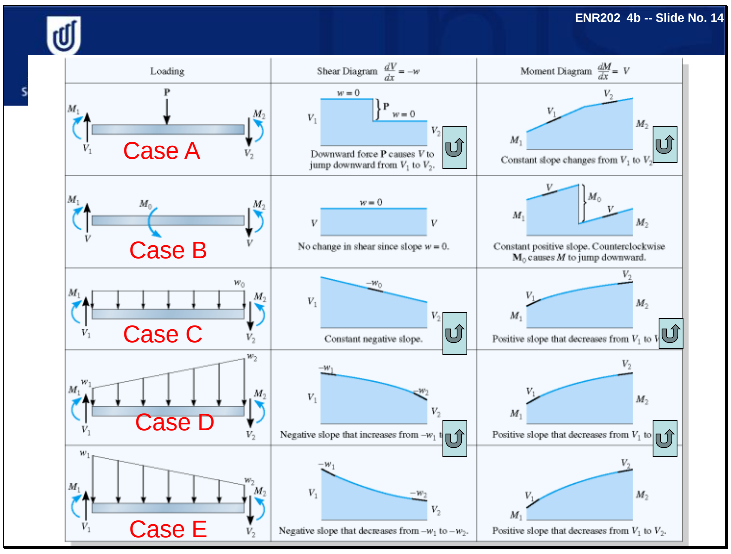
Case I: In a beam, we know the bending moment at A and we assume the area under the shear force diagram is positive. We need to calculate the bending moment at B, which is equal to the bending moment at A plus the area of the shear force diagram.

Case II: In a beam, we know the bending moment at A and we assume the area under the shear force diagram is negative. We need to calculate the bending moment at B, which is equal to the bending moment at A minus the area of the shear force diagram.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

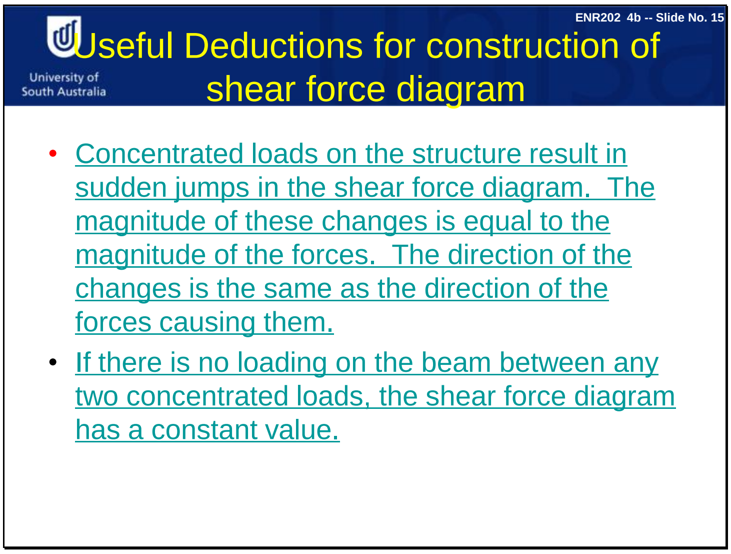
Slide 14



We will refer to these figures in next four slides. We have a point load in Case A, a concentrated moment in case B, a uniformly distributed load in case C, a linearly increasing load in case D, and a linearly decreasing load in case E. We have a shear force diagram and a bending moment diagram in each case, as shown in the figures.

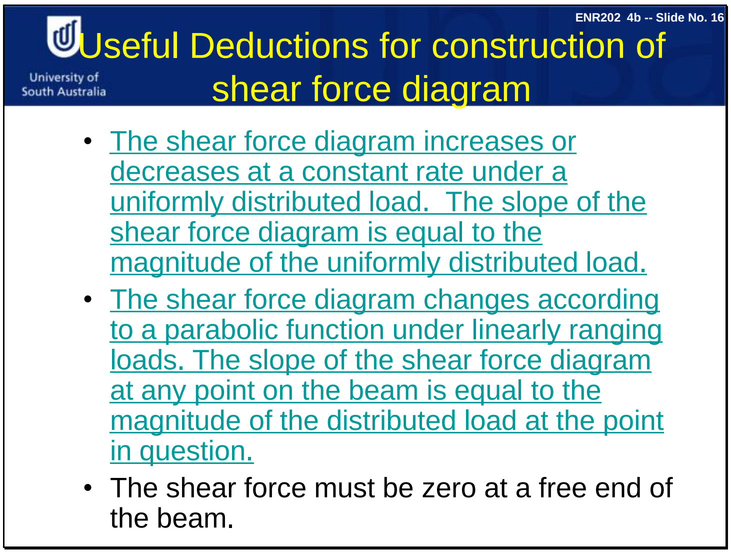
Let's consider a beam segment with a left side cross section denoted as point 1 and a right side cross section denoted as point 2.

Slide 15



In case A, if a point load or a concentrated load is acting on a beam segment, there is a sudden jump in the Shear Force Diagram at that point load. If there is no load acting on the beam segment between any two point loads, the Shear Force Diagram has a constant value.

Slide 16

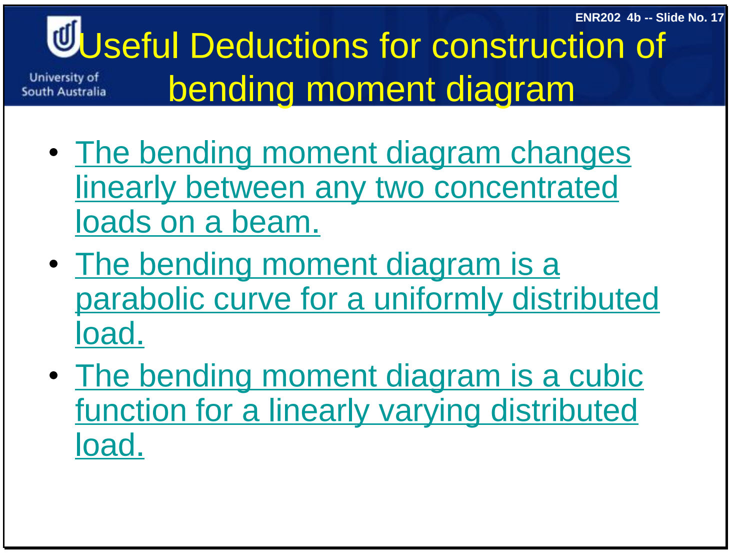


In case C, there is a uniformly distributed load acting on a beam segment. There is a linear increase or decrease in the Shear Force Diagram at the Uniformly Distributed Load.

In cases D and E, there is a linearly increasing or decreasing load acting on a beam segment. There is a parabolic function in the Shear Force Diagram under linearly ranging loads. The slope of the Shear Force Diagram at any point on the beam is equal to the magnitude of the distributed load at that point.

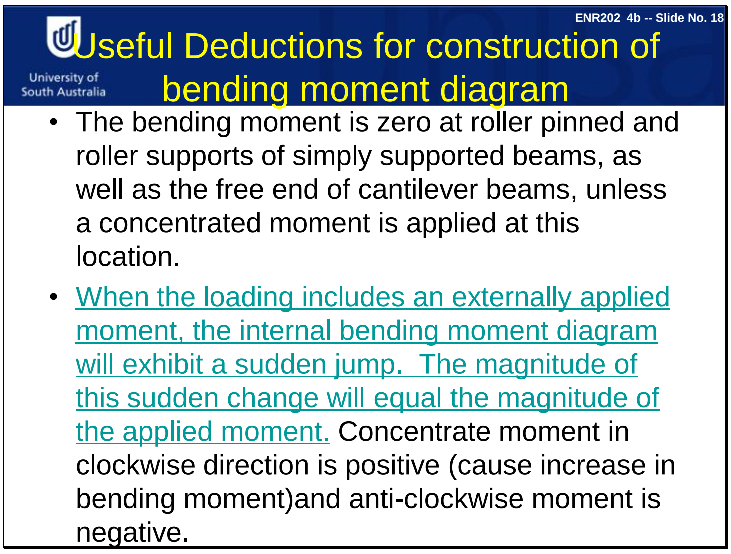
The shear force must be zero at the free end of the beam, because there is no shear force at the free end of the beam.

Slide 17



In case A, if there is a point load or concentrated load acting on a beam segment, there is a linear variation in the Bending Moment Diagram.
In case C, if there is a uniformly distributed load acting on a beam segment, there is a parabolic variation in the Bending Moment Diagram.
In cases D and E, if there is a linearly varying distributed load acting on a beam segment, there is cubic variation in the Bending Moment Diagram.

Slide 18

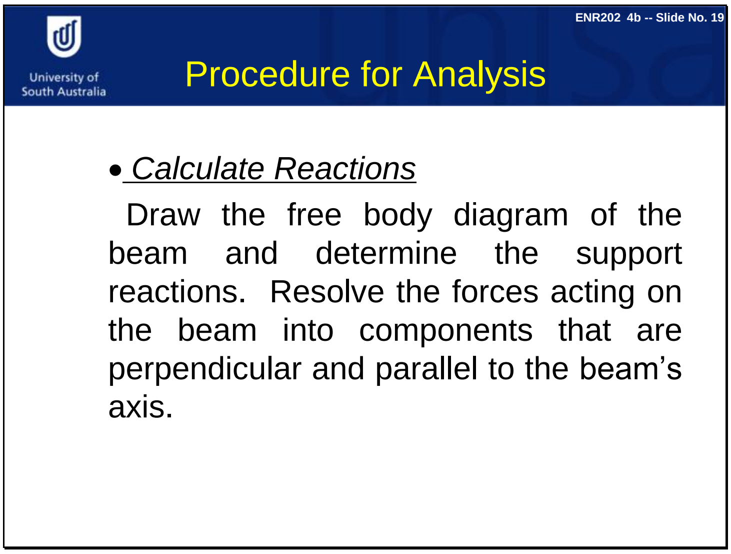


The bending moment is zero at the roller supports and hinged supports of a simply supported beam. There is no bending moment at the free end of a cantilever beam.

In case B, if there is a concentrated moment acting on the beam segment, there is a sudden jump in the Bending Moment Diagram. The jump is equal to the concentrated moment acting on the beam segment. If the concentrated moment is acting in an anti-clockwise direction, the jump will be downwards.

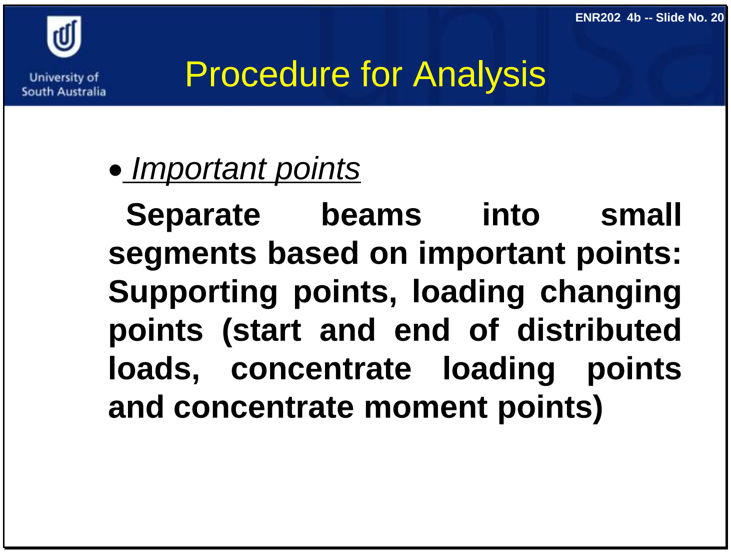
Remember these points, because they are valuable when drawing Shear Force Diagrams and Bending Moment Diagrams.

Slide 19



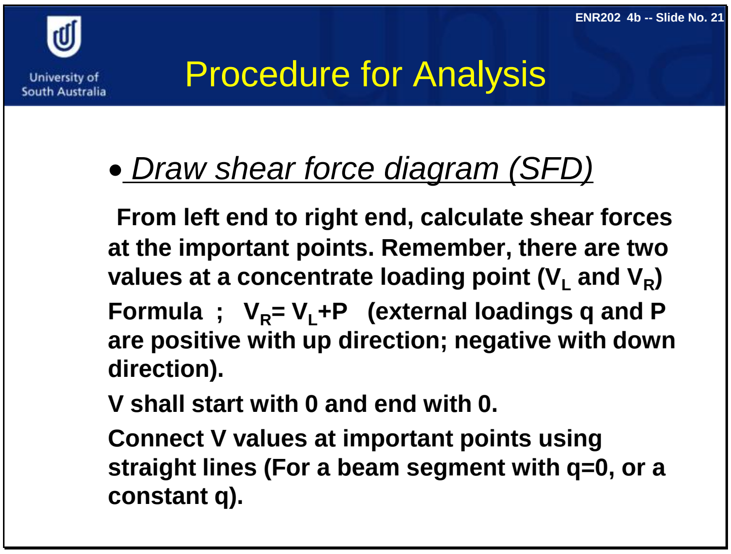
This is the procedures for drawing Shear Force Diagrams and Bending Moment Diagrams in the graphical method. As you know, the first step is to calculate the reaction forces at the supports.

Slide 20



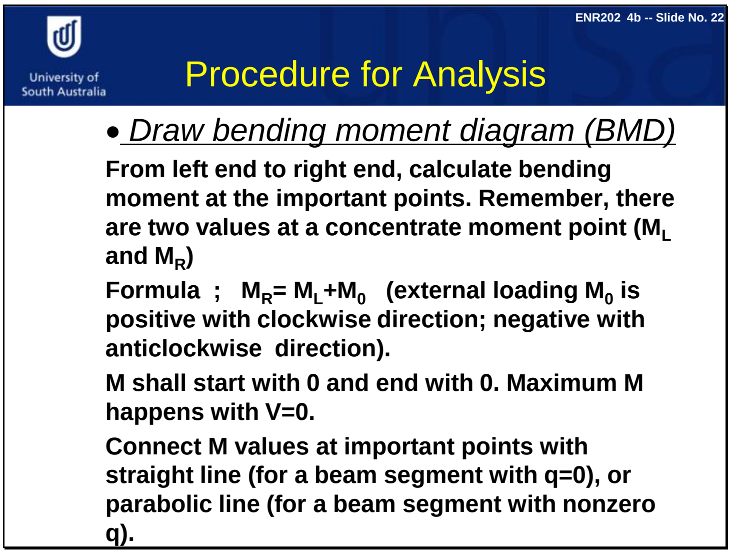
Then separate the beam into several small beam segments. The separating points should be the important points, such as the supporting points and loading changing points (this could be the start or end points for a distributed load, or the concentrated loading point, or the concentrated moment point).

Slide 21



Now calculate the shear forces at important points from the left end to the right end. We know the shear force at the left end and we can calculate the shear force at right end. Consider the left and right side of an important point if there is a concentrated load acting at that important point. Use the formula which states that the shear force at the right side of the beam segment, which is VR, is equal to the shear force at the left side of the beam segment, which is VL, plus all the forces in the beam segment. The shear force starts with zero and ends with zero. Just connect the shear forces values at important points using straight lines to get the shear force diagram.

Slide 22



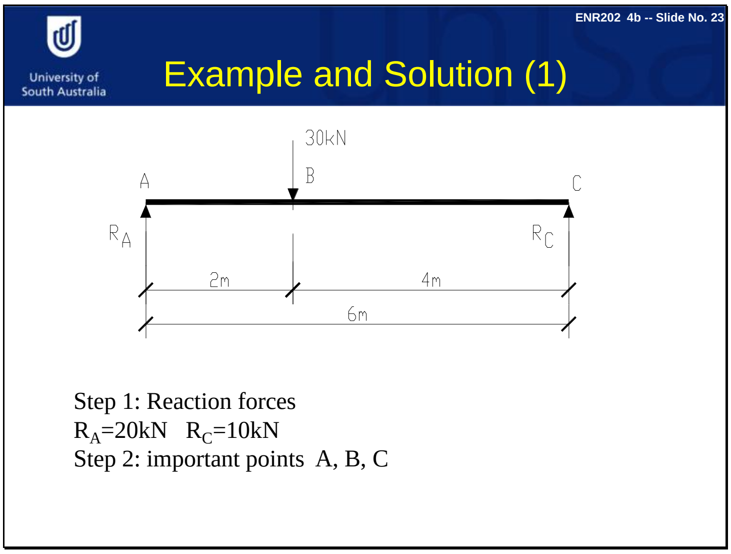
The process for the bending moment diagram is similar to that for the shear force diagram. Working from the left end to the right end, start with zero and end with zero. Calculate the bending moment at the important points, using the formula which states that the bending moment at the right side of the beam segment, which is MR, is equal to the bending moment at the left side of the beam segment, which is ML, plus shear force diagram area between the two points.

The bending moment starts with zero and ends with zero. The maximum bending moment happens when the shear force is equal to zero (this is because the first derivative of bending moment is equal to the shear force. Where the derivative is zero, there is the maximum value.)

Finally, connect the bending moment values at the important points with straight lines for the segments without any distributed load, or parabolic lines for non-zero distributed loads.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 23



Let's look back at the example which we worked through in Lecture 4a, a simply supported beam is subjected to a concentrated load or point load of 30 kilo Newtons at Point B, which is at a distance of 2 meters from point A, as shown in the figure. The total length of the beam is 6 meters.

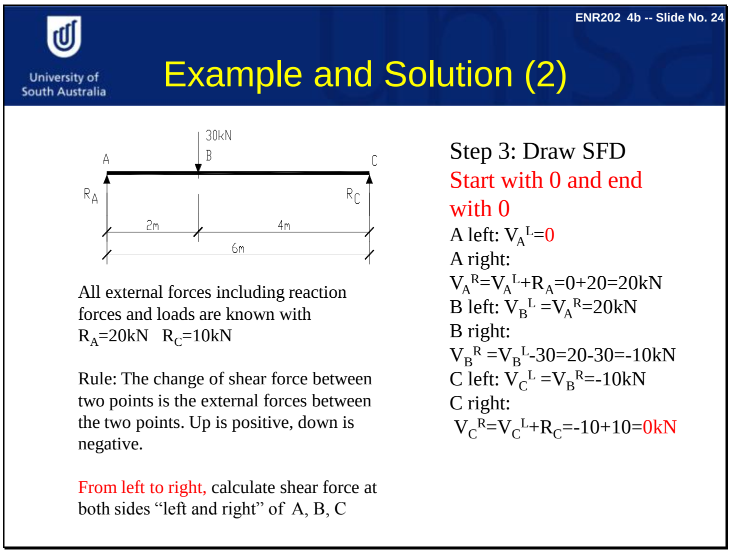
Suppose that you are required to draw a Shear Force Diagram and Bending Moment Diagram for this simply supported beam.

The first step is to calculate the reaction forces. We already calculated the reaction forces in Lecture 4a (in slide 6). (If you need to, go back and have a quick look at how we did those calculations.) The reaction force at point A is equal to 20 kilo Newtons, and the reaction force at Point C is equal to 10 kilo Newtons.

For the second step, we need to find the number and position of the important points. We have three important points. Two are support points, Point A and Point C. A further important point is the concentrated load at Point B. As we have discussed, if we have a concentrated load, you need to consider the left and rights sides of Point B to calculate the shear force.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 24

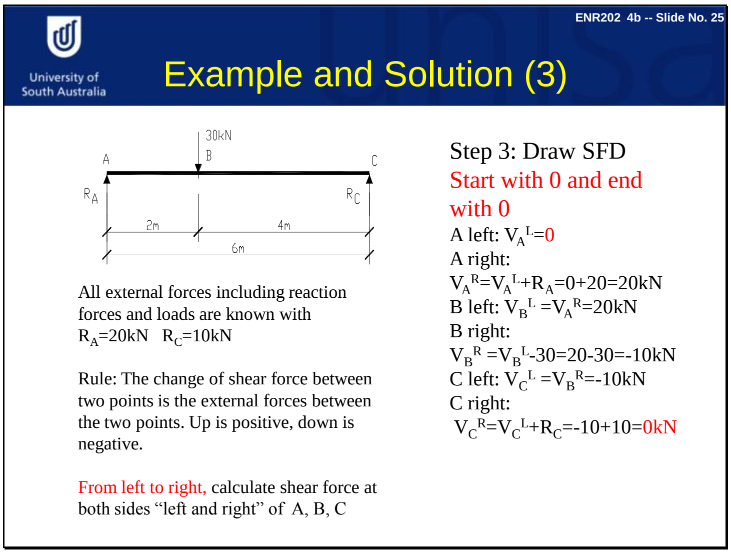


The shear force starts with zero and ends with zero, and you need to calculate shear force from left end to right end. If we have a concentrated load, you must consider the left and right sides of that point to calculate the shear force. Here, the reaction force RA and RC are also concentrated loads. So to calculate shear force, you need to consider the left and right side of point A because of the reaction force RA, and the left and right side of point B because of the point load at B, and the left and right side of point C because of the reaction force RC.

If the external forces or reaction forces between the two points act in an upward direction, they are positive, and if they are in a downward direction, they are negative.

The reaction force RA is 20 Kilo Newtons, the concentrated load is 30 Kilo Newtons, and the reaction force at C, which is RC, is 10 Kilo Newtons.

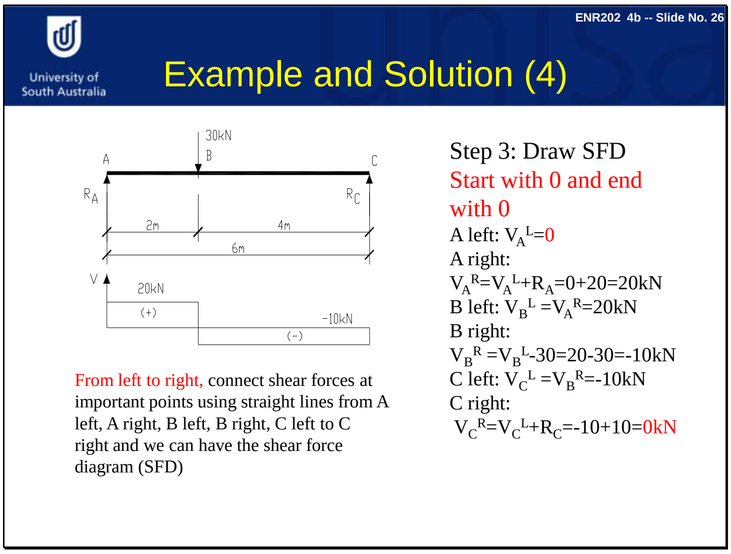
Slide 25



We have three important points: A, B and C. To draw the shear force diagram, we will start with the left end. Consider just to the left of Point A. At Aleft we have zero shear force because the shear force starts with zero. Then we come to Aright. Between Aleft and Aright, we have a concentrated load. That is, RA equals 20 Kilo Newtons in an upward direction. We know that upward is positive. Finally we get Aright equals 20 Kilo Newtons.

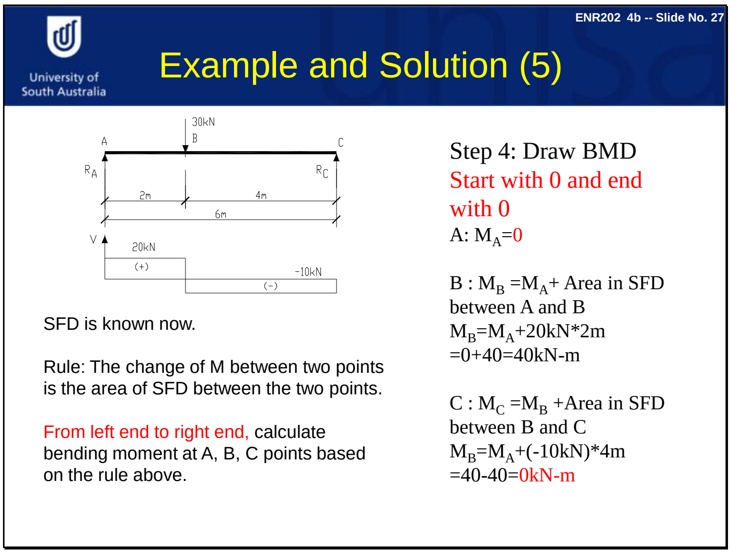
When we come to Bleft, we don't have any vertical force between Aright to Bleft. So Bleft equals 20 kilo Newtons. Move on to Bright. We should have a jump of 30 Kilo Newtons because we have a concentrated load of 30 kilo Newtons. Bright equals 20 kilo Newtons minus 30 kilo Newtons, which is negative 10 Kilo Newtons. We come to Cleft. We don't have any vertical force from Bright to Cleft. So Cleft equals minus 10 kilo Newtons. Move on to Cright. We should have a jump of 10 Kilo Newtons, because we have a reaction force RC which is 10 kilo Newtons acting in an upward direction. Cright equals minus 10 kilo Newtons plus 10 kilo Newtons, which is zero. We can see that the shear force ends with zero. Therefore, we know that the values are correct.

Slide 26



Connect all the values at the important points, and we get shear force diagram as shown in the figure here. Try to draw the shear force diagram on your own and check your work against this figure.

Slide 27



In the previous slides, we worked out the shear force diagram for this example. We are now going to draw the bending moment diagram based on the shear force diagram.

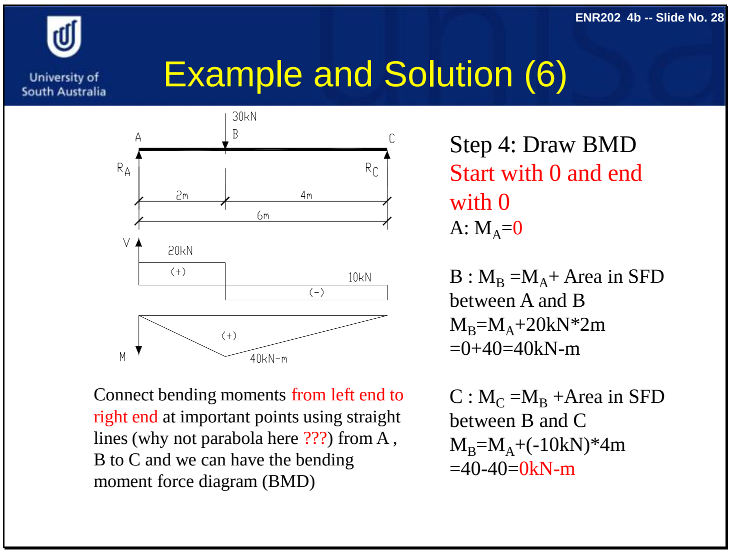
Again, we start with the left end and then go back to the right end. We have three important points, A,B and C. We already know that there is no bending moment at the roller support nor at the hinged supports. Therefore, the bending moment at Points A and C is equal to zero.

We also know that bending moment starts with zero and ends with zero. Therefore, the bending moment at A is equal to zero. The bending moment at B is equal to the bending moment A plus the area under the shear force diagram between Points A and B because the Shear Force Diagram is positive. The shear force area between A and B is 20 kilo Newtons times 2 meters, which is equal to 40 kilo Newton meters. This shear force diagram is positive between A and B. Finally, we calculate the bending moment at B which is 40 kilo Newton meters.

Then we move on to Point C. The bending moment at C is equal to the bending moment at B minus the area under the shear force diagram between Points B and C because the Shear Force Diagram is negative. The shear force area between B and C is 10 kilo Newtons times 4 meters, which is 40 kilo Newton meters. This shear force diagram is negative between B and C. We calculate the bending moment at C using 40 kilo Newton meters at Point B minus 40 kilo Newton meters which is the area of the Shear Force Diagram. Therefore, the bending moment at C equal to zero. Bending moment should end with zero, and the bending moment at point C (which is the last point) is also zero. That means our calculations are correct.

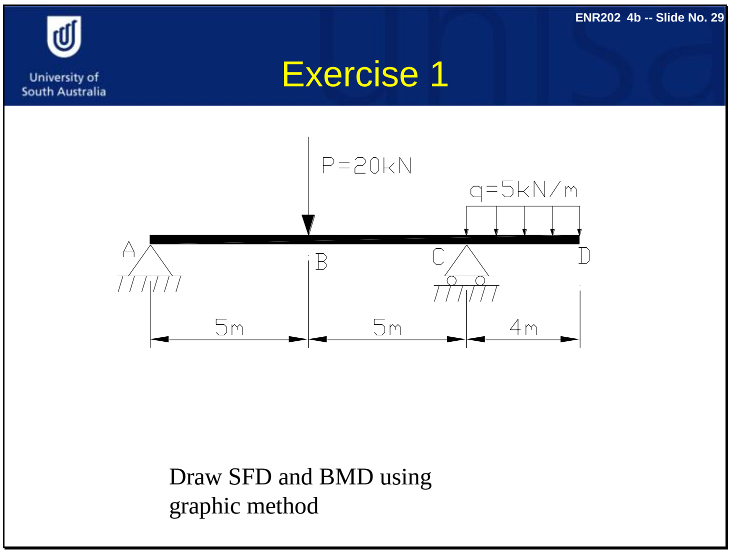
ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 28



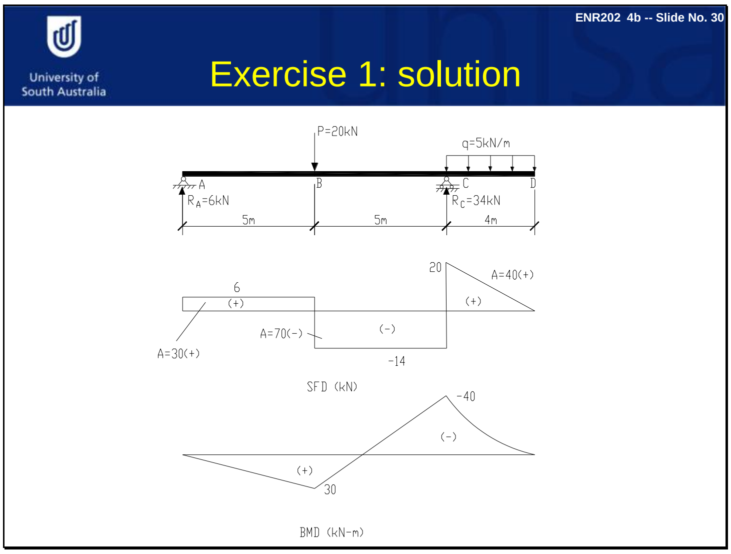
Connect the bending moment values at the important points from the left end to the right end using a straight line from A to C, and we get the bending moment diagram shown in the figure. Try drawing the bending moment diagram yourself and checking it against this figure.

Slide 29



Now pause this presentation and try to work through this exercise. You must draw a shear force diagram and a bending moment diagram for an overhang beam subjected to a uniform distributed load of 5 kilo Newtons per meter on the overhang portion that is between Point C and D, and the point load acting at point B as shown in the figure. Draw the Shear Force Diagram and Bending Moment Diagram using the graphical method. The solution is on the next slide.

Slide 30



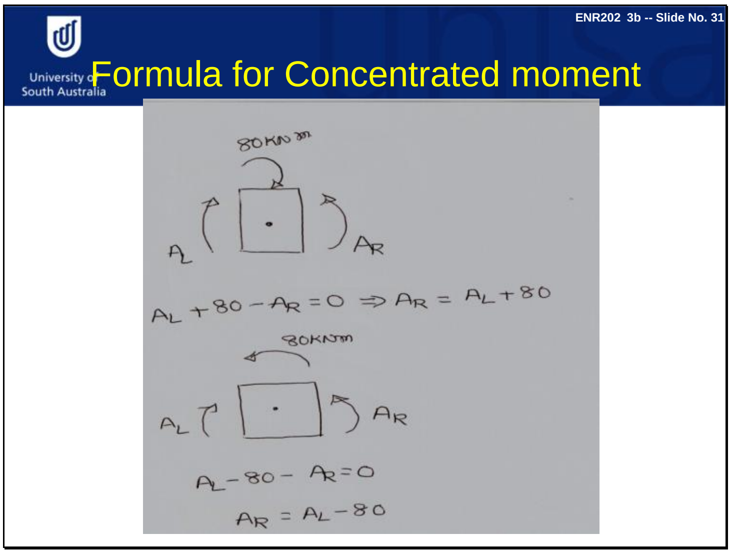
First, calculate the reaction forces at the supports at points A and C. Next, work out the number and position of the important points. There are four important points.

The shear force starts with zero and ends with zero. You need to calculate the shear force from the left end to the right end. If we have a concentrated load, you must consider the left and right side of that point to calculate the shear force. Here, the reaction forces RA and RC are also concentrated loads. So to calculate shear force, you need to consider the left and right side of point A because of the reaction force RA, the left and right side of point B because of the point load at B, and the left and right side of point C because of the reaction force RC,.

Once you know shear force values at important points, connect them from the left end to the right end using a straight line from A to D. You will get the shear force diagram as shown in the figure.

Now, calculate the bending moment values at important points, and connect all the bending moment values to draw the Bending Moment Diagram.

Slide 31

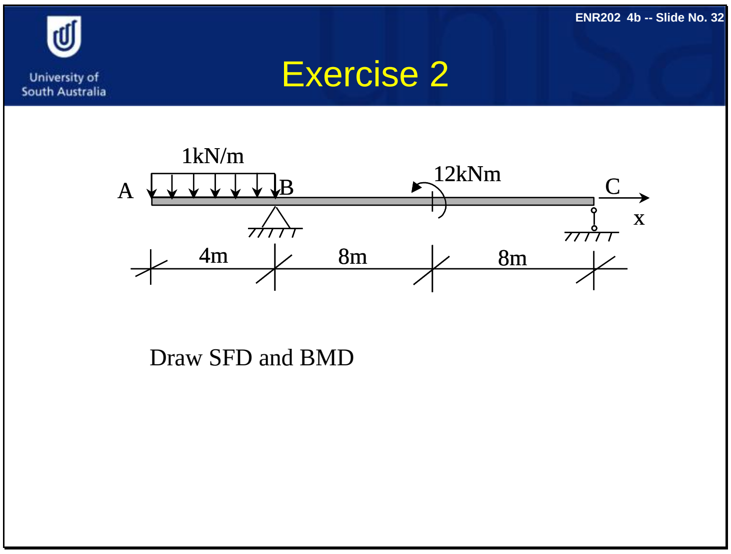


Now let's look at the case of a concentrated moment acting on a beam segment.

Suppose that we have a concentrated moment of 80 kilo Newton meters in a clockwise direction acting on the beam segment as shown in the top figure. The left side cutting cross section has a positive bending moment AL that is in a clockwise direction. The right side cutting cross section has a positive bending moment AR that is in an anti-clockwise direction. We know the value of the bending moment in the left side cross section. We need to calculate the value of the bending moment in the right side cross section. We use the equation which states that AR is equal to AL plus 80 kilo Newton meters. If there is a clockwise concentrated moment acting on the beam segment, we use a positive sign to calculate the right side moment.

There is a concentrated moment of 80 kilo Newton meters in an anti-clockwise direction acting on the beam segment as shown in the bottom figure. The left side cutting cross section has a positive bending moment AL that is in a clockwise direction. The right side cutting cross section has a positive bending moment AR that is in an anti-clockwise direction. We know the value of the bending moment in the left side cross section. We need to calculate the value of the bending moment in the right side cross section. We use the equation which states that AR is equal to AL minus 80 kilo Newton meters. If there is an anti-clockwise concentrated moment acting on the beam segment, we use a negative sign to calculate the right side moment.

Slide 32

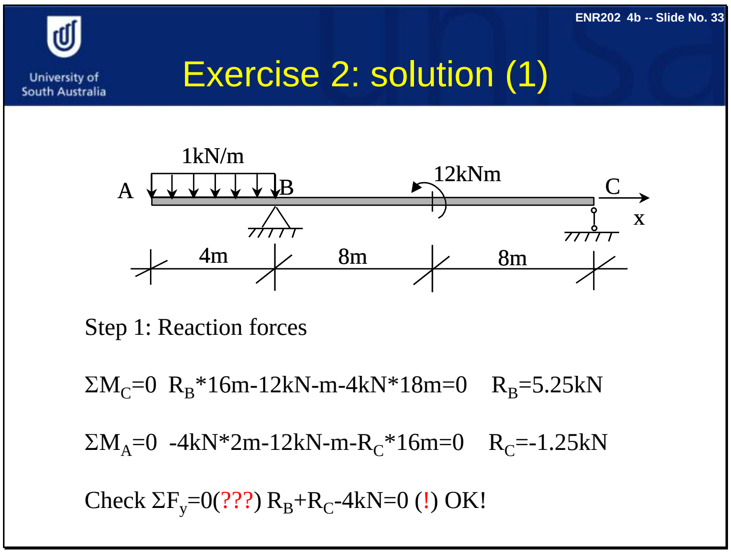


Now pause this presentation and try to work through this exercise. An overhang beam is subjected to a uniform distributed load of 1 kilo Newton per meter between Point A and B as shown in the figure, and a concentrated moment of 12 kilo Newton meters acting in the middle between B and C. Draw the Shear Force Diagram and the Bending Moment Diagram.

Hint: The first step is to calculate the reaction forces at the supports at Points B and C. Next, work out the number and position of the important points. Then calculate the shear force and bending moment values at the important points. Then connect all shear force values to draw the Shear Force Diagram and connect all bending moment values to draw the Bending Moment Diagram.

The solution is given in the next four slides.

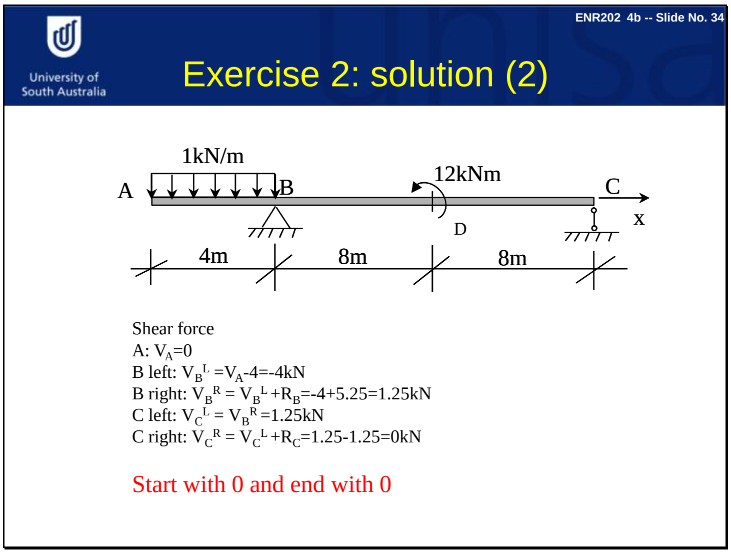
Slide 33



We know the first step is to calculate the reaction forces at the supports. There are two reaction forces at B and C, which are RB and RC, so we need two equilibrium equations.
The first equilibrium equation is that the sum of all force moments about C is equal to zero. That means that you consider Point C as a lever, as we covered in lecture 3a. We have four force moments about C. The first is the moment of reaction force RB about C, which is RB times 16 meters in a clockwise direction. The second is the moment of the uniform distributed load of 1kilo Newton per meter about C . That means the resultant UDL is 1kilo Newton per meter times 4 meters, which is 4 kilo Newtons times 8 meters plus 8 meters plus 2 meters, which equals 18 meters in an anti-clockwise direction. The third is a concentrated moment of 12 kilo Newtons meters in an anti-clockwise direction. Finally, we can calculate reaction force RB which is 5.25 kilo Newtons. The next equilibrium equation is that all forces in a vertical direction are equal to zero. The downward force is the resultant UDL of 4 kilo Newtons. We already know the reaction force RB is 5.25 kilo newtons acting in an upward direction. Finally, we see that the reaction force at C is equal to minus 1.25 kilo newtons. That means the RC is down.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 34



For the next step, we again need to find the number and positions of the important points. We have four important points. Point A is the beginning of the UDL, point B is the end of the UDL, and also a supporting point. Point D is a concentrated moment, and point C is another supporting point.

You have already calculated reaction forces. You can draw a shear force diagram based on vertical forces. These vertical forces include external loads and reaction forces. Based on the shear force diagram, you can draw a bending moment diagram from the left end to the right end. It starts with zero and ends with zero.

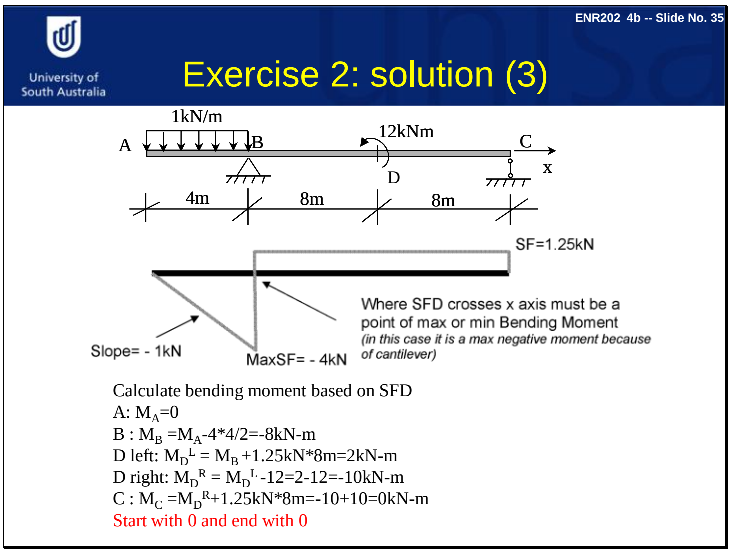
We start with Point A. We have zero shear force at point A. Then we come to Point B. The total UDL is 4 Kilo Newton. We need to consider the left side of point B and the right side of point B, because of reaction force RB.

Consider just to the left side of Point B. Bleft equals minus 4 kilo Newtons. Then we come to Bright. Between Bleft and Bright, we have a concentrated load of PB, which is equal to 5.25 Kilo Newtons in an upward direction. We know that upward is positive. We work out that Bright equals 1.25 Kilo Newtons.

When we come to Cleft, we don’t have any vertical force between Bright to Cleft. So Cleft equals 1.25 kilo Newtons. Move on to Cright. We should have a jump of 1.25 Kilo Newtons. We have a reaction force of 1.25 kilo Newtons acting in a downward direction. Cright equals 1.25 kilo Newtons minus 1.25 kilo Newtons, which is equal to zero. The shear force ends with zero, so we know that the values are correct.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 35



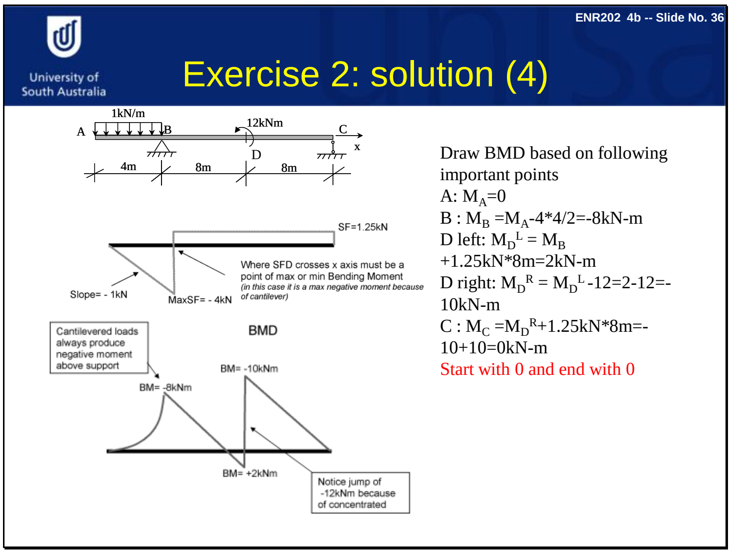
Connect the shear force values at all the important points from left end to right end, you will have a shear force diagram.

To work out the bending moment diagram, start from Point A and end with Point C. Start with a zero bending moment and end with a zero bending moment. Start from the left side of the beam and end with the right side of the beam.

We have zero bending moment at Point A. Then we come to Point B. The total area of the shear force diagram between A and B is negative 8 Kilo Newton meters, which means that the area of the triangle is negative 8 Kilo Newton meters. Therefore the bending moment at B is negative 8 Kilo Newton meters. Then we come to Point D; we have concentrated moment, so we have to find the bending moments at Dleft and Dright. The area of the shear force diagram between B and D is 1.25 kilo Newtons times 8 meters, which is 10 kilo Newtons. This is a positive Shear Force Diagram, so the bending moment at Dleft equals negative 8 Kilo Newton meters plus 10 Kilo Newton meters, which equals 2 Kilo Newton meters. From Dleft to Dright we have a concentrated moment of 12 Kilo Newton meters jump, and this is a negative concentrated moment because it is an anti-clockwise moment. Therefore Dright equals 2 Kilo Newton meters minus 12 Kilo Newton meters, which is negative 10 Kilo Newton meters. From Dright to Point C, we have a positive shear force diagram, so the bending moment at C equals negative 10 Kilo Newton meters plus 10 Kilo Newton meters, which is zero.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

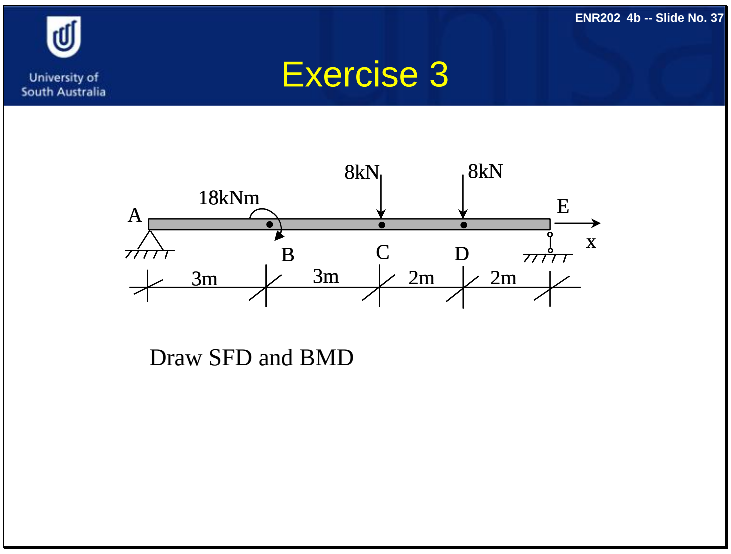
Slide 36



Connect the bending moment values at the important points from the left end to the right end using a straight line from B to C and between A and B. We have a Uniformly Distributed Load, so the bending moment diagram between A and B is parabolic. The top part AB will be negative and the bottom part AB will be positive, because the bending moment at B is negative and because the AB part is a cantilever beam. Try to draw the bending moment diagram on your own and check your work against this figure.

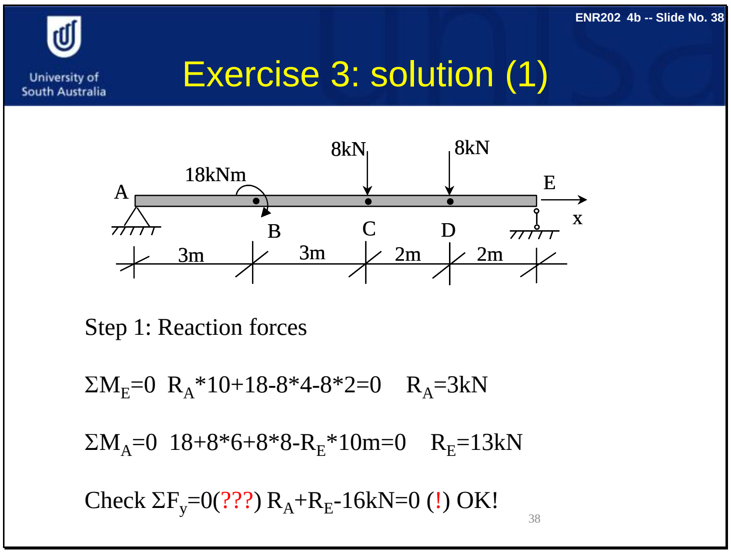
ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 37



Pause the presentation and work through this example. A simply supported beam is subjected to a concentrated moment of 18 kilo Newton meters acting at B at a distance of 3 meters from the left support A, as shown in the figure. One point load is 8 kilo Newtons acting at Point C, at a distance of 6 meters from the left support, and another point load is 8 kilo Newtons acting at Point D at a distance of 8 meters from the left support, as shown in the figure. The total length of the beam is 10 meters. Draw the Shear Force Diagram and the Bending Moment Diagram. The solution is on the next six slides.

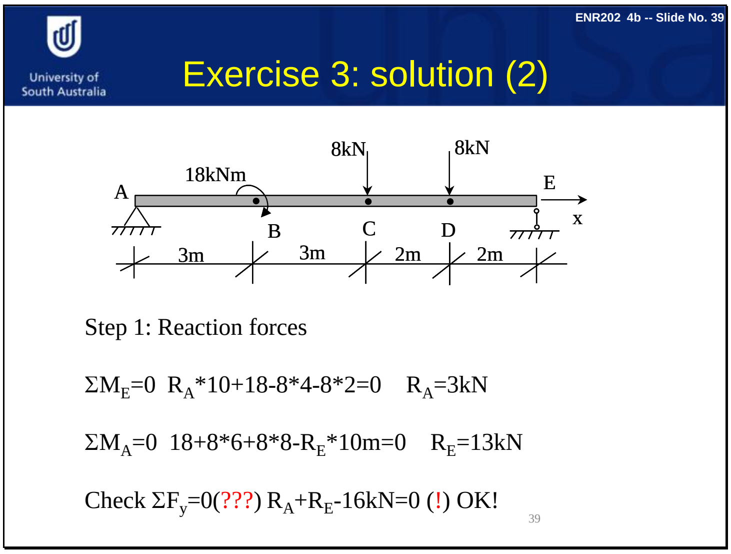
Slide 38



Firstly, calculate the reaction forces at the supports at points A and E.
Next, work out the number and position of the important points.
Next, calculate the shear force and bending moment values at the important points.
Finally, connect all shear force values to draw the Shear Force Diagram and connect all bending moment values to draw the Bending Moment Diagram.

If you are still unsure of how to do this, the solution is given in more detail in the following slides.

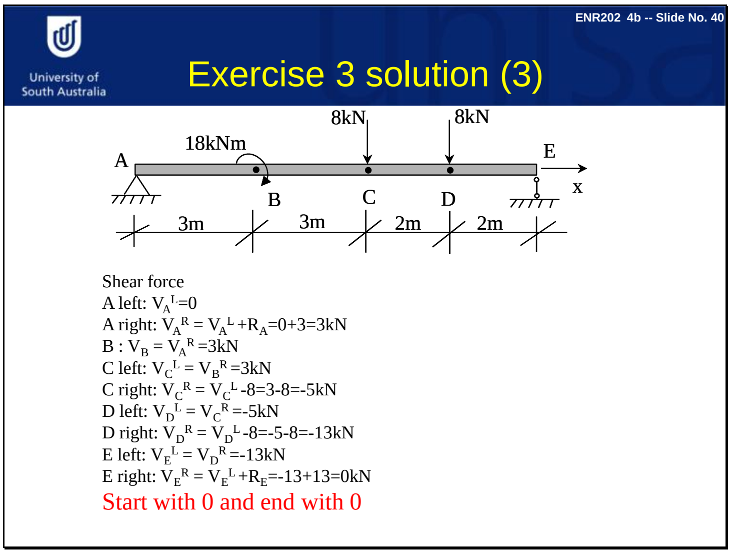
Slide 39



We know the first step is the calculation of the reaction forces at the supports. Calculate the reaction forces at A and E. We have two reaction forces, RA and RE. We need two equilibrium equations. The first equilibrium equation is that the sum of all force moments about E is equal to zero. That means that you consider point E as a lever, as we explained in lecture 3a. We have four force moments about C. The first is the moment of reaction force RA about C, which is is RA times 10 meters in a clockwise direction. The second is a concentrated moment of 18 kilo Newton meters in a clockwise direction. The third is the moment of concentrated load of 8 kilo Newtons at C about point E, which is 8 kilo Newtons times 4 meters. The fourth is the moment of another concentrated load of 8 kilo Newtons at D about point E, which is 8 kilo Newtons times 2 meters. Finally we calculate the reaction force RA which is 3 kilo Newtons.

The next equilibrium equation is that all forces in a vertical direction are equal to zero. The downward force is 8 kilo Newtons plus 8 kilo Newtons, which is equal to 16 kilo Newtons. We already know that the reaction force RA is 3 kilo newtons acting in an upward direction. Finally, the reaction force at E is equal to minus 13 kilo newtons acting in an upward direction.

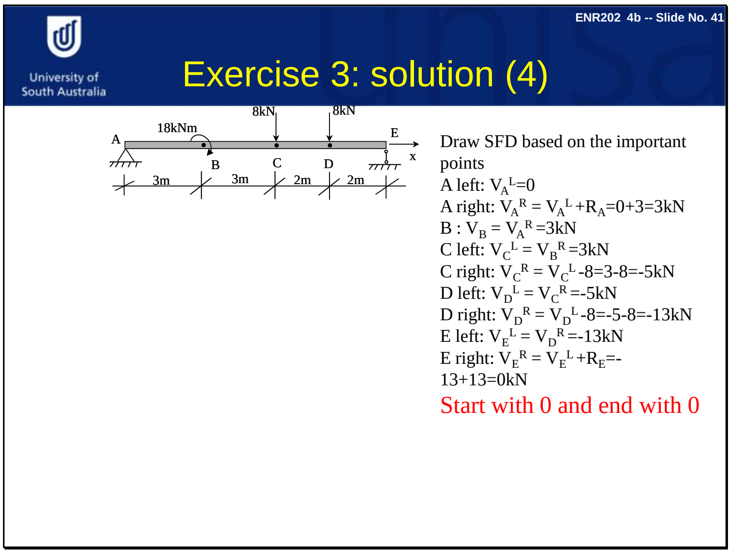
Slide 40



Based on the reaction forces and vertical forces, we can draw the shear force diagram. We have five important points. Point A is a support point, point B is a concentrated moment point, point C is a point load, point D is another point load, and point E is a support point.

First we start with zero shear force and start from left end. So, Aleft is zero. When we come to Aright we have a reaction force RA, so Aright is 3KN, so we have a jump in the Shear Force Diagram of 3 Kilo Newtons. From A to C, we don't have any vertical force, so we have a concentrated shear force. So, Cleft is equal to Aright , which is 3 Kilo Newtons. At Point C, we have a concentrated downward vertical force, so Cright equals 3 kilo Newtons minus 8 kilo Newtons, which is negative 5 kilo Newtons. From C to D, we don't have any distributed force, so Dleft is the same as Cright, which is negative 5 kilo Newtons. We have a concentrated downward force of 8 Kilo Newtons, and Dright equals negative 5 minus 8, which is negative 13 Kilo Newtons, so we have a jump in the shear force diagram at D of 8 Kilo Newtons. From Point Dright to Point Eleft, we don't have any vertical force. So Eleft is the same as Dright, which is 13 kilo Newtons. From Eleft to E right, we have a reaction force RE which is 13 Kilo Newtons acting in an upward direction. Therefore Eright equals negative 13 kilo Newtons plus 13 kilo Newtons, which equals zero.

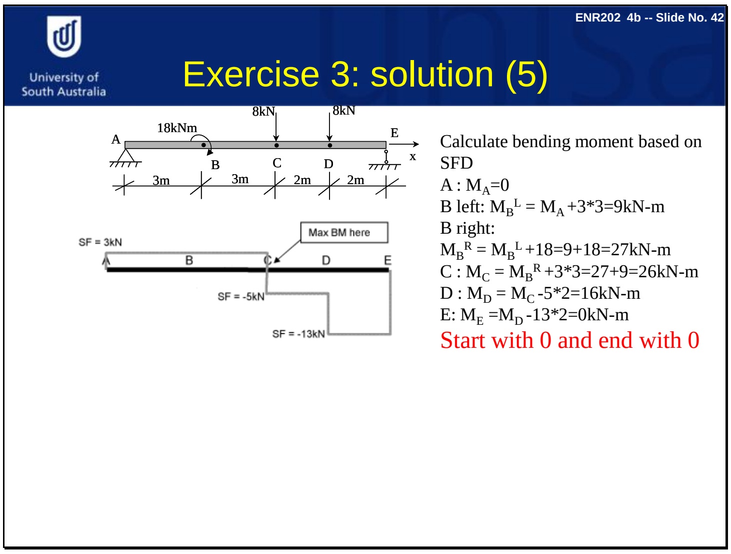
Slide 41



Connect all the shear force values at the important points from the left end to the right end to get the shear force diagram, which you will see in the next slide.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

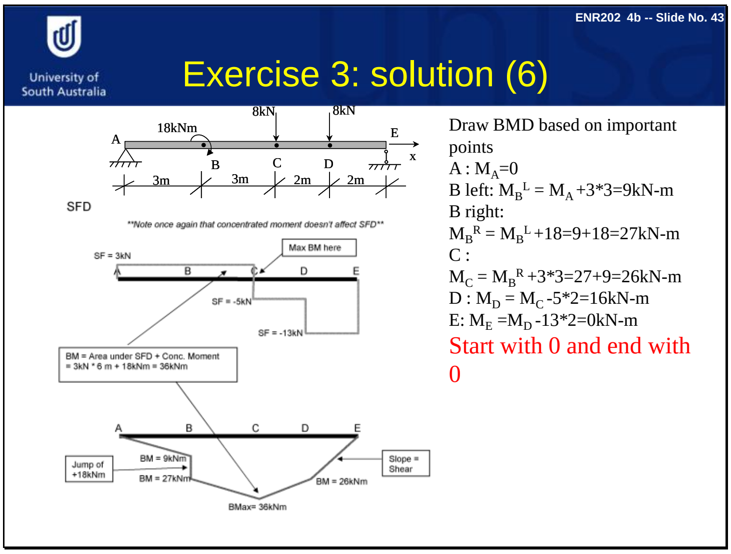
Slide 42



Compare your work to this Shear Force Diagram. If it is the same, well done.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 43

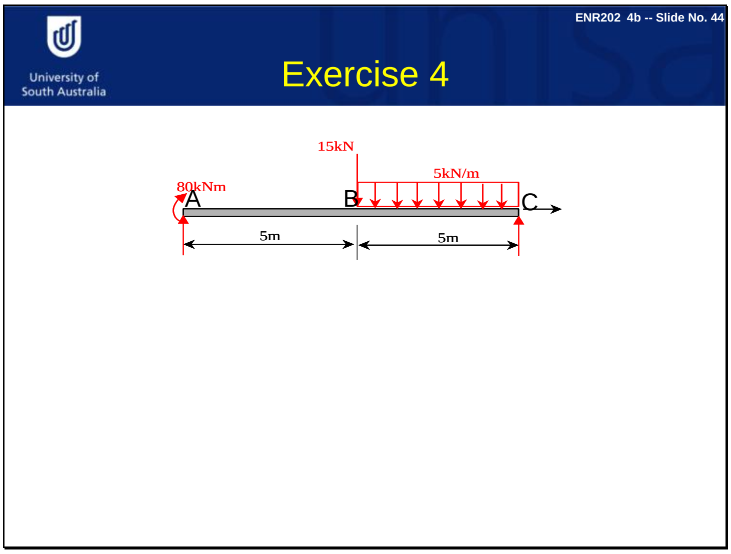


Based on the shear force diagram, we can get the bending moment diagram. Start from the left side and end with the right side. The bending moment at Point A is zero, and the area of the shear force diagram between A and B is 9 Kilo Newton meters. We have a concentrated moment at B of 18 Kilo Newton meters acting in a clockwise direction, so the bending moment at Bleft is 9 Kilo Newton meters, and the bending moment at Bright is 9 Kilo Newton meters plus 18 Kilo Newton meters, which is equal to 27 Kilo Newton meters. From Bright to Point C, we can say that the total area of the shear force diagram between Points B and C is 9 Kilo Newton meters, so the bending moment at C equals 27 Kilo Newton meters plus 9 Kilo Newton meters, which is 36 Kilo Newton meters. From Point C to Point D, we have a negative area shear force diagram of 10 Kilo Newton meters, so the bending moment at D equals 36 Kilo Newton meters minus 10 Kilo Newton meters, which is 26 Kilo Newton meters. We have another negative area of shear force diagram between D and E of 26 Kilo Newton meters, so the bending moment at Point D is 26 Kilo Newton meters minus 26 Kilo Newton meters, which equals zero.

Connect all the bending moment values at important points to get the bending moment diagram.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

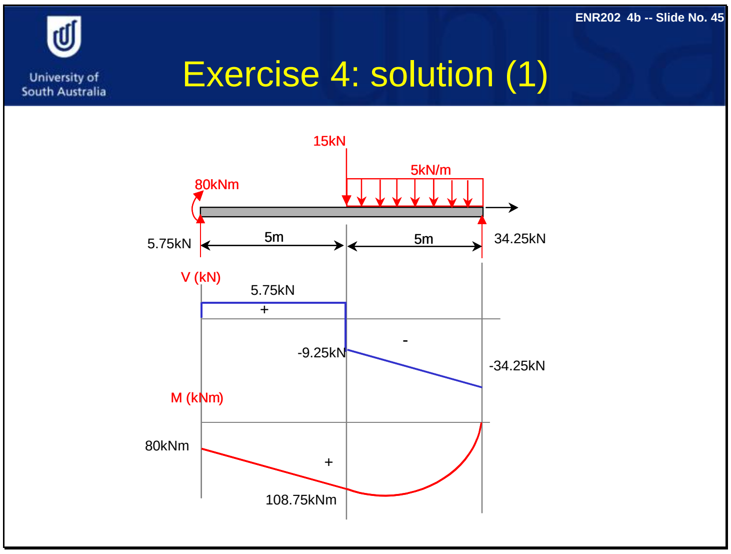
Slide 44



Pause the presentation and work out your solution to this exercise. A simply supported beam is subjected to a concentrated load 15 kilo Newtons acting at the middle of the beam, a uniformly distributed load of 5 kilo Newtons per meter at the right half length of the beam, and a concentrated moment acting of 80 kilo Newtons meters acting at A, as shown in the figure. Draw the Shear Force Diagram and the Bending Moment Diagram.

The solution is in the next three slides.

Slide 45

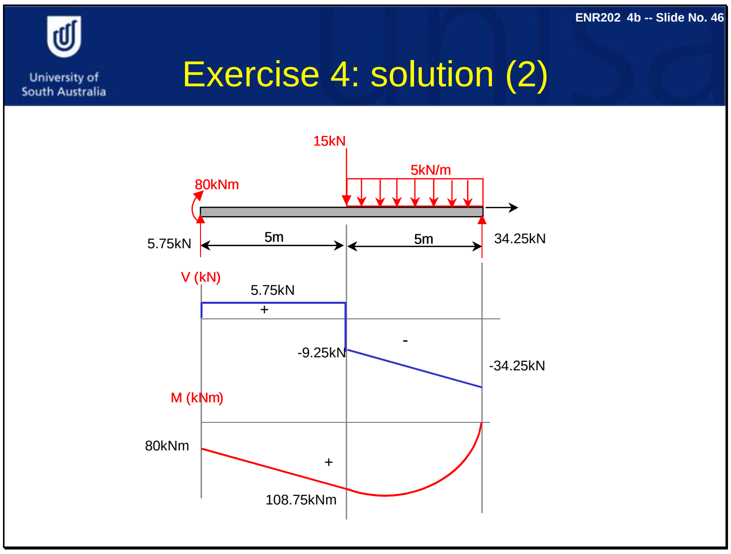


Let’s have look at solution for this example. This example is for a simply supported beam. The concentrated moment acting on left support is 80 kilo Newton meters, and the concentrated load acting in the mid span of the beam is 15 Kilo Newtons, with a distributed load of 5 Kilo Newtons per meter acting on the right half span of the beam. You are required to calculate the shear force diagram and the bending moment diagram.

We have three important points. Point A is a support, point B is a concentrated load and also beginning of the Uniformly Distributed Load. Point C is the end of the Uniformly Distributed Load and is also another support. The first thing you have to calculate is the reaction forces. Consider that all force moments at B equal zero; how many moments are there? There is one moment from RA, one concentrated moment, one moment from the Uniformly Distributed Load, and one moment from the concentrated load. All four moments together equal zero. In this equation, we have only one unknown, which is RA. So we can work out the RA is 5.75 Kilo Newtons. You can also calculate RC, which is our only unknown. All forces in the y-direction are zero. So, we can work out that RC is 34.25 Kilo Newtons.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

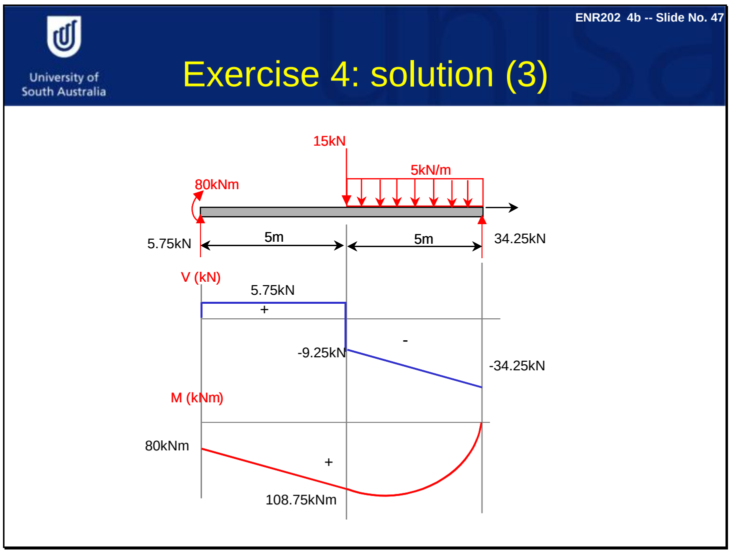
Slide 46



Now lets look at how to draw the shear force diagram. The reaction force RA is 5.75 Kilo Newtons, we have a concentrated moment of 80 Kilo Newton meters, a concentrated load of 15 Kilo Newtons, and a Uniformly Distributed Load of 5 Kilo Newtons per meter. RC is 34.25 Kilo Newtons, and we have three important points, A, B and C. To draw the shear force diagram, we will start with the left end. Consider just to the left of Point A. At Aleft we have zero shear force and zero bending moment. Then we come to Aright. Between Aleft and Aright, we have a concentrated load RA, which is equal to 5.75 Kilo Newtons in an upward direction, so Aright equals 5.75 Kilo Newtons.

When we come to Bleft, we don't have any vertical force between Aright and Bleft. So Bleft equals 5.75 Kilo Newtons. Move on to Bright. We should have a jump of 15 Kilo Newtons of concentrated load. Bright equals 5.75 kilo Newtons minus 15 kilo Newtons, which is negative 9.25 Kilo Newtons. We come to Cleft. The total UDL is 25 Kilo Newtons acting down. Cleft equals negative 9.25 kilo Newtons minus 25 kilo Newtons, which is negative 34.25 Kilo Newtons. From Cleft to Cright, we have a concentrated load RC. Cright is equal to negative 34.25 kilo Newtons plus 34.25 kilo newtons, which is zero. Connect all the values at the important points from the left end to the right end, and we will get the shear force diagram.

Slide 47



Now let us look at the bending moment diagram. There is a concentrated moment 80KNm acting at A. We have other vertical forces and reaction forces. Again, we start with the left end and then back to the right end. We have 3 important points, A,B and C.

We start with Aleft. We have zero bending moment at Aleft. But we have a concentrated moment of 80 Kilo Newton meters acting at A, so we have a bending moment jump of 80 Kilo Newton meters in a clockwise direction. This is a positive bending moment as per the graphical method. Aright equals 80 Kilo Newton meters.

The shear force area between A and B is 5.75 kilo Newtons times 5 meters. It is a positive Shear Force Diagram. The moment at B equals 80 Kilo Newton meters plus 5.75 kilo Newtons times 5 meters, which is 108.75 Kilo Newton meters. From Point B to Point C, we have to include the shear force area between B and C, which is negative, and is equal to 108.75 Kilo Newton meters.

The moment at C equals 108.75 Kilo Newton meters minus 108.75 Kilo Newton meters, which is zero. Finally, the bending moment at C is back to zero. Connect all the moment values at the important points. Connect with a straight line between A and B. From B to C we have a Uniformly Distributed Load, so connect with a parabolic line between B and C. All these values are positive. Finally we get bending moment diagram as shown in the figure.

ENR202 Mechanics of Materials Lecture 4B Slides and Notes

Slide 48

Thank you for your attention. In the next lecture summary, we start centroids, and moments of inertia, and bending stresses in beams.