ENR202 Mechanics of Materials Lecture 5A Slides and Notes

Slide 1

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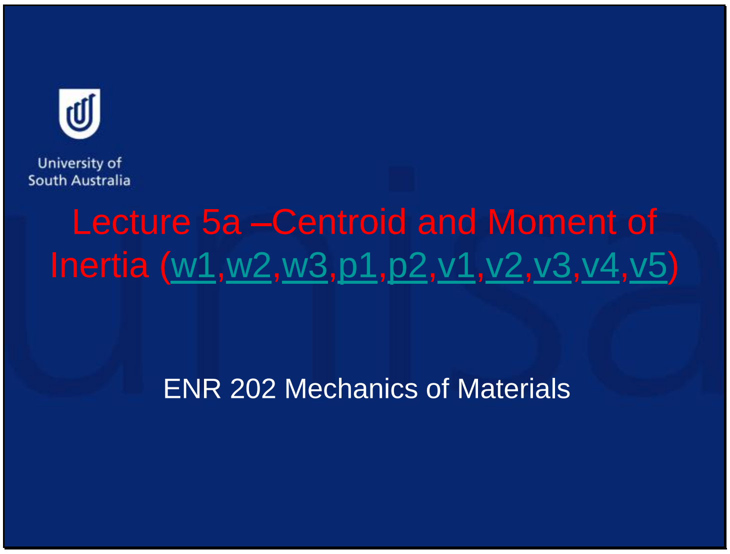
Slide 2



Welcome to Lecture summary 5A, which will concentrate on centroid and moments of inertia of cross sectional shape of members.

Note that throughout all the lecture summaries for Mechanics of Materials, you will see live links, denoted by the letters W, P and V. These links point to web pages, presentations and videos which will enhance your understanding of the content. You can pause the presentation at any time to access these links, and then go back to the presentation when you have finished looking at them.

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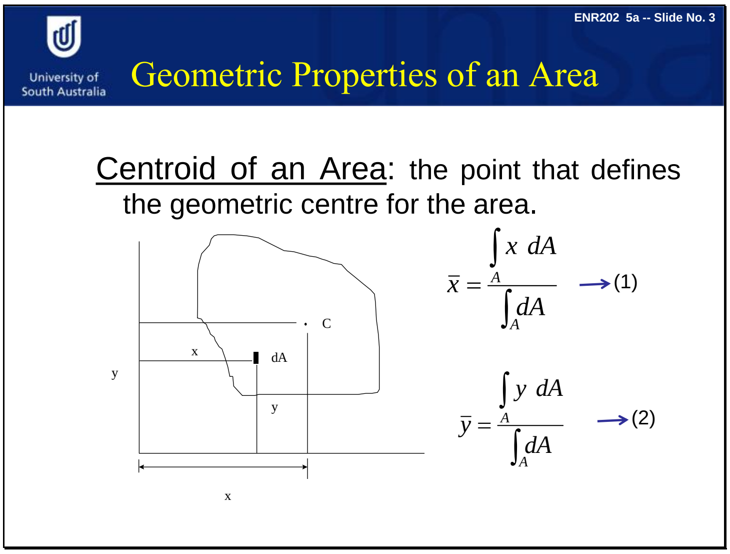

In the same way that we can have a centre of gravity, a centre of weight, a centre of mass and a centre of volume, we can have a centre of area. The centre of area is called the centroid. It is also defined from the first moment of an area (the term moment comes from the moment of a force about a point). We have already covered the moment of a force about a point when we looked at Shear Force Diagrams and Bending Moment Diagrams. We use same concept to calculate the first moment of an area. You can see the formulas in the slide to calculate the geometric centre for the given area. You use the integral formula to calculate the centroid from the y axis (the x bar) in Equation 1 and from the x axis (the y bar) in Equation 2. The integration x times dA or y times dA is the first moment of an area. Here dA is the infinitesimal element area as shown in the figure. The integration of dA is the area of the irregular shape.

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Centroid of Composite Areas (v1)

In engineering work, often an area can be divided into several parts, each part having a familiar geometric shape (centroid and area of each of these part are known), then the centroid and area of the whole area can be calculated by using:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i} \longrightarrow (3) \qquad \bar{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i} \longrightarrow (4)$$

where Ai is the area of the ith part,
x_i and y_i are the coordinates of the centroid of the ith part,
and n is the number of parts.

In Engineering, we have structural members with regular shapes such as squares, rectangles, circles, triangles, square hollow sections, rectangular hollow sections, circle hollow sections, ‘I’ shape sections, Channel shape sections, ‘T’ sections and angle shape sections. Often an area can be divided into several parts, each part having a regular geometric shape. For example, suppose you divide a cross section into a number of parts (‘n’). The centroid from the ‘y’ axis is the x bar, which is equal to the sum of the first moment of area, which is ‘x’ times the area of the shapes divided by the sum of all areas of the shapes. ‘x’ times A is Area moment.

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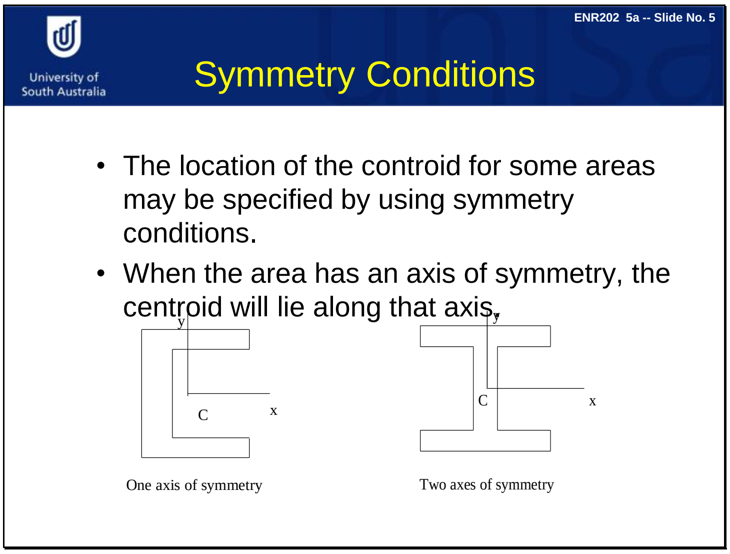

You can use advantage of symmetry shapes to find the centroid. For example, the left figure is a channel section which has horizontal symmetry. The I shape section in the right figure has both horizontal and vertical symmetry. The centroid lies on the symmetric axis for both. So, in the channel section, we have an x-axis which is also a symmetric axis. The centroid lies on this x-axis. For the I section, the second figure, the centroid lies on the intersection of the x-axis and y-axis, as you can see in the figure.

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Composite-area Procedure for Locating the Centroid

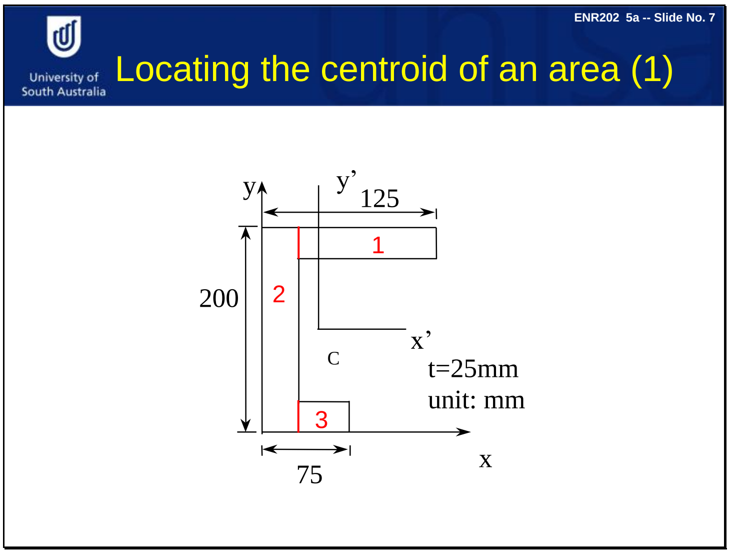
1. Divide the composite area into simpler areas for which there exist formulas for area and for the coordinates of the centroid. (see the table inside the front cover.)
2. Establish a convenient set of reference axes (x, y).
3. Calculate the coordinates of the composite centroid, (x, y), using Eq. (A-2).

To calculate the centroid of a composite-area shape, follow these three steps. Firstly, divide the composite area into regular shape areas for which you will be able to find the centroid, using the x-axis and y-axis approach. You could divide the shape into squares, rectangles, triangles or circles – we can easily find the centroids for all of these shapes.

The second step is to draw x and y reference axes wherever is convenient.

The third step is to calculate the centroid of the composite area shape using the formulas which we gave in slide 4. Remember that Equation 3 is the centroid from the y axis, which is the x bar. Equation 4 is the centroid from x axis, which is the y bar.

Slide 7



In this example, we have an unsymmetrical channel section. Suppose that you need to locate the centroid of this channel shape. For the first step, separate the channel section into regular shapes. We have three rectangular shapes (indicated by red lines in the figure). The first rectangle is 125 millimetres by 25 millimetres. The second rectangle is 200 millimetres by 25 millimetres. The third rectangle is 50 millimetres by 25 millimetres, as you can see in the figure.

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Locating the centroid of an area (2)

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i} = \frac{75\times 100\times 25 + 12.5\times 200\times 25 + 50\times 50\times 25}{100\times 25 + 200\times 25 + 50\times 25}$$

$$= \frac{312500}{8750} = 35.7 \ mm$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i} = \frac{187.5\times 100\times 25 + 100\times 200\times 25 + 12.5\times 50\times 25}{100\times 25 + 200\times 25 + 50\times 25}$$

$$= \frac{984375}{8750} = 112.5 \ mm$$

In the third and final step, you calculate the centroid of the composite area. The centroid of the first rectangle, from the y axis, is 25 millimetres plus in brackets (100 millimetres divided by 2), which is 75 millimetres. The centroid of second rectangle, from the y axis, is 25 millimetres divided by 2, which is 12.5 millimetres. The centroid of the third rectangle, from the y axis, is 25 millimetres plus in brackets (50 millimetres divided by 2), which is 50 millimetres. The area of the first rectangle is 100 millimetres by 25 millimetres. The area of the second rectangle is 200 millimetres by 25 millimetres. The area of the third rectangle is 50 millimetres by 25 millimetres. So, we can calculate that the centroid from y axis is the x bar, which is 35.7 millimetres.

Now you need to calculate the centroid from the x axis, which is the y bar, as shown in the bottom calculations. The centroid of the first rectangle, from the x axis, is 200 millimetres minus in brackets (25 millimetres divided by 2), which is equal to 187.5 millimetres. The centroid of the second rectangle, from the x axis, is 200 millimetres divided by 2, which is 100 millimetres. The centroid of the third rectangle, from the x axis, is 25 millimetres divided by 2, which is 12.5 millimetres. So, we can calculate that the centroid from the x axis is the y bar, which is 112.5 millimetres

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Moment of Inertia for an Area (1)

There are some topics in mechanics of materials that require evaluation of an integral of the second moment of an area, this integral is referred to as the moment of inertia.

When we work out a centroid, we use the integral of first moment of the area. Now, we will look at how to calculate an integral of second moment of an area. This integral is also know as the moment of inertia.

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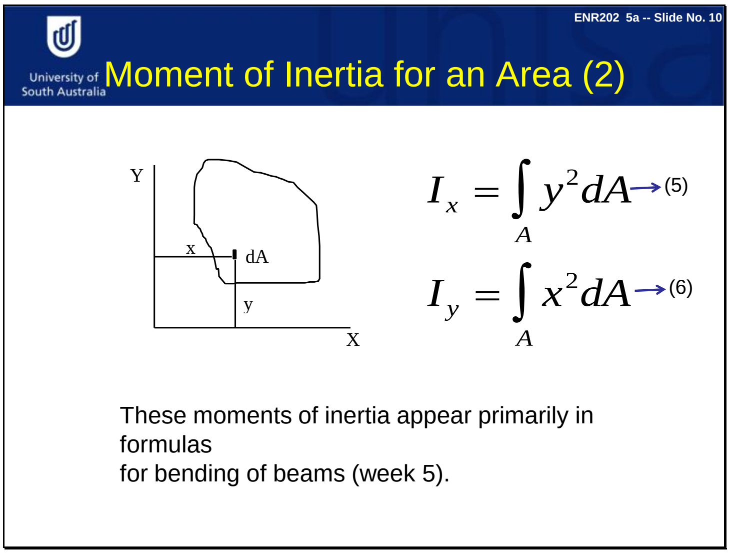

Here is the process for calculating the integral of second moment of area. Equation 5 calculates the moment of inertia about the x-axis. Equation 6 calculates the moment of inertia about the y-axis. Note that the moment of inertia about the centroid x-axis is Ix, the moment of inertia about the centroid y-axis is Iy. The Ix and Iy values will be used in bending of beams (we will look at this in the next lecture summary). Ix is the integral of y squared times dA over the area. Iy is the integral of x squared times dA over the area.

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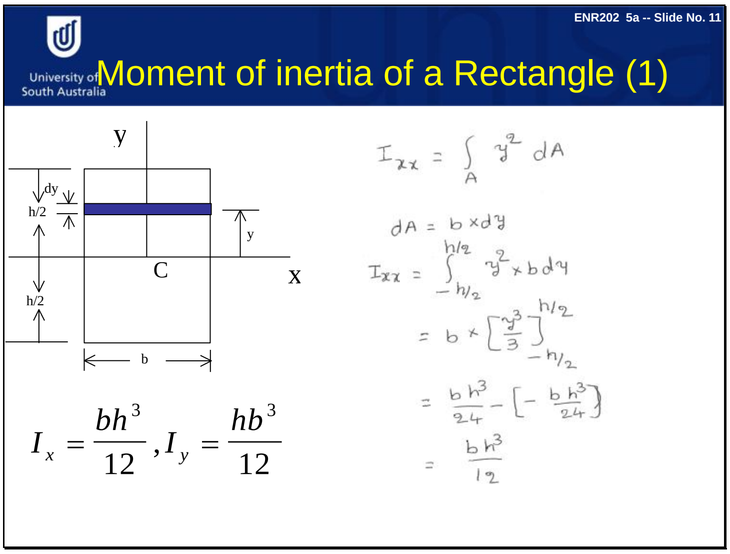

Suppose that you need to calculate the moment of inertia of a rectangle about a centroid about the x axis and y-axis. Start with the x-axis. The formula for the centroid moment of inertia about the x axis is the integral of y squared times dA. dA is equal to the breadth of the rectangle times ‘dy’, as shown in the figure. dA is the area of the shaded area. ‘y’ is the distance between the centroid of the shaded portion and x-axis, as shown in the figure. Ix is the integral of y squared times b times dy. The limits of integration are from minus half height of the rectangle to positive half height of the rectangle. So, we can work out the moment of inertia about the centroid x axis is b times h cubed divided by 12.

You can calculate the moment of inertia about the centroid y axis using the same procedure. Pause this presentation, and try to work that out now. The answer is on the next slide.

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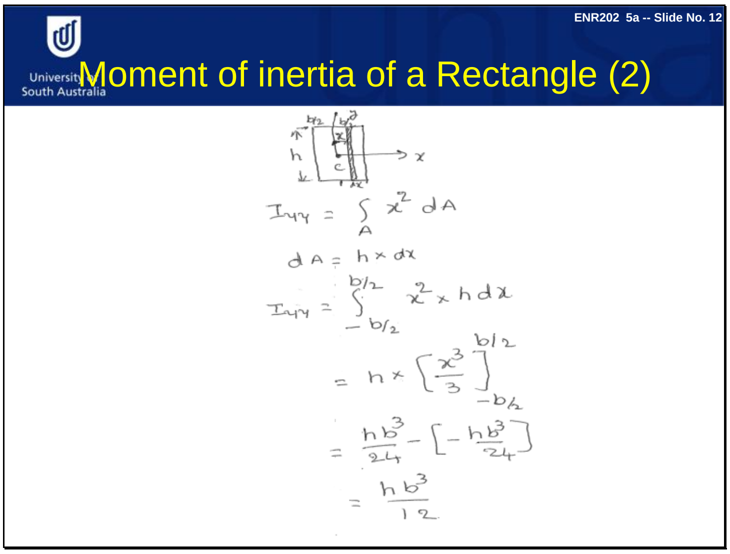

So, here is the solution. The problem is to calculate the moment of inertia of a rectangle about the centroid about the y-axis. The formula for the centroid moment of inertia about the y axis is the integral of x squared times dA. dA is the height of the rectangle times ‘dx’, as shown in the figure. dA is the area of the shaded area. ‘x’ is the distance between the y-axis and the shaded portion, as shown in the figure. Iy is the integral of x squared times h times dx. The limits of integration are from minus the breadth of the rectangle divided by 2 to positive the breadth of the rectangle divided by 2. So we can calculate the moment of inertia about the centroid y axis is h times b cubed divided by 12.

How does this answer compare with the result which you got?

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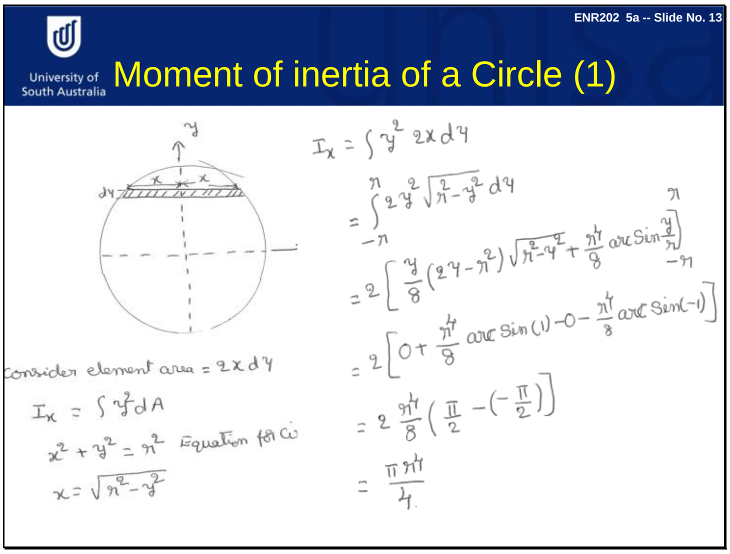

Now, let's look at how to calculate the moment of inertia of a circle about the centroid about the x-axis. The formula for the centroid moment of inertia about the x axis is equal to the integral of y squared times dA. dA is equal to 2 times x times dy, as shown in the figure. dA is the area of the shaded area. Ix is the integral of y squared times 2 times x times dy. The limits of integration are from minus the radius of the circle to plus the radius of the circle. So, the moment of inertia about the centroid x axis is pi times r to the power of 4, divided by 4.

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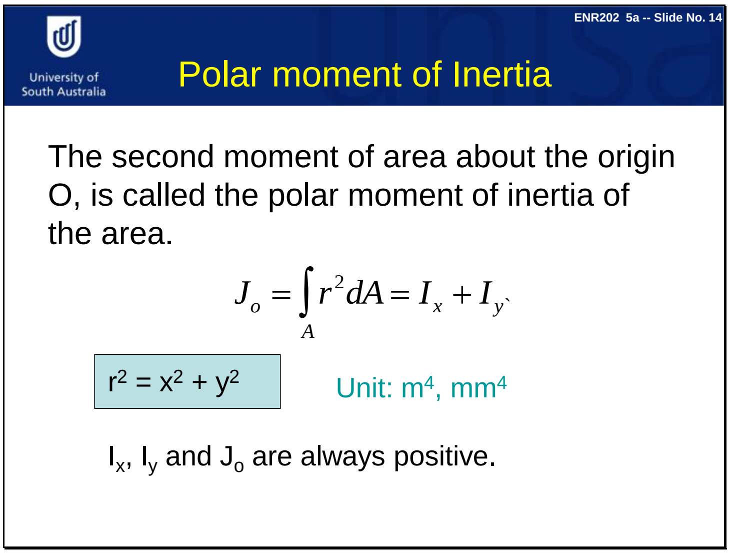

Now we have looked at how to calculate the moment of inertia about the x-axis and y-axis. What about the moment of inertia about the origin (that means the intersection of the x-axis and y-axis). The second moment of area about the origin is called the polar moment of inertia of the area. The polar moment of inertia about the centroid is the integral of ‘r’ squared times dA, which is the sum of moment of inertia about the centroid x-axis and the moment of inertia about the centroid y-axis. ‘r’ is the square root of the sum of ‘x’ squared plus ‘y’ squared. The units of the moment of inertia are meters to the power of 4 or millimetres to the power of 4.

Remember that the moment of inertia about the centroid x-axis is Ix, the moment of inertia about the centroid y-axis is Iy, and the moment of inertia about the origin is J0. These are always positive values.

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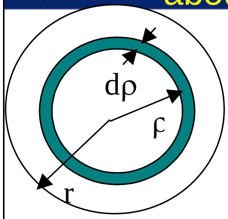
Polar Moment of inertia of a circle about its centre (p1,p2,p3,w1)



$$J_o = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

Polar moment of inertia will be used for torsion design(week10)

Shaded area = $2\pi \rho \cdot d\rho$

$$J_o = \int \rho^2 \, dA$$

$$= \int_0^r \rho^2 \, 2\pi\rho \, d\rho$$

$$= 2\pi \int_0^r \rho^3 \, d\rho$$

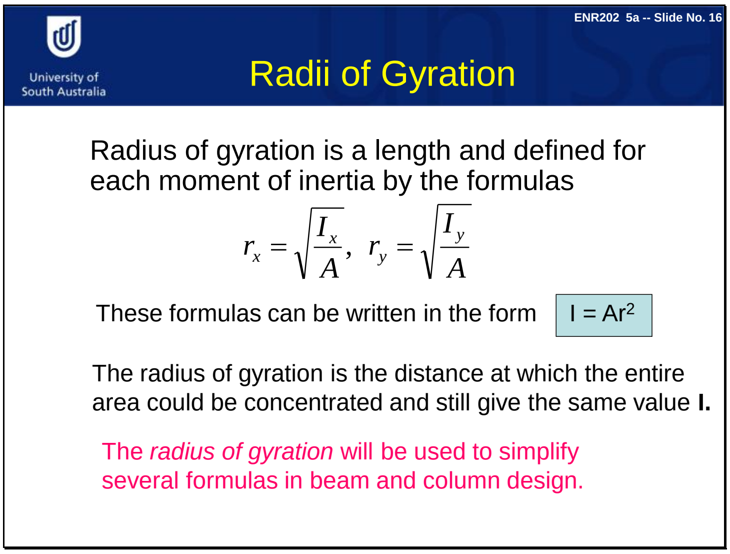
$$= 2\pi \left[\frac{\rho^4}{4}\right]_0^r$$

$$= 2\pi \frac{r^4}{4}$$

$$= \frac{\pi r^4}{2}$$

We know already the moment of inertia about the centroid x-axis is pi times r to the power of 4 divided by 4. The same value for the moment of inertia about the centroid y-axis is pi times r to the power of 4 divided by 4. The polar moment of inertia is the sum of the moment of inertia about the centroid x-axis and the moment of inertia about the centroid y-axis equal to pi times r to the power of 4 divided by 2. This formula will be used for torsion design, which we will cover later in the unit. You can see the derivation for the polar moment of inertia for the circle here.

Slide 16



There is one more technical term which you will need to know. It is the Radius of gyration. The radius of gyration about the centroid x-axis is a length unit. It is defined as the square root of the moment of inertia about the centroid x-axis divided by the area. The radius of gyration about the centroid y-axis is also a length unit. It is defined as the square root of the moment of inertia about the centroid y-axis divided by the area.

This radius of gyration will be used to simplify several formulas in beam and column design.

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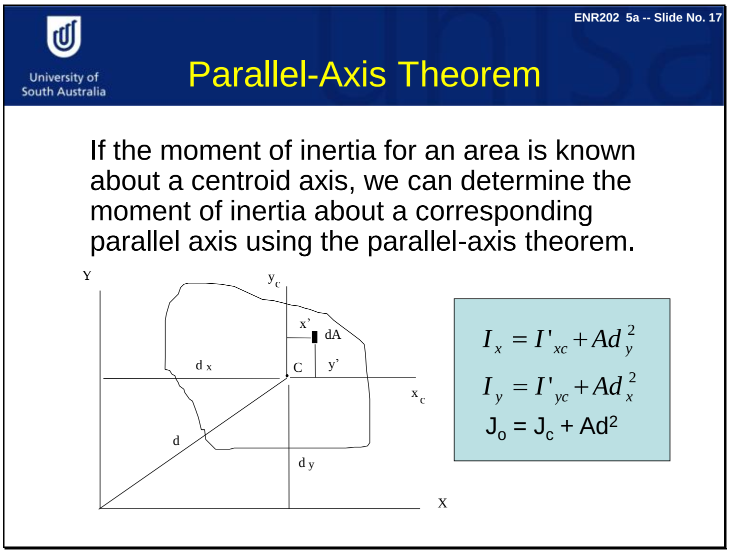

If the moment of inertia about the centroid axis is known for an area, we can determine the moment of inertia about a corresponding parallel axis by parallel-axis theorem. Point C is the intersection of the ‘xc’ and ‘yc’ axis. ‘xc’ is the centroid x-axis and ‘yc’ is the centroid y-axis, as shown in the figure. That means that we know moment of inertia about the centroid ‘xc’ axis and ‘yc’ axis.

In the first equation, if we know I dash xc (the moment of inertia about the centroid x axis), we can calculate the moment of inertia about the ‘x’ axis which is parallel to the centroid x axis. In the second equation, if we know I dash yc, which is the moment of inertia about the centroid y axis, we can calculate the moment of inertia about the ‘y’ axis which is parallel to the centroid y axis. In the third equation, if we know Jc, which is the polar moment of inertia about the centroid axis, we can calculate the polar moment of inertia Jo about the intersection of the ‘x’ axis and the y-axis, which is the origin of the x and y axes.

dx’ is the distance between the y axis and the centroid y axis, and dy is the distance between the x axis and the centroid x axis. ‘d’ is the distance between the centroid to the origin of the x-axis and y-axis. A is the area of the given shape.

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Moments of Inertia of Composite Areas

The moments of inertia of a composite area may be computed by summing the contributions of the individual areas:

$$I_x = \sum_i (I_x)_i, \quad I_y = \sum_i (I_y)_i$$

Now we have studied the centroid moment of inertia about the x-axis and the y-axis for a rectangle and a square. If you combine rectangles, we can work out T shapes, Channel shapes and I shape sections. The moment of inertia of a composite area is calculated by summing the contributions of the individual rectangles.

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Composite-area Procedure for Calculating Second Moments

1. Divide the composite area into simpler area for which there exist formulas for centroidal coordinates and moments of inertia. (see the table inside the front cover of the text book)
2. Locate the centroid of each constituent area and establish centroidal reference axes (x_c, y_c) parallel to the given (x, y) axes.
3. Employ Eqs (A-5,A-6,A-7) to compute the moments of inertia of the constituent areas with respect to the (x, y) axes and sum them.

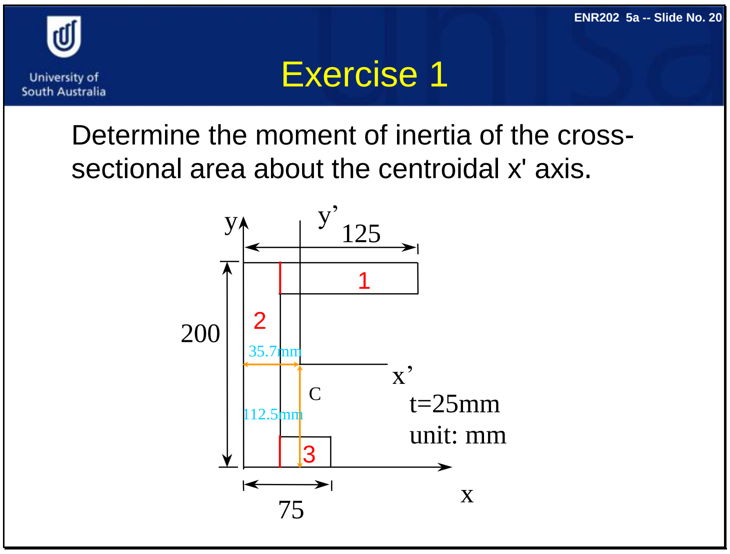
In this slide, we will talk about the procedure for calculating the moment of inertia for composite area shapes.

The first step is to divide the composite area into simple areas for which you know the centroid and moment of inertia formulas.

The second step is to find the centroid of each simple area and establish the centroid reference axes ‘xc’ and ‘yc’, and show axes parallel to the centroid reference axes, the ‘x’ and ‘y’ axes.

The third step is to compute the moment of inertia for the simple areas about the centroid reference axes ‘xc’ and ‘yc’ of the composite area and sum them. From this, we can get the moment of inertia of a composite area. In the next slides, we will look at an example of this in action.

Slide 20



Earlier on in this lecture summary, we calculated the centroid of the channel section above (slide 7). The centroid from the x axis is the y bar, which is equal to 112.5 millimetres. The centroid from the y axis is the x bar, which is equal to 35.7 millimetres. So we have three simple rectangular shapes, as you can see in the figure. Pause this presentation and try to determine the moment of intertia of the cross-sectional area about the x axis. The solution is on the next three slides.

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Exercise 1: Solution (1)

$$I_x = \bar{I}_{x'} + Ad_y^2 = \frac{1}{12}\times 100\times 25^3 + 100\times 25\times(187.5-112.5)^2$$

$$+\frac{1}{12}\times 25\times 200^3 + 25\times 200\times(100-112.5)^2$$

$$+\frac{1}{12}\times 50\times 25^3 + 50\times 25\times(12.5-112.5)^2$$

$$= 44.208\times 10^6 \ mm^4$$

Here you can see how we calculate the centroid moment of inertia for the composite area. The channel section is separated into three simple rectangles. The contribution of the first rectangle moment of inertia is calculated by using parallel axes theorem. We calculate the moment of inertia of the first rectangle about the centroid of composite area ‘xc’ axis. The first rectangle is 100 millimetres wide and 25 millimetres high. The area of this rectangle is 100 millimetres times 25 millimetres. The centroid moment of inertia of this rectangle is 100 times 25 to the power of 3 divided by 12. ‘dy’ is the distance from the centroid of this rectangle to the centroid of the composite area from the x axis, which is 187.5 millimetres minus 112.5 millimetres.

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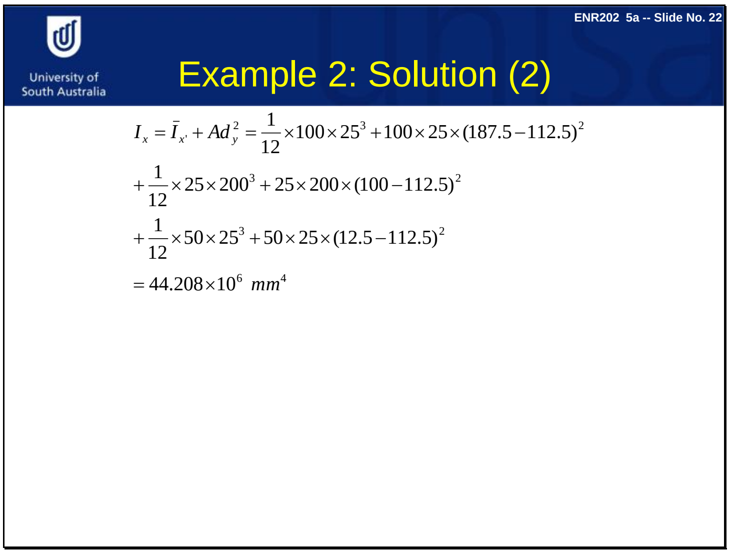
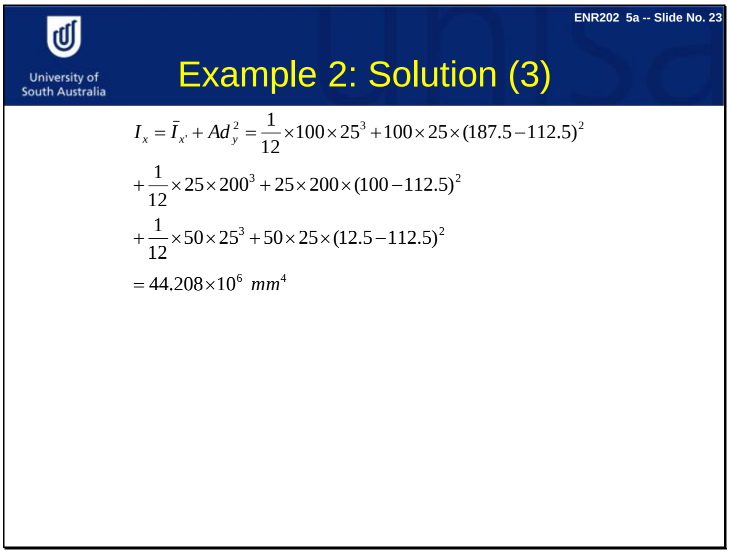


In the same way, the second rectangle is 25 millimetres wide and 200 millimetres high. The area of this rectangle is 25 millimetres times 200 millimetres. The centroid moment of inertia of this rectangle is 25 times 200 to the power of 3 divided by 12. ‘dy’ is the distance from the centroid of this rectangle to the centroid of the composite area from the x axis, which is 100 millimetres minus 112.5 millimetres.

Slide 23



The third rectangle is 50 millimetres wide and 25 millimetres high. The area of this rectangle is 50 millimetres times 25 millimetres. The centroid moment of inertia of this rectangle is 50 times 25 to the power of 3 divided by 12. ‘dy’ is the distance from the centroid of this rectangle to the centroid of the composite area from the x axis, which is 12.5 millimetres minus 112.5 millimetres. So we can calculate that the centroid moment of inertia for the channel section is 44.208 times 10 to the power of 6 millimetres to the power of 4.

Now try to calculate the centroid moment of inertia for this channel section about the ‘y’ axis (that is, the centroid y axis of the channel section). The answer is on the next slide.

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Exercise 2: Solution

$$I_x = \bar{I}_{x'} + Ad_y^2 = \frac{1}{12}\times 100\times 25^3 + 100\times 25\times(187.5-112.5)^2$$

$$+\frac{1}{12}\times 25\times 200^3 + 25\times 200\times(100-112.5)^2$$

$$+\frac{1}{12}\times 50\times 25^3 + 50\times 25\times(12.5-112.5)^2$$

$$= 44.208\times 10^6\ mm^4$$

$$I_y = \bar{I}_{y'} + Ad_x^2 = \frac{1}{12}\times 25\times 100^3 + 25\times 100\times(75-35.7)^2$$

$$+\frac{1}{12}\times 200\times 25^3 + 200\times 25\times(12.5-35.7)^2$$

$$+\frac{1}{12}\times 25\times 50^3 + 50\times 25\times(50-35.7)^2$$

$$= 9.41\times 10^6\ mm^4$$

The answer is 9.41 times 10 to the power of 6 millimeters to the power of 4. How does your answer compare with this answer? If you didn't get the answer right, the detailed calculations are on this slide.

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Thank you for your attention.