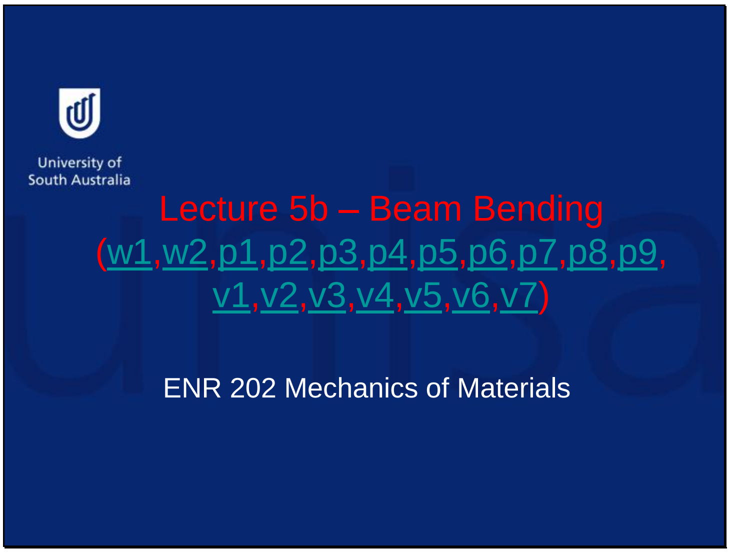
ENR202 Mechanics of Materials Lecture 5B Slides and Notes

Slide 1



Welcome to this lecture summary on beam bending, a new topic.

Note that throughout all the lecture summaries for Mechanics of Materials, you will see live links, denoted by the letters W, P and V. These links point to web pages, presentations and videos which will enhance your understanding of the content. You can pause the presentation at any time to access these links, and then go back to the presentation when you have finished looking at them.

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Slide 2

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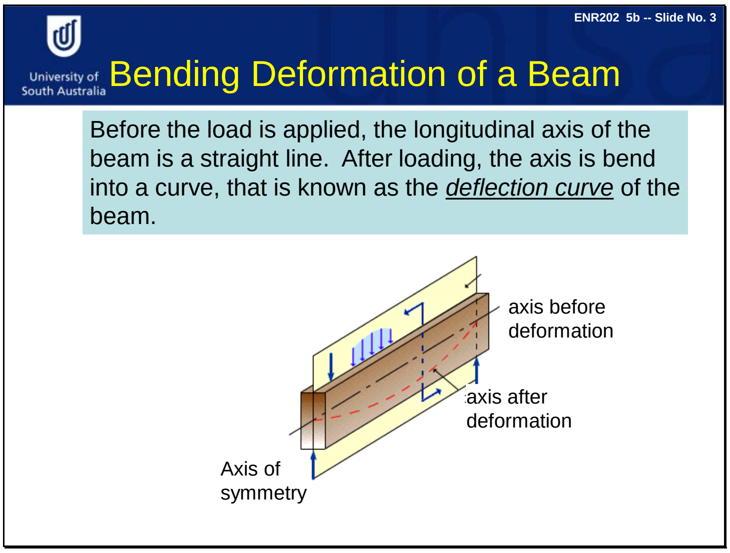
Slide 3



Beam bending refers to the application of vertical forces on a beam. Before force is applied, the beam is in a straight line. After applying the loads, the straight axis deforms into a curve. This deformation is known as bending.

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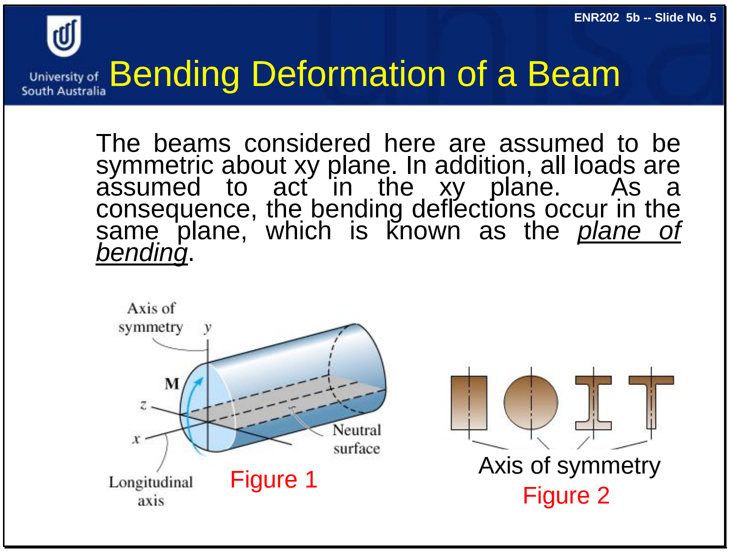
Slide 4

For example, in this image you can see a beam which has deformed into a curve after an application of load.

Slide 5



In this course, we are only studying the phenomenon of plane bending. We are considering beams with symmetrical cross-sections. For example, there is a symmetrical plane about y axis on the cross section as shown in the figure 1, and all loadings are applying to this symmetric plane, so the deformed beam axis will be limited within the symmetric plane. We called this type of bending *plane bending*. Here, all vertical loads must be applied within the symmetric plane, otherwise the beam might bend in two directions, or torsion may happen. You can see axis of symmetry for various shapes in figure 2.

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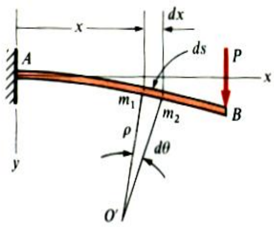
Radius of Curvature



A line normal to the tangent to the deflection curve was drawn at each of the points m_1 and m_2. These normals intersect at point O' (centre of curvature). The length of a normal is called the *radius of curvature* ρ.

After deformation, the beam deforms into a curved line. An important parameter here is the curvature. We can draw two perpendicular lines to the tangent lines of two points m1 and m2 of the deformed shape, and at the intersection of those perpendicular lines is O dash, which will be the centre of curvature, as shown in the figure. The length of this line is denoted by rho. (that is, the length from the centre of curvature O dash to the deformed point m1 or m2, which is called the radius of curvature.

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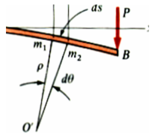
Curvature

The curvature κ is equal to: $\kappa = \dfrac{1}{\rho}$ → (1)



Also, from the geometry of the figure:

$\rho d\theta = ds$ → (2)

If the deflections are small, assume ds=dx:

$\kappa = \dfrac{1}{\rho} = \dfrac{d\theta}{ds} = \dfrac{d\theta}{dx}$ → (3)

Will be used to obtain the strains in a bent beam

The relationship between curvature, which we denote as kappa and rho, is as follows: kappa is equal to 1 divided by rho, as shown in the equation 1. From the geometry of the figure, we can see that the radius 'rho' and the centre of the curve is O dash. The distance between m1 and m2 is 'ds' which is equal to rho times 'd theta', as shown in the equation 2.

Therefore, if kappa is 1 divided by rho as per equation 1, then kappa equals dtheta divided by ds, as per equation 2 and as shown in equation 3. As ds is approximated equal to horizontal distance dx, kappa equals 'd theta' divided by dx. This is the geometric relationship for curvature.

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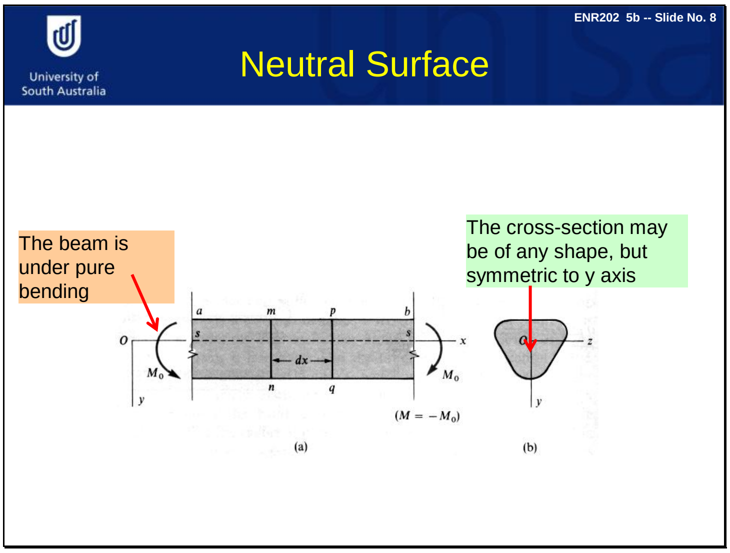
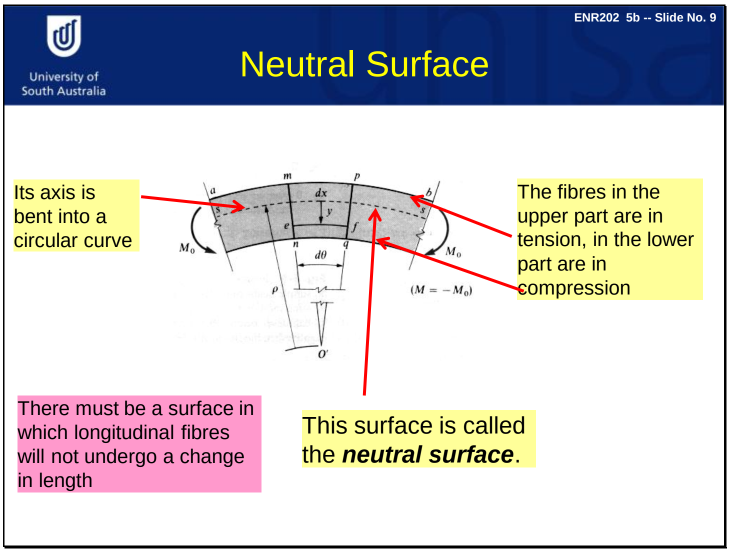


As shown in the figure, before the deformation we have a straight beam. The cross sections m-n and p-q located at distance 'dx' are also straight lines, as shown in figure a. Here, we consider the symmetry cross section about the y-y axis, as shown in figure b. We applied pure bending with negative bending moment 'Mo'.

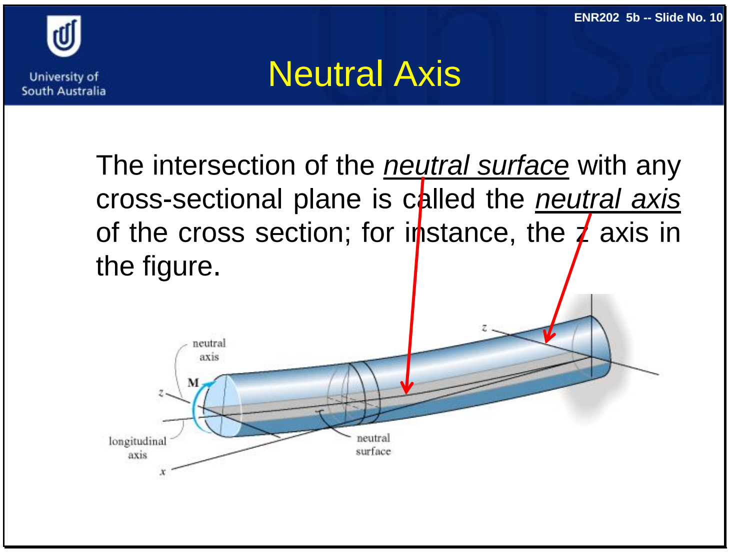
Slide 9



Once you apply bending moment, the beam deforms into a curve, as shown in the figure. We assumed the plane cross sections remain plane before bending and after bending. That means that the cross sections m-n and p-q lines are straight before bending and after bending. Here we consider the symmetry cross section, with the axis y-y bent into a circular curve, as shown in the figure, and we can say that this bending causes the top part of the beam to be in tension and the bottom part of the beam to be in compression, because we applied negative bending moment. Real bending moment should be negative M0. Because the top fibre of the beam is in tension and the bottom fibre of the beam is in compression, somewhere in between the top fibre and the bottom fibre will not change. Assume that there is no compression and no tension at the s-s surface. That means that there is no normal strain in this surface. We have tension in the top surface and compression in the bottom surface and there is no change at the s-s. S-s is called a neutral surface.

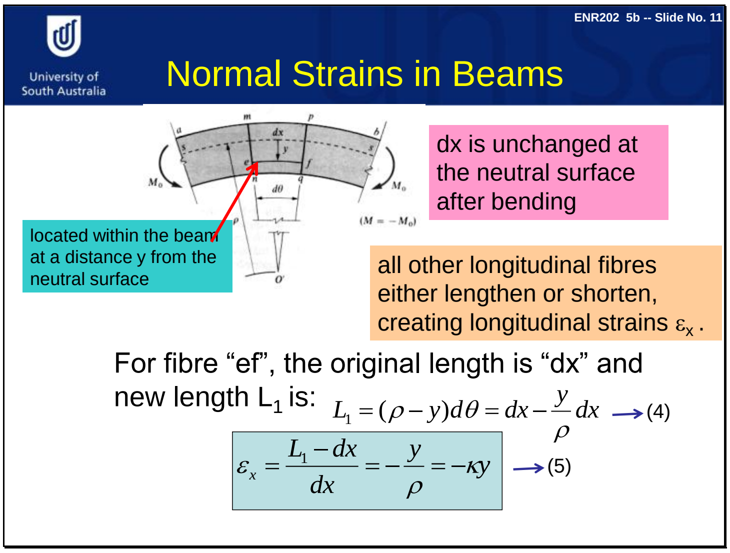
In other words, if tension is considered as positive and compressive as negative, that means that there must be somewhere with zero normal strain. That zero surface is called the neutral surface.

Slide 10



The intersection of the neutral surface with any cross section plane is called the neutral axis. For example, we have an s-s neutral surface, and we have this cross section in the beam. The z-line is a neutral axis as shown in the figure. In a neutral axis, we don't have any longitudinal deformation, we don't have any normal or normal stress.

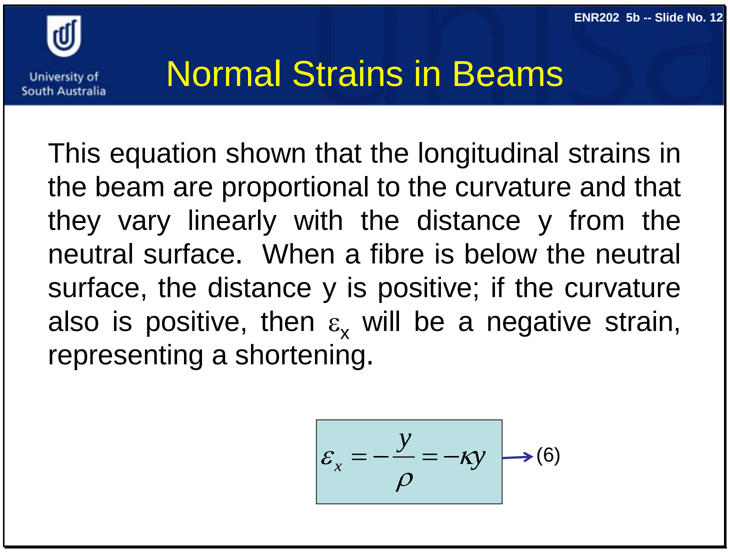
Slide 11



If we have normal strain in a beam after deformation, the beam changes into a curve. The O dash is the centre of the circle, as shown in the figure. Rho is the radius of the circular curve. We can say that for e-f, the original length is dx (that is, the original length before bending), and the deformed length (that is, the length after bending) is equal to rho minus y. That is the distance between the neutral axis and the e-f line multiplied by 'd theta'.

So, the original length of e-f line is 'dx' and the deformed length is 'L1' which is equal to rho minus 'y' times 'd theta'. If we substitute 'd theta' equal to 'dx' over rho, we will get equation 4. Now think back to the definition of strain, which was explained in lecture 1b slide 10. Strain is the change in length divided by the original length. The change in length of the e-f line is L1 minus 'dx' and the original length is 'dx'. Substitute the equation 4 into equation 5, based on the strain definition, and we will get that the strain is equal to minus y divided by rho. Considering the curvature is equal to one divided by rho, the strain is equal to minus curvature 'kappa' times 'y', as shown in equation 5. That means the distribution of strain along the height of the beam is a linear function.

Slide 12



Equation 6 shows that the longitudinal strains in the beam are proportional to the curvature and that they vary linearly with the distance y from the neutral surface. When a fibre is below the neutral surface, the distance y is positive; if the curvature also is positive, then strain will be a negative strain, representing a shortening. Normal strain in linear function means normal stress is also a linear function, based on Hookes law.

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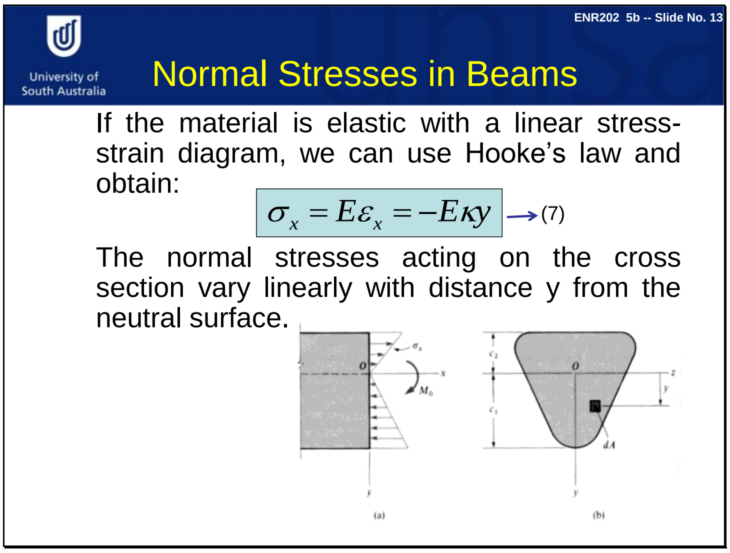

Now think back to Hooke's law, which was covered in lecture summary 2a, slide number 15. Normal stress is equal to E times normal strain, as shown in equation 7. Substitute equation 6 into equation 7, and you will get that the normal stress equals negative E times kappa times y. The distribution of normal stress is a straight line, the top part of the beam in tension, and the bottom part of the beam is in compression, as shown in figure a. This is very important. Make sure that you understand that the normal stress distribution along the height is linear. Maximum tensile stress occurs at the topmost fibre and maximum compressive stress occurs at the bottom-most fibre. In between, we have zero stress at neutral axis and this is the normal stress distribution in the beams due to bending.

Slide 14

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The Position of the Neutral Axis??

The *resultant force* produced by the stress distribution over the cross-sectional area with respect to the neutral axis must be equal to zero.

$$\int \sigma_x dA = -\int E\kappa y dA = 0 \quad \rightarrow (8)$$

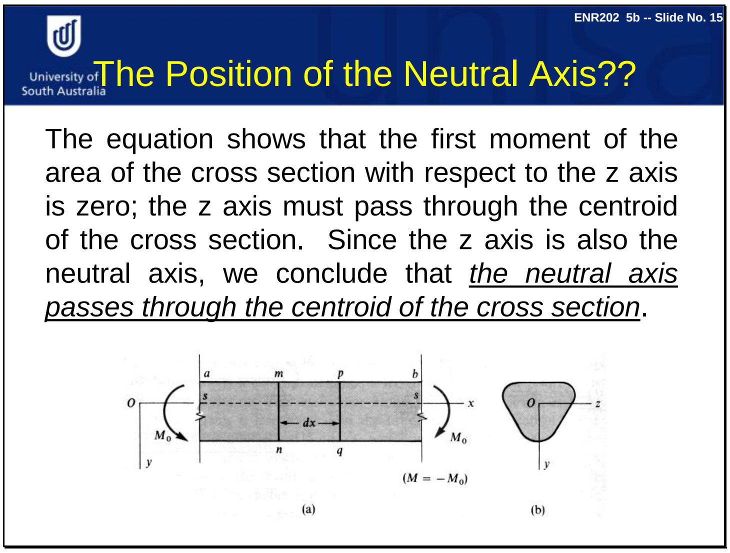
Because κ and E are constants at the cross section, we conclude that:

$$\int y dA = 0 \quad \rightarrow (9)$$

We can determine the position of neutral axis through analysing the normal force on the beam section.

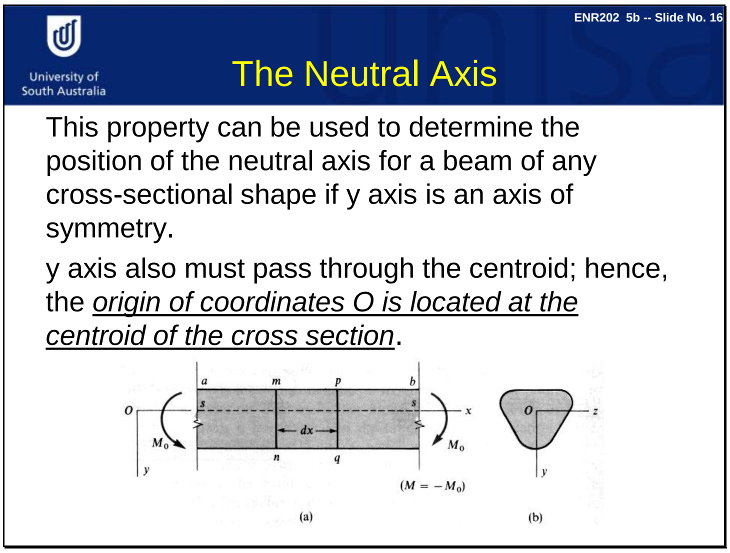
Resultant force or normal force acting on the cross section of the beam will be zero, because there is no normal force on a pure bending beam. Normal force being zero means that the resultant of normal stress all together equals zero. The integration of normal stress along the cross sectional area of the beam is zero, because normal stress is equal to negative E times kappa times y, as shown in the equation 8. Here, E and kappa are constants, so we have integration of y times dA equal to zero, as shown in the equation 9. That means the z-axis is neutral just past the centroid of the cross section (as per the definition of centroid given in lecture summary 5a, slide number 3.)

Slide 15



Equation 9 shows that the first moment of the area of the cross section with respect to the z axis is zero; the z axis must pass through the centroid of the cross section (that is, point O) as shown in figure b. Since the z axis is also the neutral axis, we conclude that the neutral axis passes through the centroid of the cross section, as shown in figure b.

Slide 16



This property can be used to determine the position of the neutral axis for a beam of any cross-sectional shape if the y axis is an axis of symmetry, as shown in figure b. The y axis also must pass through the centroid point. Therefore, the origin of coordinates O is located at the centroid of the cross section.

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A more practical formula?

$$\sigma_x = E\varepsilon_x = -E\kappa y \longrightarrow (10)$$

Can't be used as don't know κ.

Now consider the moment resultant of the stresses σ_x acting over the cross section. Its moment about the z axis is:

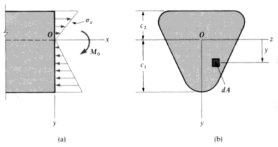
$$dM_0 = -\sigma_x y dA \longrightarrow (11)$$

$$M_0 = -\int \sigma_x y dA \longrightarrow (12)$$

Noting that $M = -M_0$

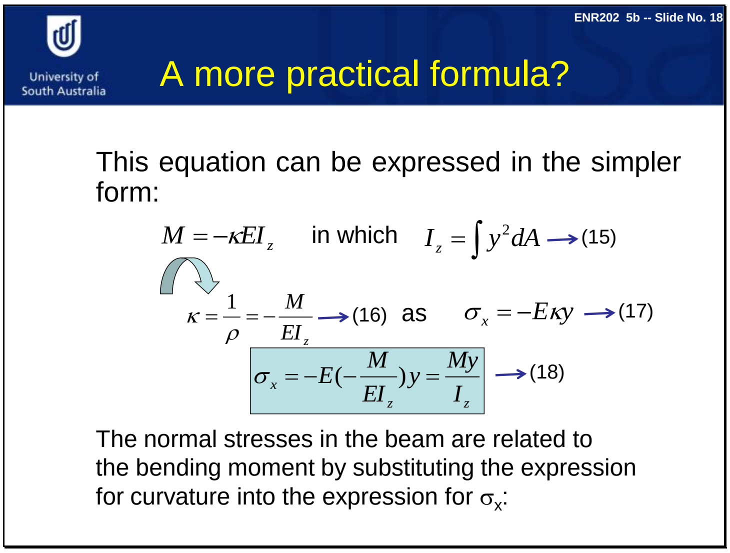
$$M = \int \sigma_x y dA = -\kappa E \int y^2 dA \longrightarrow (13)$$

$$M = -\kappa E I_z \longrightarrow (14)$$



The formula for normal stress is negative E times kappa times y, as shown in equation 10. We can have y if the neutral axis is determined and E if the material is known. However, we don't know kappa, so we have to calculate kappa first. The normal stress distribution is a result of all normal stresses. That means that the resultant moment about the neutral line is equal to total moment. Take the moment of any normal stress about the z-axis, dMo equals negative 'sigma x' times y times dA, as shown in equation 11. Mo equals the integration of negative 'sigma x' times y times dA, as shown in equation 12. Substituting equation 10 into equation 12, we can have that M equals negative kappa times E times integration y square dA, as shown in equation 13. This relationship depends only on the integration formula (the shape of the cross section). We know that the integration y square dA is equal to the second moment of inertia about the centroid axis z (if you are not sure about this, look back at lecture summary 5a, slide number 10). Finally, we will get the bending moment M in terms of kappa, E, and Iz, as shown in equation 14.

Slide 18



We already know the moment acting on the cross section. M equals negative kappa times E times Iz, as shown in equation 14. The Iz value is very important. Iz is equal to the integration y squared dA, as shown in equation 15. Kappa is equal to 1 divided by rho, which is equal to M divided by EIz, as shown in equation 16. We already know that 'sigma x' is equal to minus E times kappa times y, as shown in equation 17. Substituting equation 16 into equation 17, we will get equation 18 (that is, sigma x is equal to M times y divided by Iz). The maximum bending stress happens at maximum y, because M and Iz are constants about the cross section.

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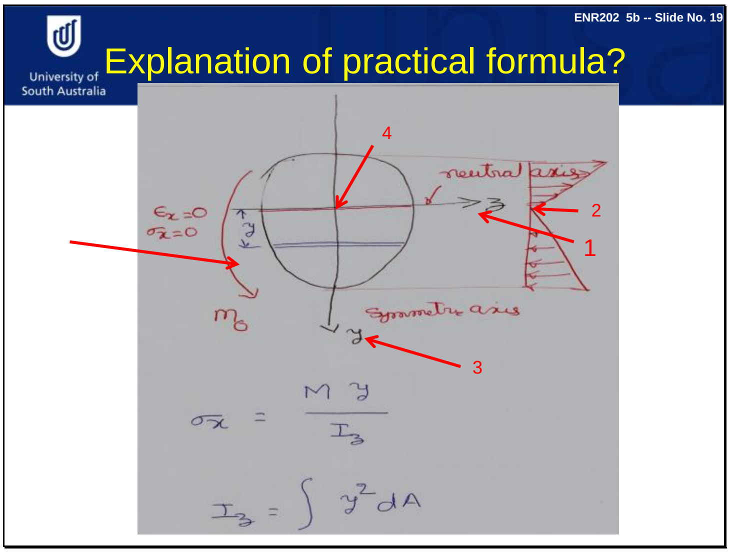
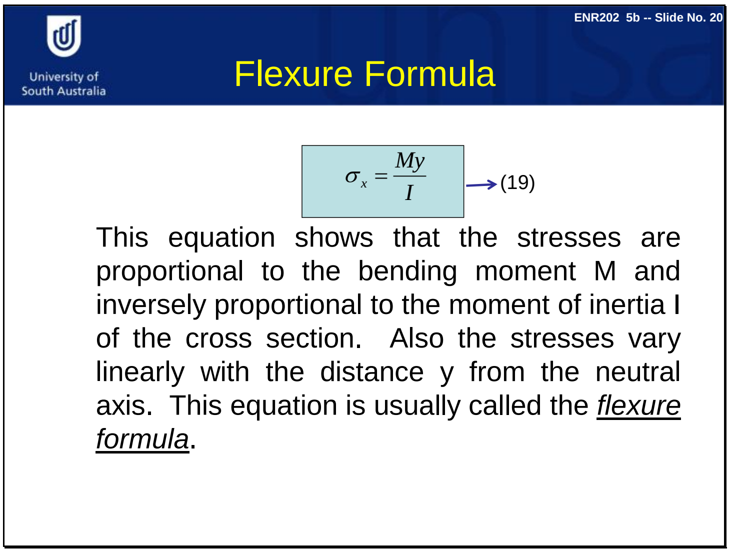


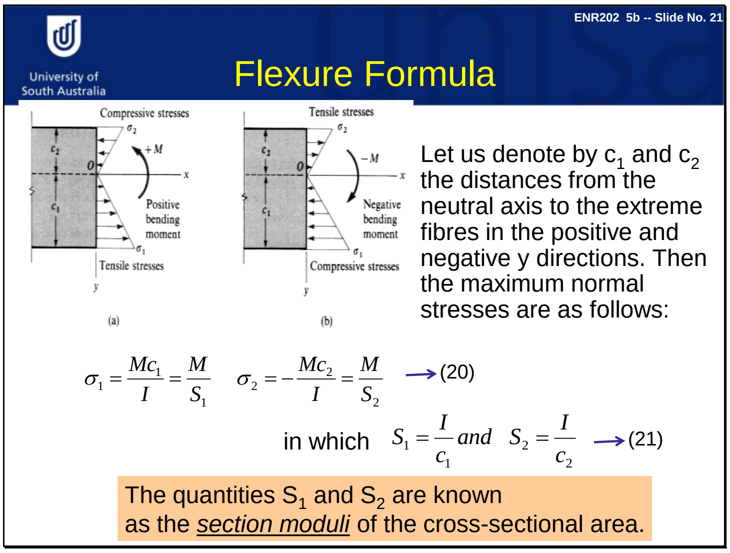
Coming back to the neutral axis, if we have a cross section as shown in the figure. The z-axis is the neutral axis, because for the points on the z-axis line as shown in arrow 1, we have zero normal stress as shown in arrow 2 or zero normal strain, so this z-axis line is called the neutral axis. The y-axis is the symmetry line of this cross section as shown in the arrow 3. The intersection of the y-axis and z-axis is the centroid of this cross section as shown in arrow 4. If we have bending moment acting on the cross section, top part of the beam in tension and bottom part of the beam in compression, the distribution of stress will be a linear variation along the cross section as shown in the normal stress distribution diagram. we have maximum tension at the topmost surface of the cross section, maximum compression at bottommost surface of the cross section. This is the distribution of normal stress on the cross section.

Slide 20



Equation 19 is the flexure formula. This equation shows that the stresses are proportional to the bending moment M and inversely proportional to the moment of inertia I of the cross section. Also, the stresses vary linearly with the distance y from the neutral axis.

Slide 21



Suppose that C1 is the distance between the bottom fibre to the neutral axis, and C2 is the distance between the top fibre to the neutral axis (as shown in figures a and b). If we have positive moment as shown in the figure a, the top is in compression and the bottom is in tension. For maximum tension at the bottom, sigma1 is equal to M times C1 divided by I, as shown in equation 20. If S1 equals I divided by C1, as shown in equation 21, then sigma1 is equal to M divided by S1. For maximum compression at the top, sigma2 is equal to M times C2 divided by I, as shown in equation 20. If we consider S2 as equal to I divided by C2, as shown in equation 21, then sigma2 is equal to M divided by S2. S1 and S2 are known as section moduli of the cross section.

If negative bending moment is acting on the beam, the top part of the beam is in tension and the bottom part of the beam is in compression, as shown in figure b.

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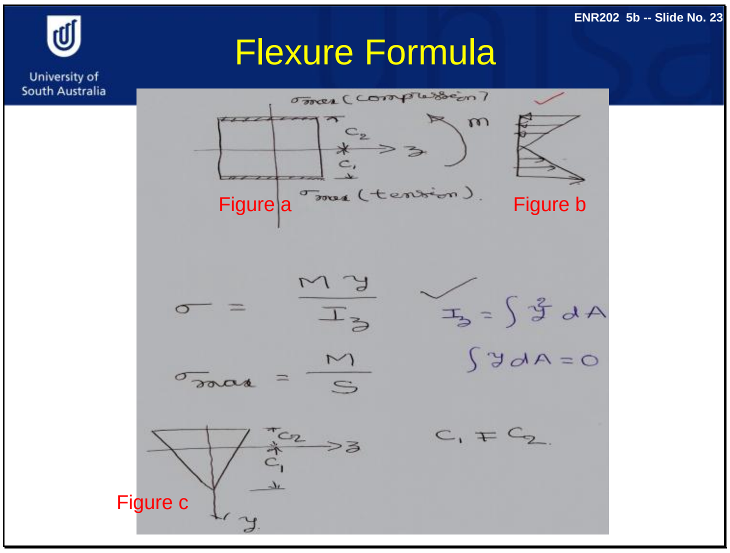
Flexure Formula

If the cross section is symmetric with respect to the z axis, then $c_1=c_2=c$, and the maximum tensile and compressive stresses are equal numerically:

$$\sigma_1 = -\sigma_2 = -\frac{Mc}{I} = \frac{M}{S} \quad \rightarrow (22)$$

If we have a double symmetric cross section with C1 equal to C2, we can consider that S1 equals S2. Finally, we have sigma1 equals sigma2, as shown in equation 22.

Slide 23

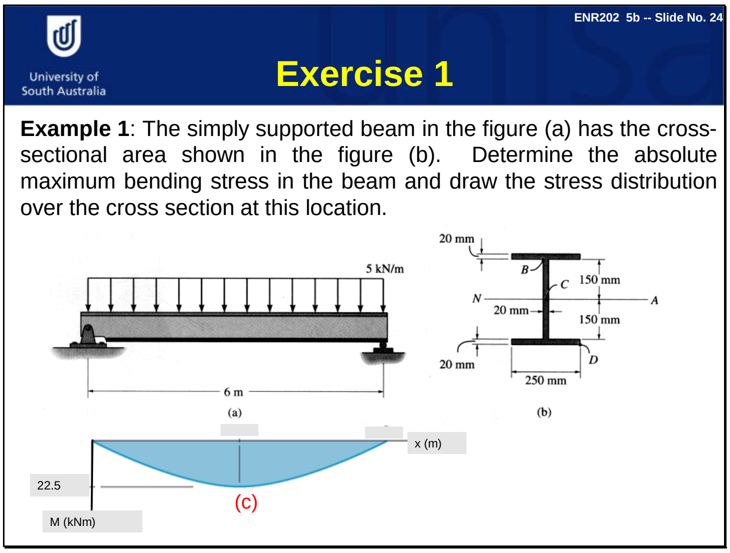


For maximum compression on the top and maximum tension on the bottom, you have to remember some of the formulas.

Firstly, sigma is equal to M times y divided by Iz. Don't use the formula that sigma is equal to M divided by S, because in some cross sections, C1 is not equal to C2. You have to calculate them separately, and there is a greater chance of making a mistake. In figure b, we have zero normal stress at the z-axis, maximum compression stress on the top, and maximum tension on the bottom. In this formula, you have to know that Iz is equal to the integration of y square times dA.

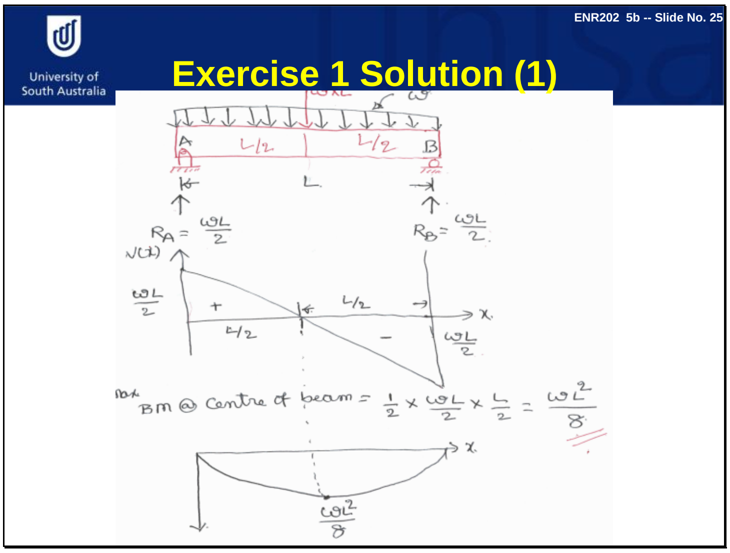
Now we will study how to calculate this integration. This depends only on the cross section. It doesn't have any relationship with moment and the length of the beam. There are several steps for the calculation of Iz. First, the Integration of y times dA equals 0. That means that the co-ordinating axis of z and y-axis pass through the centroid of the cross section, as shown in the figure c. We need firstly calculate the centroid of the cross section and then Iz. Based on Iz, we can then calculate normal stress by using flexure formula.

Slide 24



In this example, we have a simply supported beam subjected to a uniformly distributed load of 5 kilo Newtons per meter over the total length of the beam, which is 6 meters, as shown in figure a. This beam has an I shaped cross section, as shown in figure b. You need to determine the absolute maximum bending stress in the beam, and draw the stress distribution over the cross section where the maximum bending moment occurs.   Pause the presentation and try to do this now. The answer is on the next five slides.

Slide 25



First step, you need to draw the shear force diagram and bending moment diagram. (Go back to lecture summary 4a, slide 17 if you need revision in this). The maximum bending moment occurs where the shear force is zero, which is at the center of the beam. Therefore, the bending moment will be maximum at the middle of the beam, and is equal to the area of shear force from the support to the middle of the beam, which is equal to ‘w’ times L squared divided by 8, as shown in the bending moment diagram.

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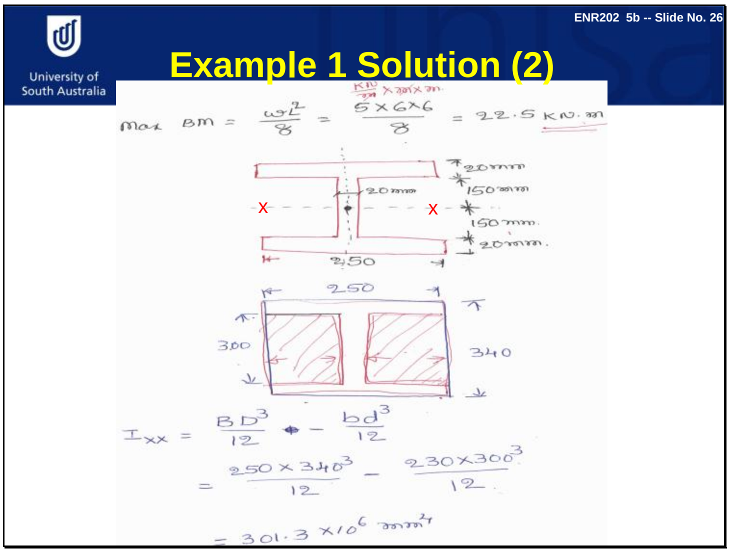
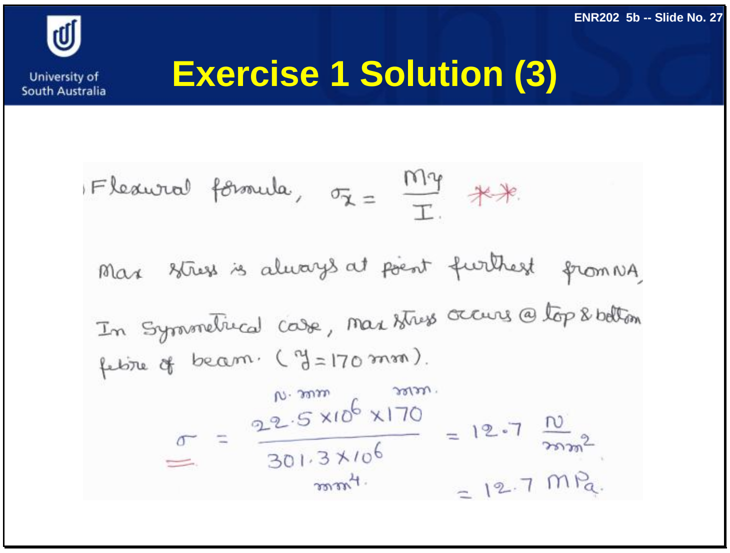


The maximum bending moment is equal to a uniform distributed load of 5 kilo Newton per meter times 6 meters times 6 meters divided by 8, which is equal to 22.5 kilo Newton Meters.

Now, you need to calculate the moment of inertia about the x-x axis for the I cross section. The I-shaped cross section is a double symmetric cross section, as shown in the figure. The flanges are 250 millimetres wide and 20 millimeters thick. The web is 300 millimeters high and 20 millimeters thick. Because of double symmetry, the centroid will be at the center of height of the beam. You can use the negative area method to calculate the moment of inertia about the x-x axis.

The moment of inertia about the x-x axis of the big rectangle (250 millimeters wide and 340 millimeters deep) is 250 millimeters times 340 millimeters cubed divided by 12. The two small rectangles (shaded in the figure) are 115 millimeters wide and 300 millimeters deep. The moment of inertia about the x-x axis of the is 230 millimeters times 300 millimeters cubed divided by 12. The moment of inertia of the I cross section is equal to the moment of inertia about the x-x axis of the big rectangle minus the moment of inertia about the x-x axis of the two small rectangles, which is equal to 301.3 times 10 to the power of 6 millimeters to the power of 4.

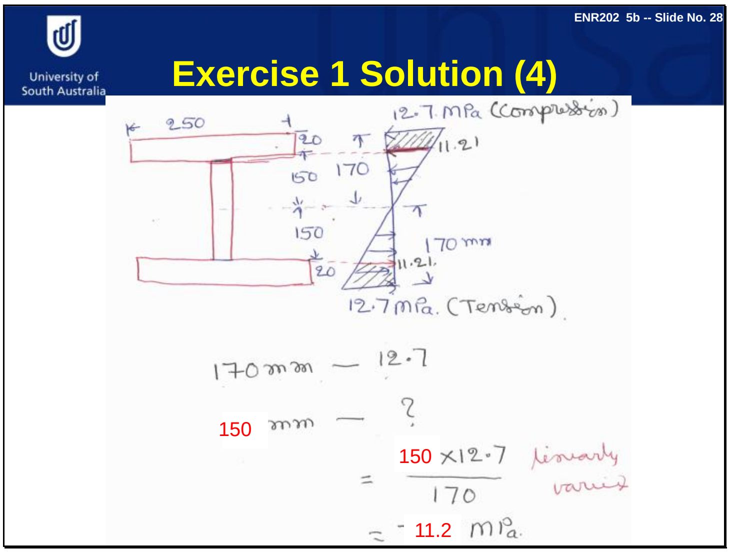
Slide 27



Use the flexure formula to calculate normal stress, which is equal to a bending moment of M times y divided by I.

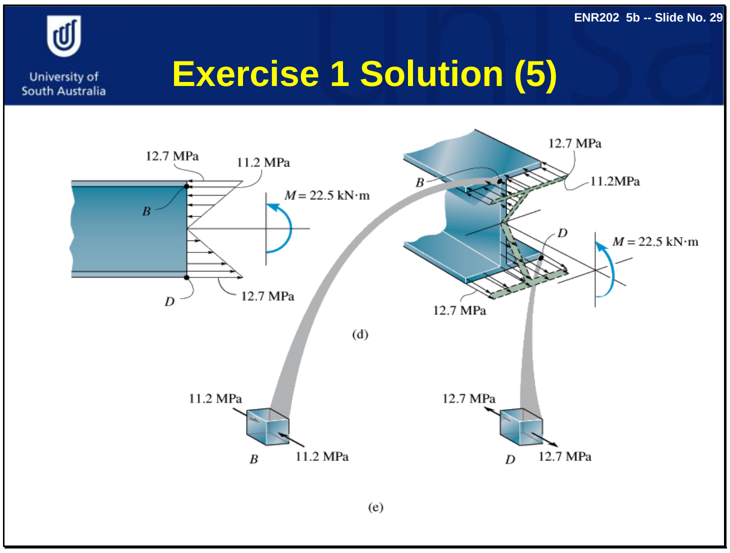
That means you will get maximum normal stress where the maximum bending moment occurs. The maximum bending moment occurs at the middle of the beam and is equal to 22.5 times 10 to the power of 6 Newton millimeters. The moment of inertia about the x-x axis is equal to 301.3 times 10 to the power of 6 millimeters to the power of 4. y means that the distance between the neutral axis or the centroid axis to the point where you need to calculate stress (that is, to the topmost or bottommost part of the beam) is equal to 340 millimeters divided by 2 which is 170 millimeters. Substitute M, y and I values into the flexure formula to calculate the maximum normal stress. You should get that the maximum normal stress is equal to 12.7 Mega Pascals or Newton per square millimeters.

Slide 28



The normal stress distribution on the I shape cross section at the middle of the beam is shown in the figure. The compressive normal stress occurs at the top part of the beam and the tensile normal stress occurs at the bottom part of the beam. The maximum compressive and tensile normal stresses equal 12. 7 Mega Pascals, as we calculated in the previous slide.   Calculate the stress at the junction of the web and the flange. This stress distribution is the linearly variation. Therefore, the normal stress at a distance of 170 millimeters from the centroid axis is equal to 12. 7 Mega Pascals and the normal stress at a distance of 100 millimeters from the centroid axis is equal to 100 millimeters times 12.7 Mega Pascal divided by 170 millimeters, which is equal to 11.2 Mega Pascals.

Slide 29



You can see the stress distribution for the I shape cross section in three dimensional view here, in figure d. The compressive normal stress cube at point B (the junction of the web and the flange) is equal to 11.2 Mega Pascals, as shown in figure e. The tensile normal stress cube at point D (the bottom most point) is equal to 12.7 Mega Pascals, as shown in figure e.

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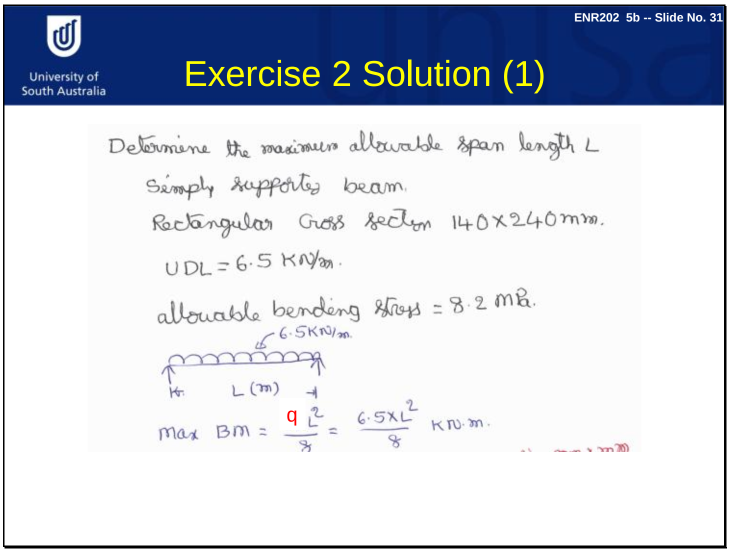
Exercise 2

Determine the maximum allowable span length L for a simple beam of rectangular cross section (140mm x 240mm) subjected to a uniformly distributed load q=6.5kN/m if the allowable bending stress is 8.2MPa.(The weight of the beam is included in the load q).

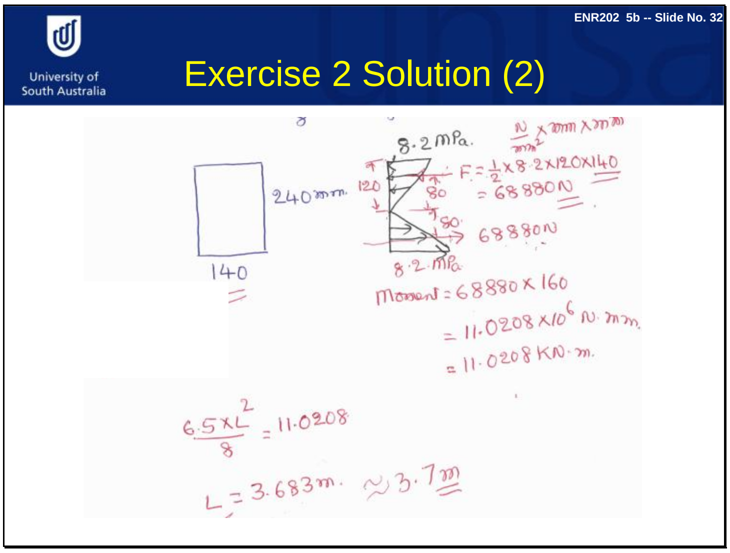
Now try this exercise. A simply supported beam with a rectangular cross section of 140 millimeters width and 240 millimeters depth is subjected to a uniformly distributed load ‘q’ of 6.5 kilo Newtons per meter. You need to calculate the maximum allowable span length ‘L’ if the allowable bending stress is 8.2 Mega Pascals. Assume that the weight of the beam is included in the uniform distributed load ‘q’. The solution is on the next two slides.

Slide 31



In exercise 1, you calculated normal stress based on the length of the beam. However, in this exercise, you need to calculate the maximum allowable span length 'L', based on allowable normal stress (that is, 8.2 Mega Pascals). This problem is a design problem, based on a simply supported beam subjected to a UDL of 6.5 kilo Newtons per meter. The beam has a rectangular cross-section 140 millimeters wide and 240 millimeters deep. You know that the maximum bending moment occurs at the middle of the simply supported beam. If the UDL acting on the total length of the beam is q times L squared divided by 8, that is 6.5 times L squared divided by 8 Kilo Newton Meters. (We are assuming the length of the beam is in meters.)

Slide 32



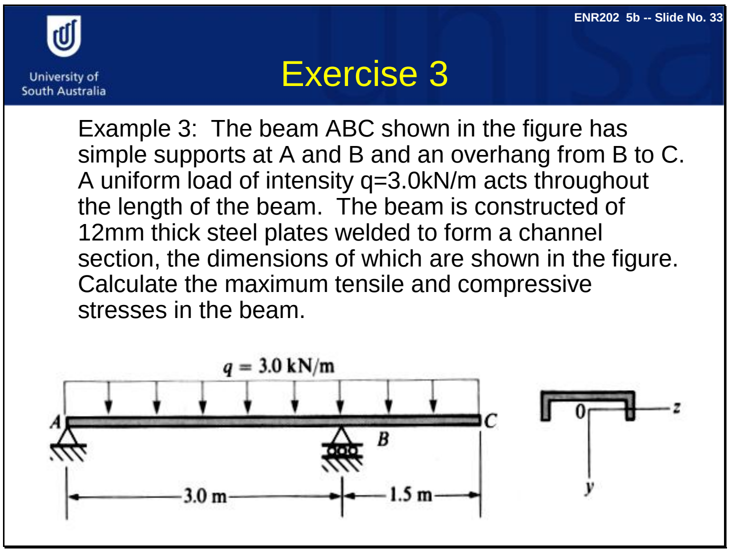
In positive bending moment, we have the top part of the beam in compression and the bottom part of the beam in tension, as shown in the normal stress distribution diagram. We have maximum compressive and tensile normal stress 8.2 Mega Pascals, as shown in the figure.
First, we calculate the compressive normal force 'F' at the top part of the beam. The top part of the beam has triangular stress distribution. The area of the triangle times the width of the beam gives the compressive normal force.
F is equal to 8.2 Mega Pascal or Newtons per millimeters squared times 120 millimeters times 140 millimeters (the width of the beam) which is equal to 68 880 Newtons. We have the same triangular distribution in the bottom part of the beam as we do in the top of the beam. Therefore, the tensile normal force equals 68 880 Newtons.

The compressive normal force at the top part of the beam and the force of the same magnitude but in the opposite direction at the bottom part of the beam is a couple. These force couples act at a distance of 160 millimeters. How did we get 160 millimeters? The depth of the beam is 240 millimeters , which means we have 120 millimeters from the centroid axis. We have triangle stress distribution, the centroid of triangle from the centroid of the beam is two thirds times 120, which is 80 millimeters. Finally, 80 millimeters plus 80 millimeters is 160 millimeters – this is the distance between the force couples.

Force couples produce moment. The moment equals 68 880 newtons times the distance between the couples, which is 160 millimeters, which equals 11.0208 times 10 to the power of 6 Newton mille meters, which is 11.0208 kilo Newton meters. This value is equal to 6.5 times L squared divided by 8. Finally, you will calculate the allowable span length of the beam, which is equal to 3.683 meters. You can round this value to next digit; that is, 3.7 meters.

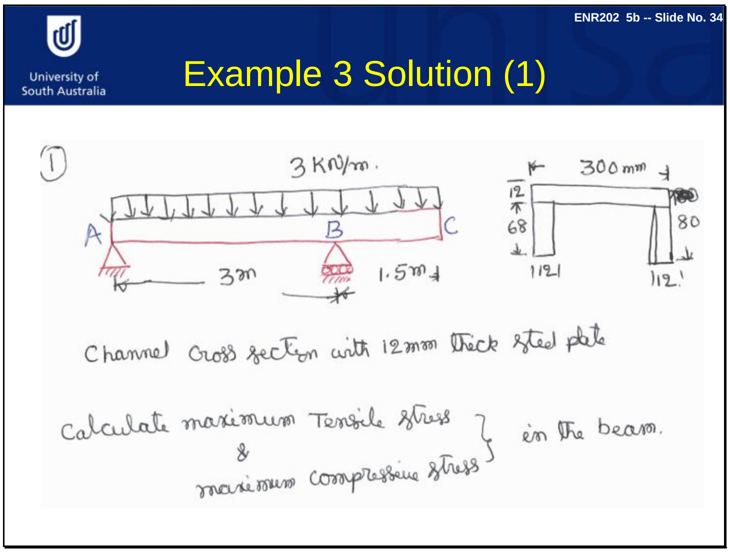
ENR202 Mechanics of Materials Lecture 5B Slides and Notes

Slide 33



Now try exercise 3. The beam ABC shown in the figure has simple supports at A and B and an overhang from B to C. A uniform load of intensity q=3.0kN/m acts throughout the length of the beam. The beam is constructed of 12mm thick steel plates welded to form a channel section, the dimensions of which are shown in the figure. Calculate the maximum tensile and compressive stresses in the beam. The length of the beam from A to B is 3 meters and from B to C is 1.5 meters. Pause this presentation and calculate the maximum tensile stress and maximum compressive stresses in the beam. The solution is on the next 16 slides.

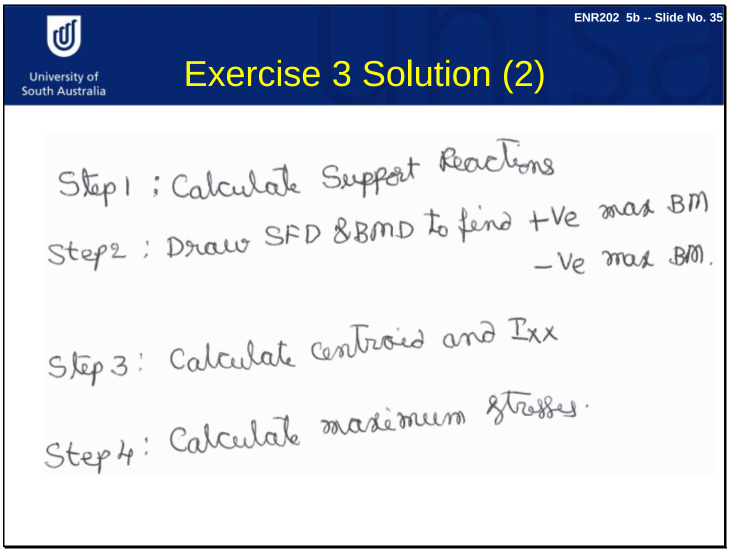
Slide 34



This overhang beam has a channel cross section of 300 millimeters by 12 millimeters, 68 millimeters by 12 millimeters, and another 68 millimeters by 12 millimeters, as shown in the figure.

The overhang beam ABC has a simply supported beam between point A and point B, and an overhang portion from B to C. A uniform distributed load of 3 kilo Newtons per meter is acting on the total length of the beam – this is ‘q’. This overhang beam has 12 millimeter thick steel plates welded to form a channel cross section of the beam. The length of the beam from A to B is 3 meters and from B to C is 1.5 meters, as shown in the figure. You need to calculate the maximum tensile stress and maximum compressive stresses in the beam.

Slide 35

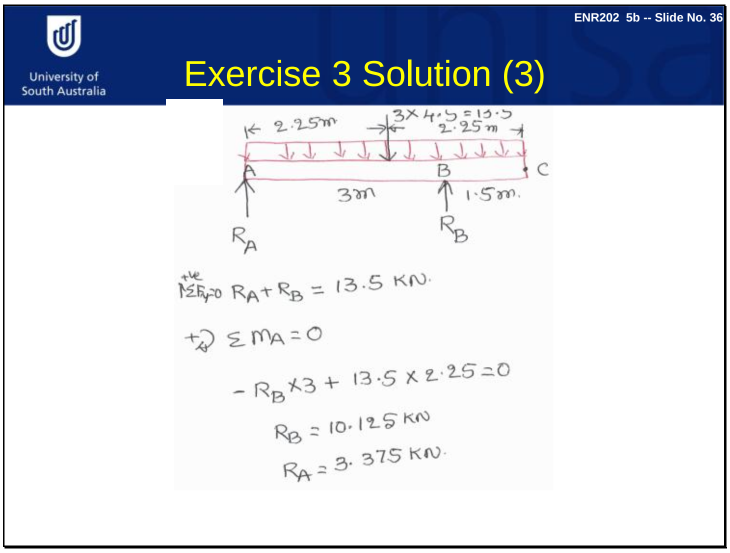


First, you need to calculate the support reactions.

Next, you need to draw the Shear force diagram and bending moment diagram, to find the maximum positive bending moment and maximum negative bending moment of the overhang beam. As you know, in positive bending moment, you will get maximum compressive normal stress at the top part of the beam and maximum tensile normal stress at the bottom part of the beam. In negative bending moment, you will get maximum compressive normal stress at the bottom part of the beam and maximum tensile normal stress at the top part of the beam.
The next step is to calculate the centroid and moment of inertia about the x-x axis (that is, the horizontal axis of the channel cross section). We covered this in lecture 5a. This channel section has symmetry about the vertical axis, but not about the horizontal axis. Therefore, you need to calculate the centroid using the centroid formula.

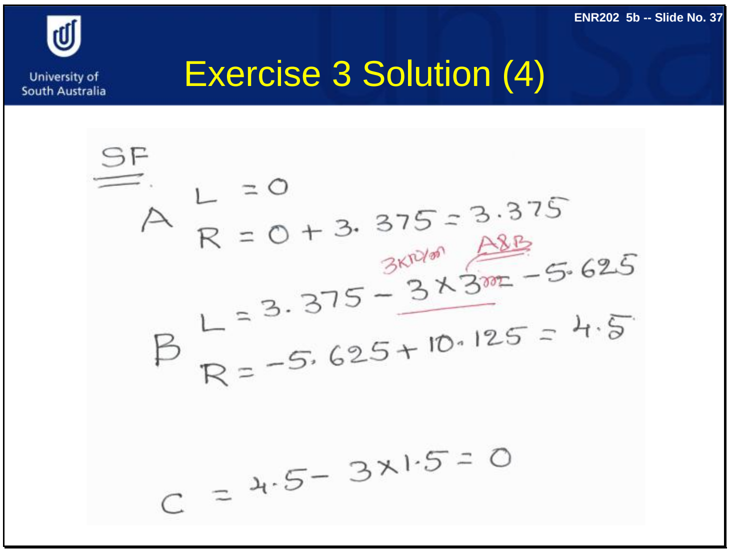
Next, you need to calculate the maximum normal stresses, using the flexure formula.

Slide 36



We have covered how to calculate reaction forces for an overhang beam (look back at lecture 4b, slide 27 if you need to). You have to use the equilibrium equation to calculate reaction forces. The first equilibrium equation is that all forces in a vertical direction are equal to zero. A second equilibrium equation is that all forces moment about point A equal zero. The answer is that the reaction force at support point A is 3.375 kilo Newtons and at support point B is 10.125 kilo Newton.

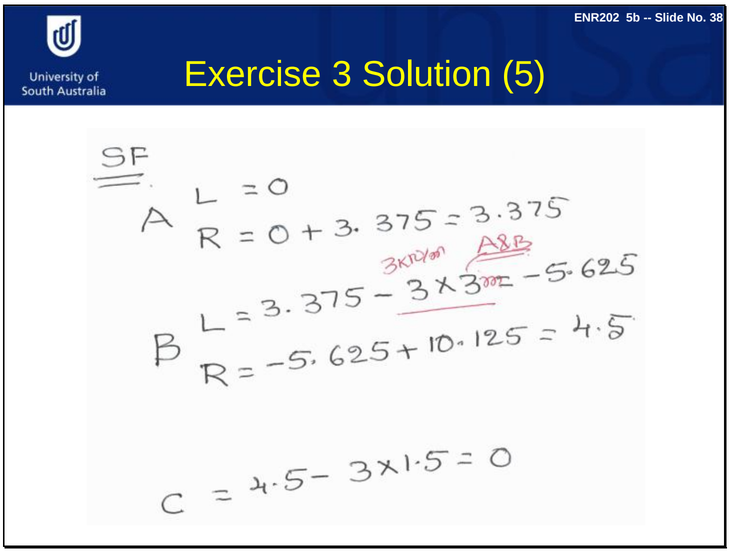
Slide 37



You now need to calculate shear force values at important points (check lecture 4b, slide 31 if you need to). We have three important points. The first is at the beginning of the UDL. The second is a supporting point, B. The third is another supporting point, C, which is at the end of the uniform distributed load.

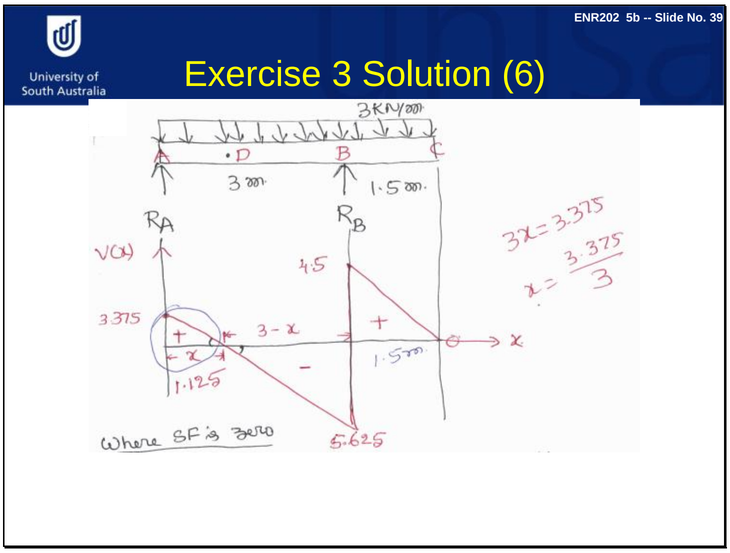
Consider just to the left of Point A. At *Aleft* we have zero shear force, because shear force starts at zero. Then we come to *Aright.* Between *Aleft* and *Aright*, we have a concentrated reaction force, RA, which is equal to 3.375 Kilo Newtons in an upward direction. We know that upward is positive. Finally, we work out that *Aright* equals 3.375 Kilo Newtons.

Slide 38



When we come to *Bleft*, we have a vertical uniformly distributed load of 3 kilo Newtons per meter between *Aright* and *Bleft*. So *Bleft* equals 3.375 minus 3 kilo newtons per meter times the distance between A and B, which equals minus 5.625 kilo Newtons. Now move on to *Bright*. We have a concentrated reaction force of 10.125 kilo Newtons at B. *Bright* equals minus 5.625 kilo Newtons minus 10.125 kilo Newtons, which is positive 4.5 Kilo Newtons. Now move on to last important point, C. We have a vertical uniformly distributed load of 3 kilo Newtons per meter between *Bright* and *Cleft*. So, the shear force at *C* equals 4.5 minus 3 kilo newtons per meter, times the distance between B and C, which is 1.5 meters. This – the shear force – is equal to zero, so we can see that the values are correct.

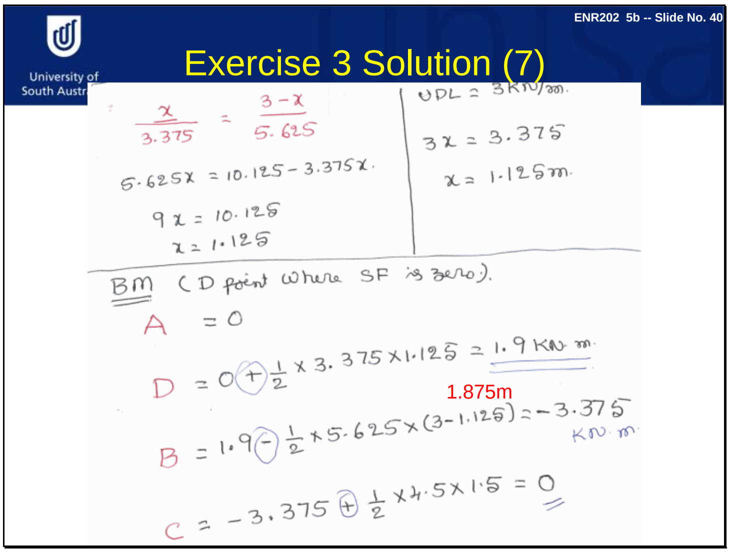
Slide 39



Connect all the values at important points, and we get shear force diagram as shown in the figure. We recommend you that draw shear force diagram on your own and check it against this figure.

Now, you need find a point between point A and B where the shear force is zero. The shear force at A is 3.375 kilo Newtons. Suppose you assume 'x' is the distance between points A and D. We have a uniformly distributed load of 3 kilo Newtons per meter. This means that the UDL times the distance between A and D, which we will call 'x', must equal 3.375 kilo Newtons. You should work out that 'x' is 1.125 meters. As you know, where the shear force zero, there you will get maximum bending moment. However, you need to draw bending moment diagram.

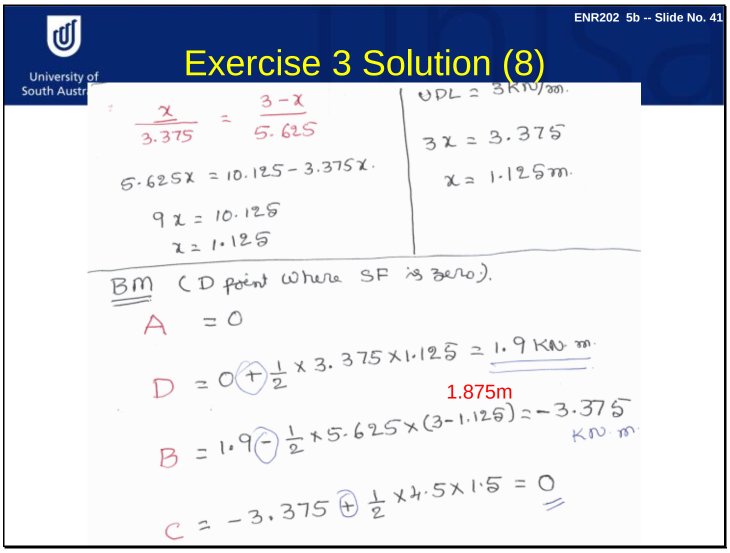
Slide 40



You can also calculate the distance ‘x’; that is, the distance between points A and D where the shear force is zero, based on a similar triangle as shown in the calculations. Once we know the shear force diagram, we can draw the bending moment diagram based on shear force diagram. Start with the left end and then go back to the right end. We have 3 important points, A,B and C.

We already know that there is no bending moment at the hinged support. Therefore, the bending moment at A is equal to zero. The bending moment at D is equal to the bending moment at A plus the area under the shear force diagram between points A and D, because the Shear Force Diagram is positive. The shear force area between A and D is 3.375 kilo Newtons times 1.125 meters divided by 2, which is 1.9 kilo Newton meters. This shear force diagram is the positive sign between A and B. Finally, we can calculate the bending moment at D as 1.9 kilo Newton meters.

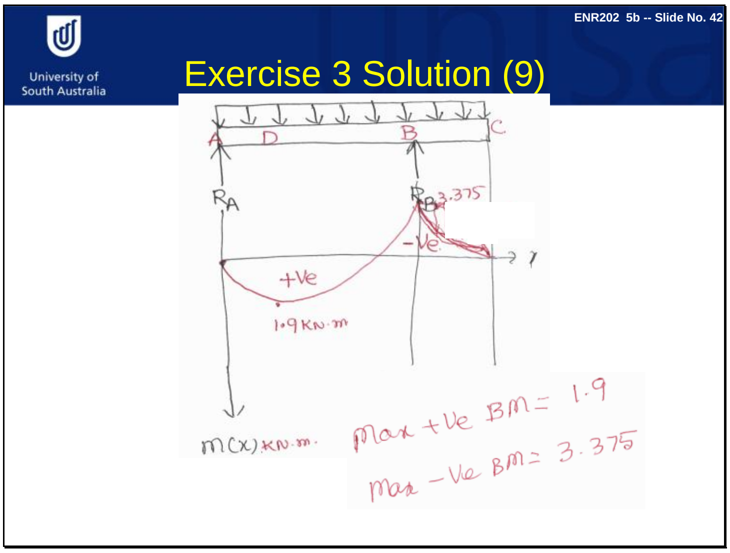
Slide 41



Then we move on to point B. The bending moment at B is equal to the bending moment at D minus the area under the shear force diagram between points D and B, because the Shear Force Diagram is negative. The shear force area between D and B is 5.625 kilo Newtons times (3 minus 1.125 meters or 1.875 meters) divided by 2, which equals 5.273 kilo Newton meters. This shear force diagram is a negative sign between D and B. We can calculate the bending moment at B to be equal to 1.9 kilo Newton meters at point D minus 5.273 kilo Newton meters (the area of the Shear Force Diagram), which equals minus 3.375kilo Newton meters.

Then we move on the point C. The bending moment at C is equal to the bending moment at B plus the area under the shear force diagram between points B and C, because the Shear Force Diagram is positive. The shear force area between B and C is 4.5 kilo Newtons times 1.5 meters divided by 2, which is 3.375 kilo Newton meters. This shear force diagram is a positive sign between B and C. We calculate the bending moment at C as being equal to minus 3.375 kilo Newton meters at point B plus 3.375 kilo Newton meters (which is the area of SFD between B and C). This is all equal to zero. Therefore, the bending moment at C is equal to zero. The bending moment ends with zero and the bending moment at C (the final point) is also zero. That means our calculations are correct.

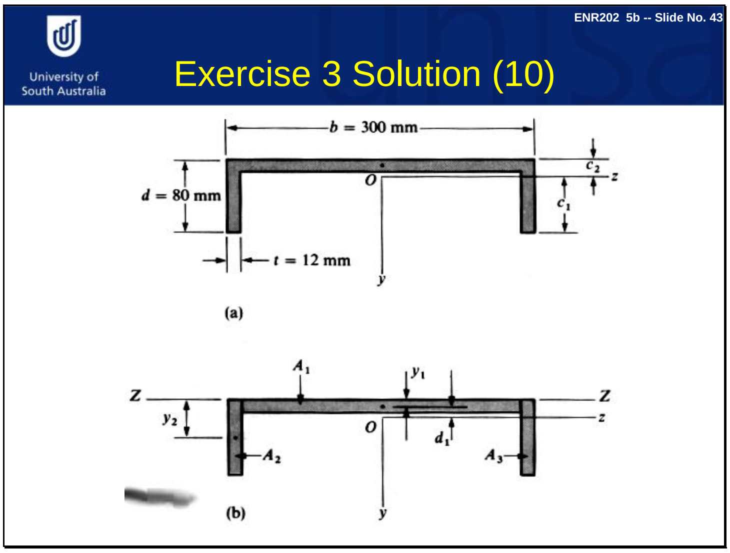
Slide 42



Connect all the bending moment values at important points, and we get the bending moment diagrams shown in the figure. You should try to draw the bending moment diagram on your own and check it against this figure.

Now, you need to find the maximum positive bending moment and the maximum negative bending moment, using the bending moment diagram. The maximum positive bending moment is 1.9 kilo Newton meters at point D where the shear force is zero. The maximum negative bending moment is 3.375 kilo Newtons per meter at point B.

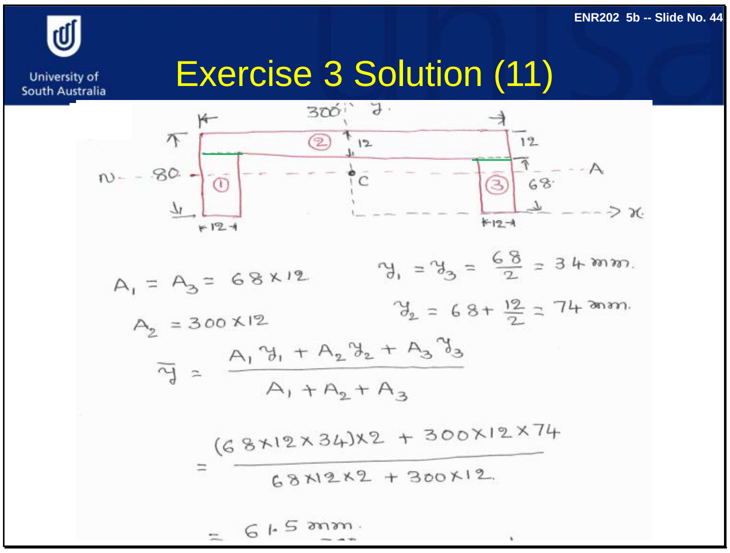
Slide 43



The cross section of the over hang beam is a channel section. You need to calculate the centroid and the moment of inertia for this channel section. The dimensions of the channel section are shown in figure a.   You then need to find the centroid axis.

ENR202 Mechanics of Materials Lecture 5B Slides and Notes

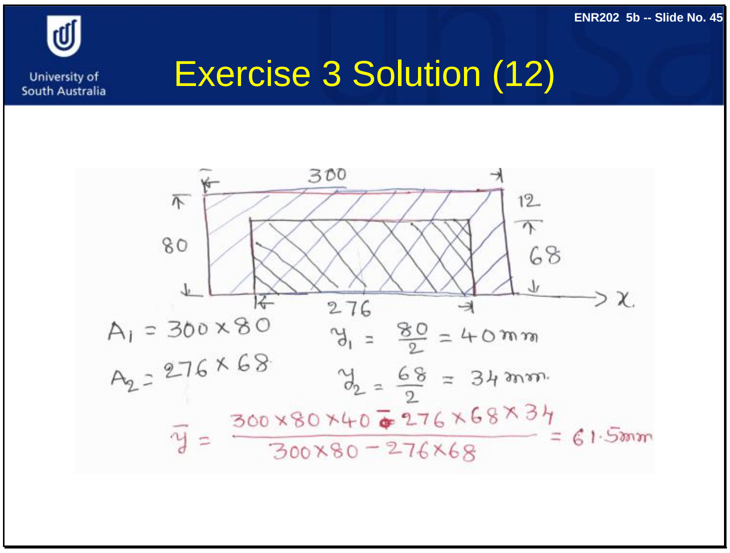
Slide 44



First, calculate the centroid of the channel section (if you are not sure about this, look back at lecture 5a, slides 7 and 8). We have an unsymmetrical channel section, and you need to locate the centroid of this channel shape. First, you need to separate the channel section into regular shapes (as we have done with the green lines in the figure). Here we have three rectangular shapes. The first rectangle is 12 millimeters by 68 millimeters, the second is 300 millimeters by 12 millimeters, and the third is 68 millimeters by 12 millimeters.

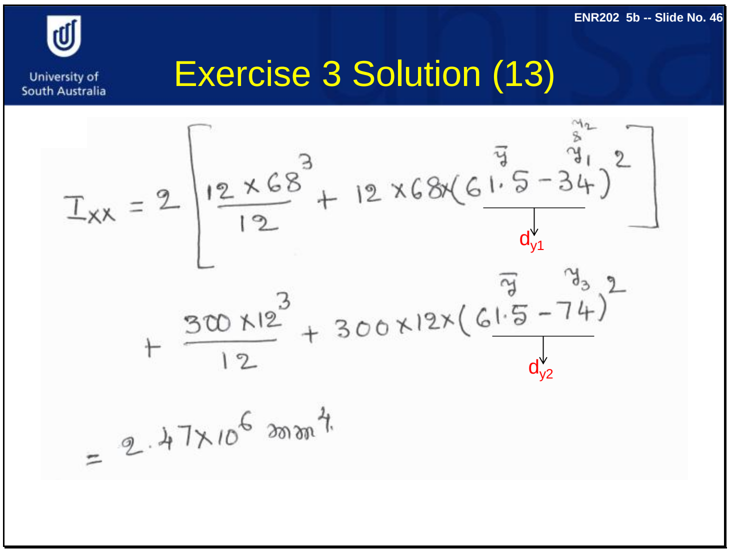
You need to locate the centroid of the given composite area. The centroid of the first rectangle from the x axis is 34 millimeters (68 millimeters divided by 2). The centroid of the third rectangle from the x axis is also 34 millimeters (68 millimeters divided by 2). The centroid of the second rectangle from the x axis is 68 millimeters plus 12 millimeters divided by 2, which is equal to 74 millimeters. The area of the first and third rectangles is 68 millimeters by 12 millimeters. The area of the third rectangle is 68 millimeters by 12 millimeters. Finally, we can calculate the centroid from the x axis y bar as 61.5 millimeters, based on the centroid formula.

Slide 45



You can also calculate the centroid of the channel section based on the negative area method. The area of the large rectangle (with the area shaded in the blue lines) is 300 millimeters by 80 millimeters. The area of the small rectangle (with the blue and black lines) is 276 millimeters by 68 millimeters. The centroid of the large rectangle from the x-axis is 80 millimeters divided by 2, which is 40 millimeters. The centroid of the small rectangle from the x-axis is 68 millimeters divided by 2, or 34 millimeters. Finally, you can work out the centroid of the channel section from the x-axis based on the negative area principle and the centroid formula. The answer will be the same as we calculated in the previous slide - 61.5 millimeters.

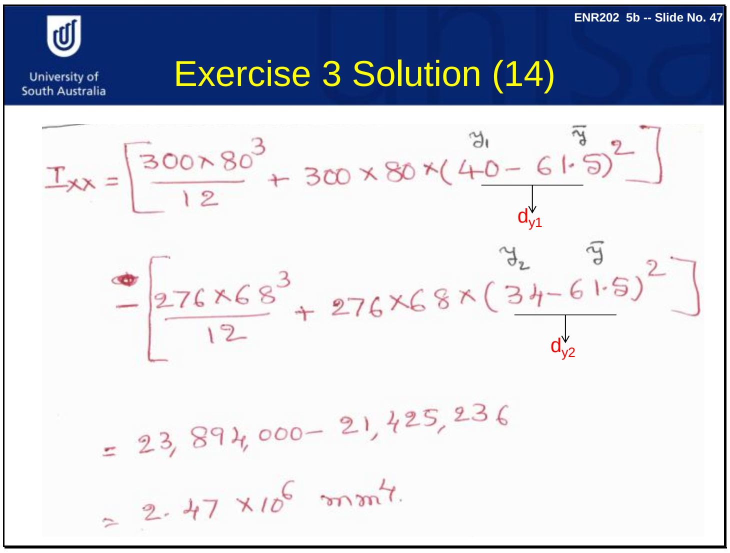
Slide 46



Now try to calculate the moment of inertia about the x-x axis of the channel cross section, and then compare your answer to our below.

The channel section is separated into three simple rectangles. The contribution of first, second and third rectangle moment of inertia is calculated by the parallel axes theorem (if you are unsure about this, go back to lecture 5a, slide 21). We are calculating the moment of inertia of the first and third rectangle about the centroid of the composite area 'xc' axis. The first and third rectangles are 12 millimeters wide and 68 millimeters high. The area of these rectangles is 12 millimeters times 68 millimeters. The self centroid moment of inertia of these rectangles is 12 times 68 to the power of 3 divided by 12. 'dy1' is the distance between the centroid of these rectangle to the centroid of the composite area from the x axis equal to 61.5 millimeters minus 34 millimeters, which is 27.5 millimeters. In the same way, the second rectangle is 300 millimeters wide and 12 millimeters high. The area of this rectangle is 300 millimeters times 12 millimeters. The self centroid moment of inertia of this rectangle is 300 times 12 to the power of 3 divided by 12. 'dy2' is the distance between the centroid of this rectangle to the centroid of the composite area from the x axis equal to 61.5 millimeters minus 74 millimeters. Finally, we can calculate the centroid moment of inertia for the channel section is 2.47 times 10 to the power of 6 millimeters to the power of 4.

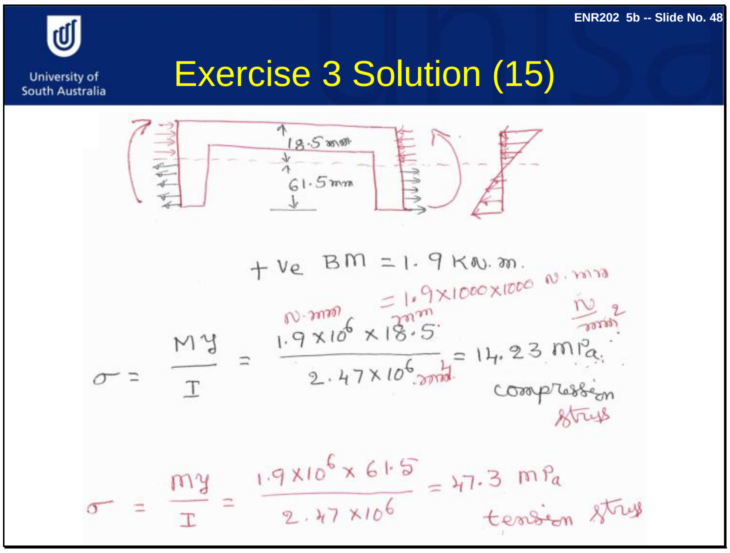
Slide 47



Try to calculate the moment of inertia about the x-x axis of the channel cross section by using the negative area principle, and then compare your answer to ours below.

To calculate the centroid moment of inertia for the composite area (the channel section about the centroid x axis), use the negative area principle. We have two rectangles (see the diagram in slide 43). Calculate the contribution of the large rectangle moment of inertia by using the parallel axes theorem. We are calculating the moment of inertia of the large rectangle about the centroid of the composite area 'xc' axis. The large rectangle is 300 millimeters wide and 80 millimeters high, so the area of the large rectangle is 300 times 80 millimeters. The self centroid moment of inertia of this rectangle is 300 times 80 to the power of 3 divided by 12. 'dy1' is the distance between the centroid of this rectangle and the centroid of the composite area from the x axis equal to 40 millimeters minus 61.5 millimeters, which is 21.5 millimeters. In the same way, the small rectangle is 276 millimeters wide and 68 millimeters high. The area of this rectangle is 276 millimeters times 68 millimeters. The self centroid moment of inertia of this rectangle is 276 times 68 to the power of 3 divided by 12. 'dy2' is the distance between the centroid of this rectangle and the centroid of the composite area from the x axis equal to 34 millimeters minus 61.5 millimeters. You need to deduct the small rectangle moment of inertia from the large rectangle moment of inertia. Finally, we calculate the centroid moment of inertia for the channel section as 2.47 times 10 to the power of 6 millimeters to the power of 4.

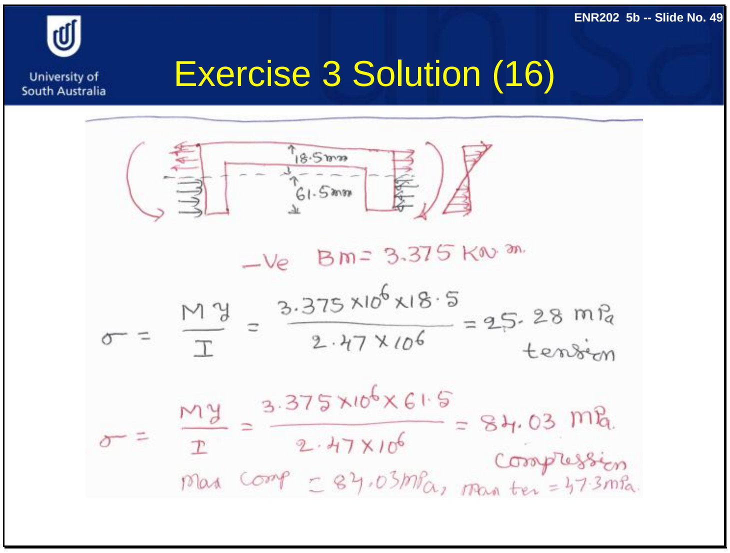
Slide 48



Now, you can calculate the bending normal stress due to the maximum positive bending moment of 1.9 kilo Newton Meters at point D where the shear force is zero. You know that positive bending moment causes compressive bending normal stress at the top and tensile bending normal stress at the bottom, as shown in the figure. You can calculate the compressive bending normal stress at the top by using the flexure formula: that is, M times y divided by I. You know the positive bending moment is 1.9 kilo Newton meters, the moment of inertia about the x-x axis is 2.47 times 10 to the power of 6 millimeters to the power of 4, and ‘y’ is the distance between the neutral axis and the topmost surface, which is 18.5 millimeters. You should calculate the compressive bending normal stress at the top surface as 14.23 Mega Pascals. Be careful with the units.

Now, calculate the tensile bending normal stress at the bottom by using the flexure formula (that is, M times y divided by I). You know the positive bending moment is 1.9 kilo Newton meters, the moment of inertia about the x-x axis is 2.47 times 10 to the power of 6 millimeters to the power of 4, and ‘y’ is the distance between the neutral axis and bottommost surface is 61.5 millimeters. Finally, you should get the value for the tensile bending normal stress at the bottommost surface as being equal to 47.3 Mega Pascals.

Slide 49



Now, you need to calculate the bending normal stress due to the maximum negative bending moment of 3.375 kilo Newton Meters at point B. You know that negative bending moment causes compressive bending normal stress at the bottom and tensile bending normal stress at the top, as shown in the figure. You can calculate the compressive bending normal stress at the bottom by using the flexure formula (M times y divided by I). You know the negative bending moment is 3.375 kilo Newton meters, the moment of inertia about x-x axis is 2.47 times 10 to the power of 6 millimeters to the power of 4, and 'y' is the distance between the neutral axis and the bottommost surface, which is 61.5 millimeters. Finally, you should get the value for compressive bending normal stress at the bottom surface as 84.03 Mega Pascals or Newton per square millimeters. Be careful with the units.

Now, calculate the tensile bending normal stress at the top by using the flexure formula (M times y divided by I). You know the negative bending moment is 3.375 kilo Newton meters, the moment of inertia about the x-x axis is 2.47 times 10 to the power of 6 millimeters to the power of 4, and 'y' is the distance between the neutral axis and the topmost surface, which is 18.5 millimeters. Finally, you will get the value for tensile bending normal stress at the topmost surface as 25.28 Mega Pascals.

Slide 50

Thank you for your attention.