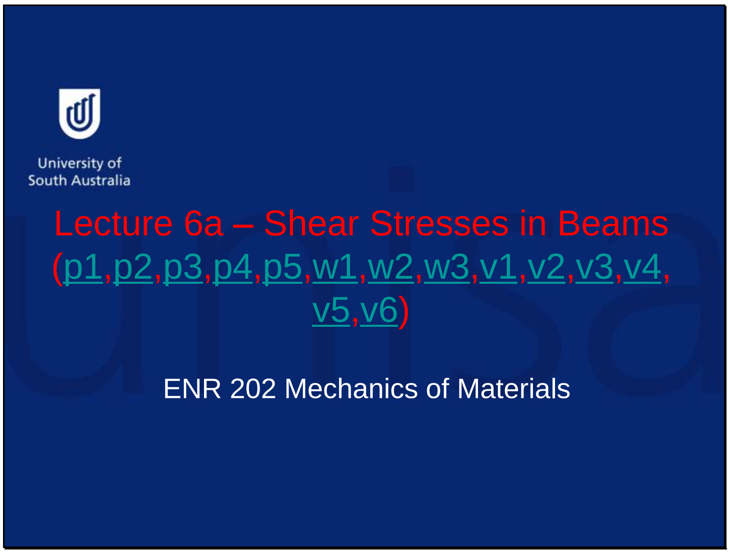
ENR202 Mechanics of Materials Lecture 6A Slides and Notes

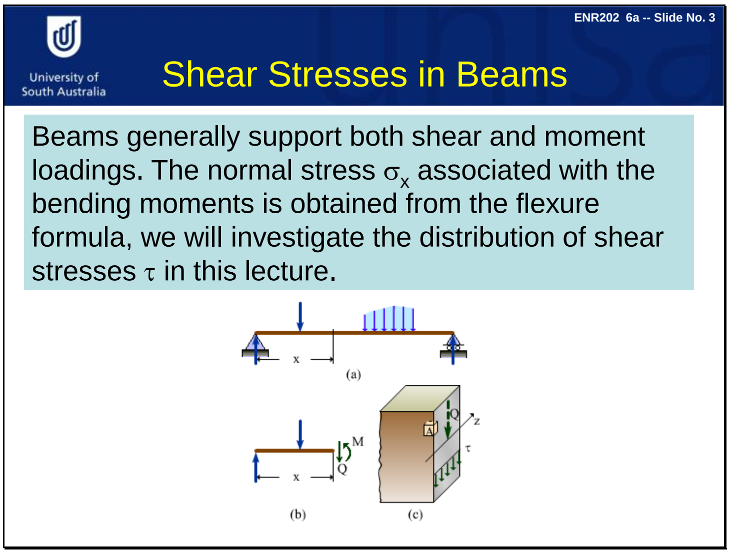
Slide 1

Slide 2



Welcome to Lecture Summary 6a. In this lecture, we will be discussing shear stresses in beams. (You can access internet links in the brackets on this slide for further information, and to help you to understand the material).

Slide 3



Suppose that a simply supported beam is subjected to lateral loads such as a concentrated load and uniformly distributed load, as shown in figure a. Suppose that you cut the beam at distance 'x' from the left support, as shown in figure b. At the cutting cross section, you have internal forces such as shear force and bending moment. We have discussed how bending moment will cause normal stress, and shear force will cause some other stresses. The shear force 'Q' is parallel to the surface as shown in figure b. The stresses 'tou' derived from the shear force are also parallel to the surface as shown in figure c, so we have shear stress acting on the cross sectional area.

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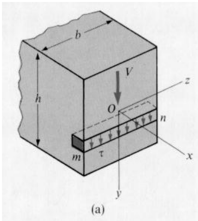
Vertical and Horizontal Shear Stresses – Rectangular Beams

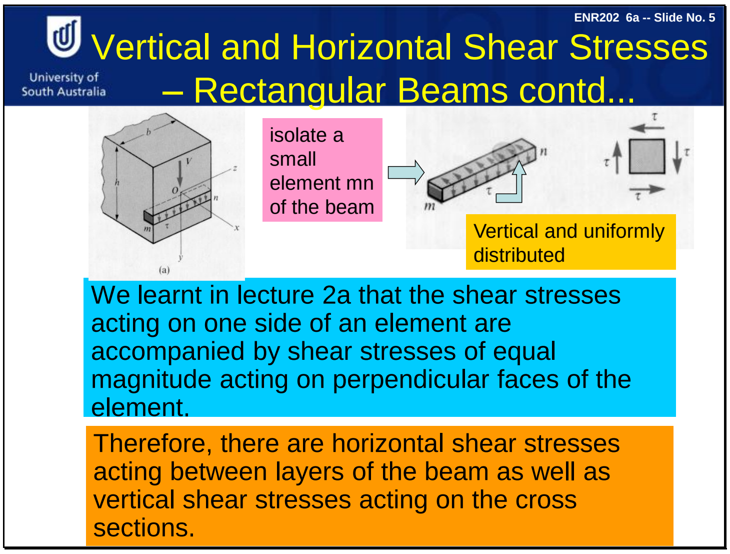


Assume:

- shear stresses τ act parallel to the shear force V;
- the distribution of shear stresses is uniform across the width of the beam.

We have some assumptions. For example, in the rectangular beam, we assume the shear stresses are acting parallel to shear force V. This means that if you have V vertical direction, and the distribution of shear stress along the width of the beam is uniform, as shown in the figure, the shear stress at that point is equal to the shear stress at the end point. We assume the shear stress is equal along the width of the beam, but in fact the shear stress changes along the height (that is, along the depth of the beam).

Slide 5



Assume that there is shear stress acting on the cross section of the rectangular beam. After isolating a small element from m-n of the beam, you can see the shear stress acting in the cross section 'tou' downward, as shown in the middle figure. You may have other shear stresses in the horizontal surface. This shear stress is equal to the vertical shear stress, as shown in the middle figure. The reason may be explained as follows. The two vertical shear forces are in fact a couple, and should cause this cross section to rotate clock-wise as shown in the right side figure, but in fact this cross section is actually in equilibrium, and doesn't rotate. This is because we have two other shear stresses in the horizontal surfaces, and these two horizontal shear stresses are causing another couple in an anti-clock-wise direction.

So this small segment is in balance with vertical shear forces and horizontal shear forces as shown in the right side figure. The shear stresses acting on one side of the element are accompanied by shear stresses of equal magnitude acting on the perpendicular faces of the element, as we discussed in lecture 2a, slide 19.

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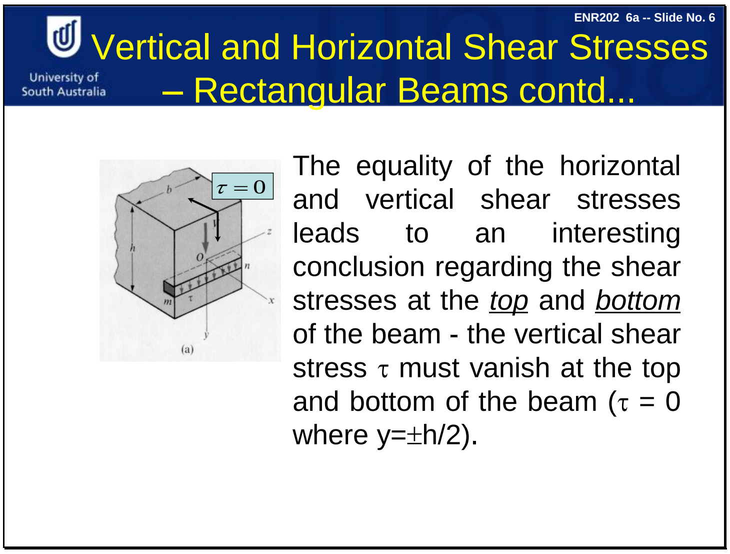

The equality of the horizontal and vertical shear stresses leads to an interesting conclusion regarding the shear stresses at the *top* most and *bottom* most parts of the beam cross section. The vertical shear stress ‘tou’ must vanish at the top and bottom of the beam. That means that the shear stress is zero where y is equal to positive or negative h/2. Y is measured from the neutral axis that is the centroid of rectangle.

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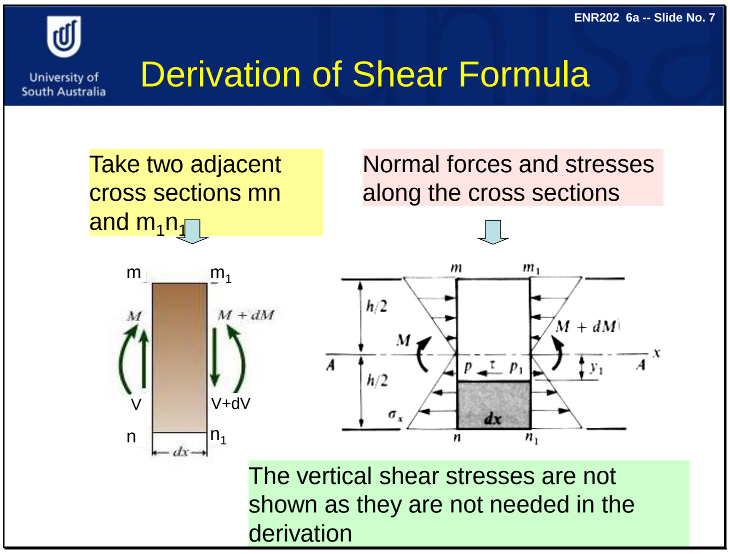
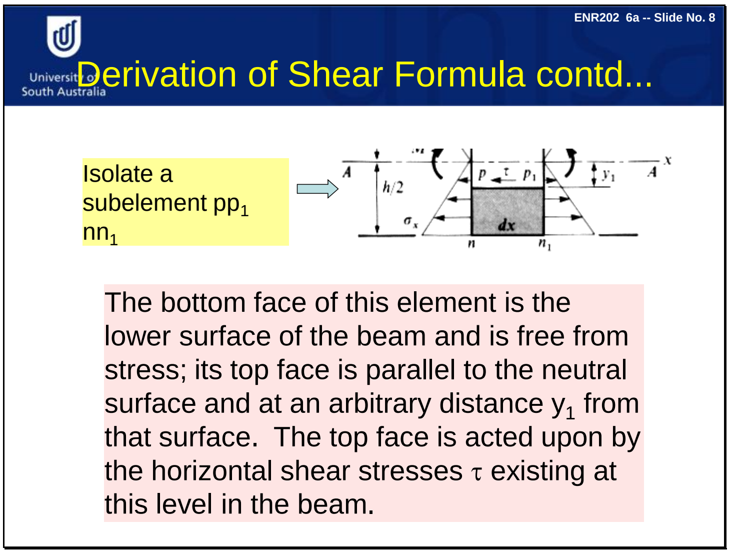


To calculate the shear stresses, consider the two cross sections m-n and m1-n1 of the small segment of the beam with length 'dx' as shown in the left side figure. On cross section m-n, we have a shear force V and bending moment M. On the other cross section m1-n1, we have a shear force 'V plus dV', and a bending moment 'M plus dM' as shown in the left side figure.

If shear force is acting on the beam, we must have bending moment change. Bending moment causes the normal stresses on the cross section. We have normal stress distributions at both cross sections m-n, and m1-n1.

Slide 8



If you isolate the sub element that is the shaded part of the diagram, you can see point p at the left cross section, and point p1 at the right cross section. We don't have any shear stresses in the bottom surface face. Consider the top surface of the sub element p-p1. What is happening in the shaded portion at a distance of 'y1' from the neutral axis to the p-p1 surface?

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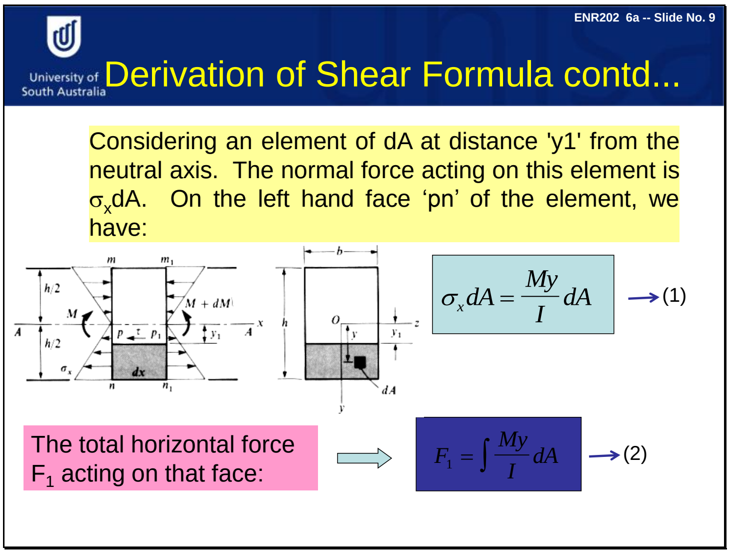

In the left side of the shaded area, normal stress is sigma x. However, in the right part of the shaded area, normal stress is different from the left side because the bending Moment on the left side is M and on the right side is M plus dM. That means the two bending moments have a difference of dM.

The normal force on the left side of the shaded area is sigma x times dA, which is equal to M times y times dA divided by I, as shown in equation 1. The total normal force on the left side of the shaded segment is F1, which is equal to the integration from y to h/2 of M times y times dA divided by I, as shown in the equation 2.

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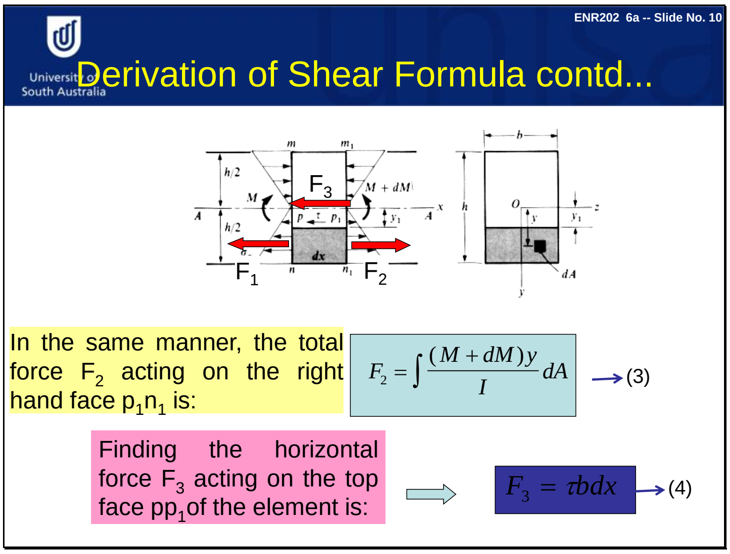
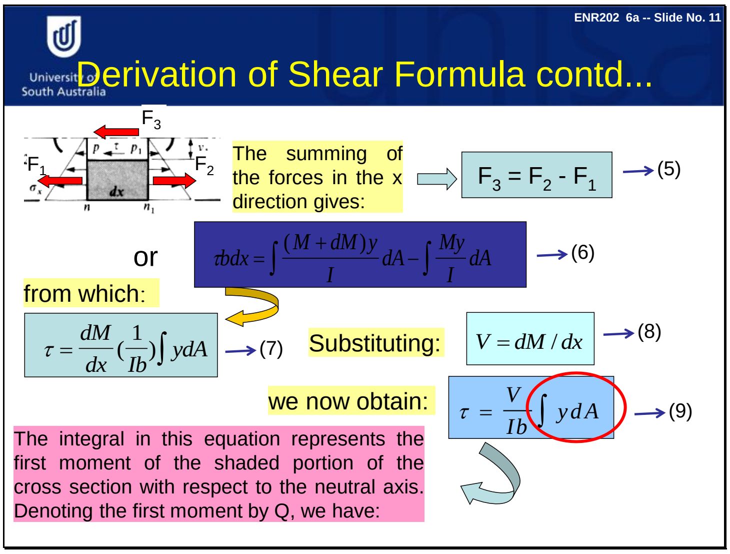


And the total normal force on the right side of the shaded segment is F2 which is equal to from y to h/2 of M plus dM times y times dA divided by I, as shown in the equation 3. The total normal force acting on the surface p-p1 is F3, which is tou times b times dx. F1 is not equal to F2 because of the different bending moment in the different cross sections. The difference normal forces of F1 and F2 are equal to F3 (based on equilibrium equation that all forces in a horizontal direction equal to zero).

Slide 11



This will lead to equation 5, that the shear force F_3 is equal to F_2 normal force on right side minus F_1 normal force on left side. F3 in fact is the shear force acting on the horizontal surface, because F3 is parallel to the horizontal surface. So, F3 is distributed uniformly along the horizontal surface p-p1, that is tou times b times dx, which is equal to the integration of M plus dM times y times dA divided by I, minus the integration of M times y times dA divided by I, as shown in the equation 6. (You can develope equation 6 by substituting equations 2, 3 and 4 into equation 5.) The shear stress formula equation 7 can be derived from equation 6; tou is equal to dM divided by dx times 1 divided by Ib times integration y dA. You know that the derivative of moment with respect 'dx' is equal to the shear force as shown in equation 8. If you substitute equation 8 into equation 7, you get the shear stress formula of tou is equal to V divided by Ib times the integration of y*dA. Here integration y*dA is equal to the first moment of the shaded area with respect to the neutral axis and is denoted as Q.

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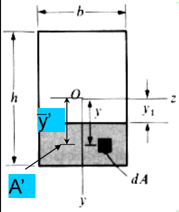
Shear Formula

$$\tau = \frac{VQ}{Ib}$$



τ = the shear stress in the member at the point located a distance y_1 from the neutral axis.

V = the internal resultant shear force, determined from the method of sections and the equation of equilibrium.

I = the moment of inertia of the entire cross-sectional area computed about the neutral axis

b = the width of the member's cross-sectional area, measured at the point where τ is to be determined

Q = $\int y dA = \bar{y}' A'$ where A' is the top (or bottom) portion of the member's cross-sectional area, defined from the section where ***b*** is measured, and $\bar{y}'$ is the distance to the centroid of A', measured from the neutral axis

Finally, we have the shear stress formula that tou is equal to V times Q divided by Ib , where V is the shear force acting on that cross section, Ib is the second moment of the area; b is the width of the cross section, Q is the first moment of the shaded area with respect to neutral axis, equal to integration y dA of the shaded area.

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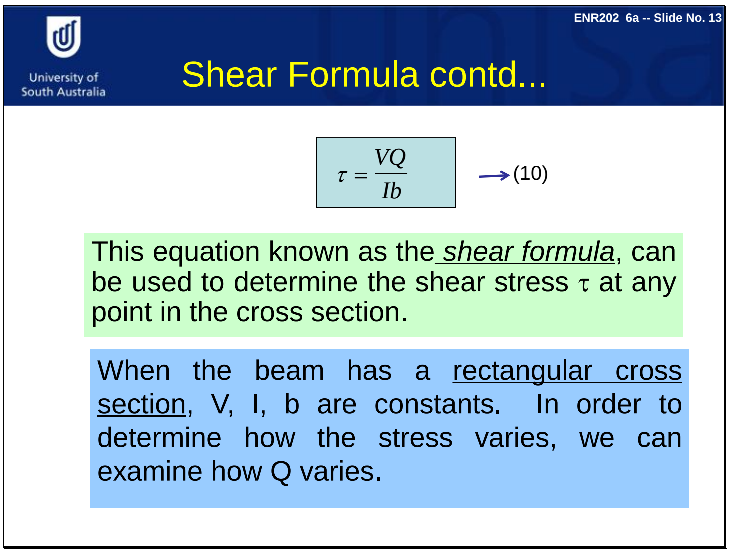

Equation 10 is the shear formula. You can get shear stress ‘tou’ at any point of the cross section based on the shear formula. The parameters of shear force V and the moment of inertia I belong to the whole cross section. However, the parameters b and Q vary from point to point where you want to calculate shear stress on that cross section. For example, if the beam is a rectangular cross section, shear force V, I, b are constants. The width of the beam is the same for the whole rectangular cross section. V and I are the same for the whole cross section. So, shear stress varies if Q varies. The shear stresses along the cross section depend on Q.

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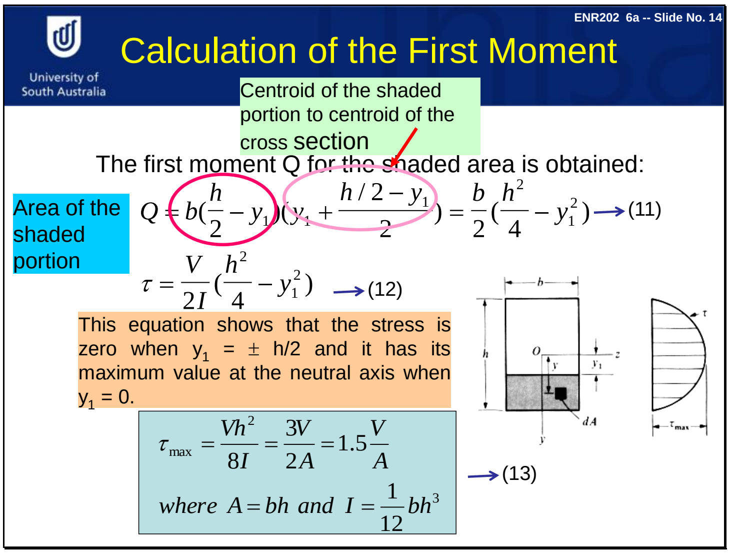

The calculation of the first moment of the shaded area (that is, parameter ‘Q’ in the shear formula for the rectangular cross section) is given here. The cross sectional area of the shaded portion is the width of the beam ‘b’ times h divided by 2 minus y1 as shown in equation 11. y1 is the distance between the neutral axis to the top of the shaded area as shown in the figure. ‘h’ is the height of the beam, b is the width of the beam. The centroid of the shaded portion to the centroid of the cross section is y1 plus h divided by 2 minus y1 whole divided by 2 as shown in equation 11. That means that the height of the shaded area is h divided by 2 minus y1. The centroid of the shaded area from the bottom is half of h divided by 2 minus y1. The first moment Q for the shaded area is the product of the area of the shaded portion and the distance from the centroid of the shaded portion to the centroid of the cross section. Finally, shear stress is a function in terms of y1. The shear stress distribution along the cross section is parabolic because we have y1 squared in the equation of shear stress.

We have zero shear stress at the bottom surfaces and the top surfaces, and at the neutral axis of the cross section, we have maximum shear stress. Maximum shear stress at the neutral axis means that y1 is equal to zero in equation 12. That is, tou max is equal to V times h squared divided by 8I. If you substitute I equal to b times h cubed divided by 12, and also substitute A equals b times h, the maximum shear stress is 1.5 times shear force ‘V’ divided by the area of the rectangular cross section as shown in the calculations in equation 13.

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Shear Formula

The formulas for shear stresses in rectangular beams are valid for beams of ordinary proportions and are subject to the same restrictions as the flexure formula for which they are derived; thus, the formulas are valid only for beams of linear elastic material with small deflections.

The formulas for shear stresses in rectangular beams are valid for beams of other cross section shapes; the condition here is the beam is of linear elastic material and also has small deflections.

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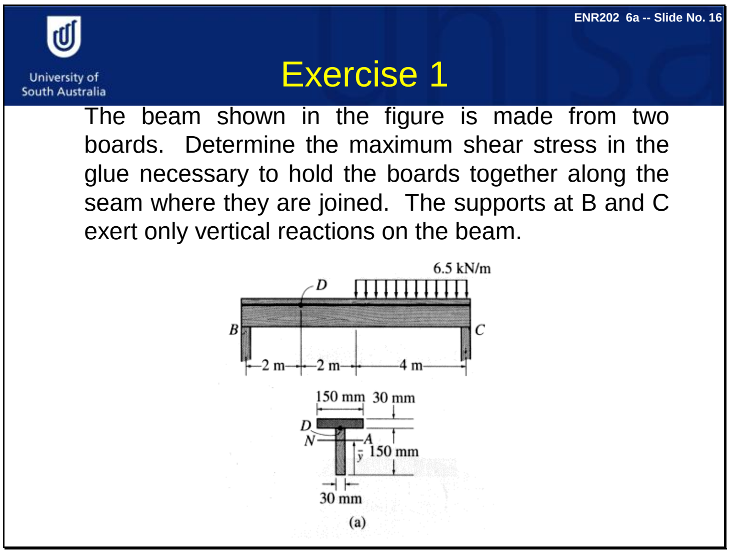
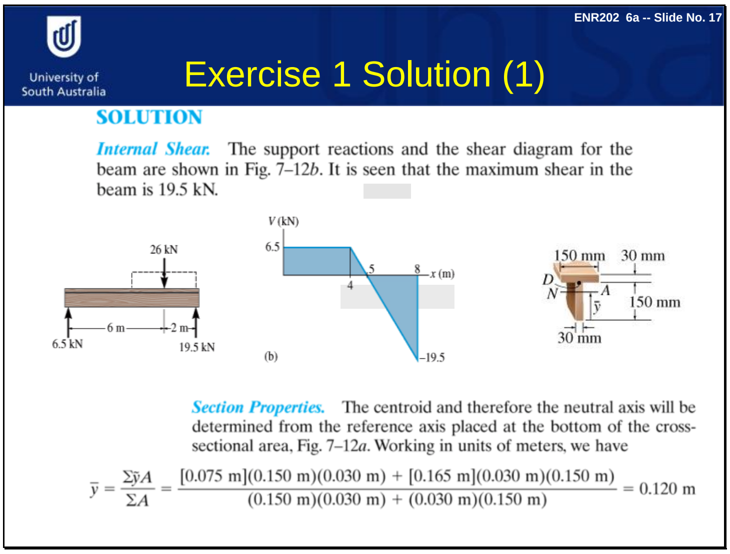


Lets look at an example of shear stress formula. Here we have the T-shaped cross section of a simply supported beam made from two boards, as shown in the figure. We want to know the maximum shear stress in the glue necessary to hold the boards together along the length of the beam where they are joined. We want to know how strong the glue is required to carry the maximum shear stress. The beam span is 8m with a Uniformly Distributed Load of 6.5 kilo Newtons per meter acting on the right half span of the beam. The board dimensions are 150 millimeters width and 30 millimeters thickness. You can try this problem for yourself if you want – just pause the presentation here. The solution is on the next five slides.

Slide 17



First, we have to calculate the reaction forces and draw the shear force diagram and find the maximum shear force happening in the beam. To do this, we need to calculate the reaction forces and then calculate shear force values at important points. Then we need to connect all shear force values at important points, and finally draw the Shear Force Diagram.

We have a reaction force at the left support of 6.5 Kilo Newtons and one at the right support of 19.5 Kilo Newtons. Thus we get the shear force diagram that you can see in the figure. The maximum shear force occurs at the right side support (that is, 19.5 kilo Newtons).

Now we need to know the centroid axis, the moment of inertia I and first moment of particular area that ‘Q’ of the cross section. The centroid of the T shape from the bottom of the cross section is Y bar equal to sigma y times A divided by sigma A. We have two rectangles in the T shape cross section. The centroid of the bottom rectangle from the bottom of the cross section is 75 millimeters, and the centroid of the top rectangle from the bottom of the cross section is 165 millimeters (that is, 150 millimeters plus 30 millimeters divided by 2). We calculate the areas of individual rectangular cross sections as 150 millimeters by 30 millimeters, which is 4500 square millimeters. Finally we can work out that the centroid of the whole cross section is 120 millimeters from the bottom .

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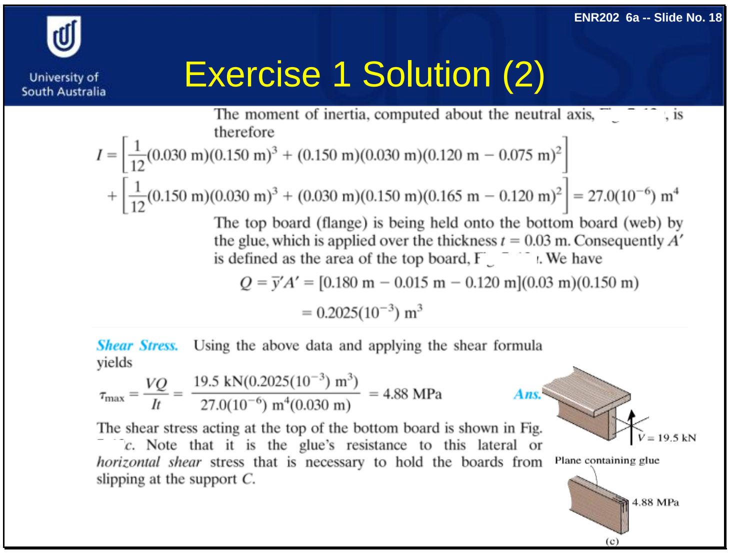

Now you need to calculate the moment of inertia, the I value (look back at lecture 5a if you are unsure). The I value includes the contribution of both rectangles, so we need do the calculation separately for each individual rectangle and then add them together. The I value for the whole cross section is 27 times 10 to the power of minus 6 meters to the power of 4.

Now, calculate the moment of inertia for the T shape cross section. The moment of inertia for each rectangle is the contribution of self moment of inertia plus Area times d1 square as per parallel axis theorem.

Now, we need the calculation of Q. The shear formula says that shear stress 'tou' is equal to V times Q divided by 'It'. We have maximum shear force of 19.5 Kilo Newtons at the right support. We calculated the I value that is 27 times 10 to the power of 6 millimeters to the power of 4.

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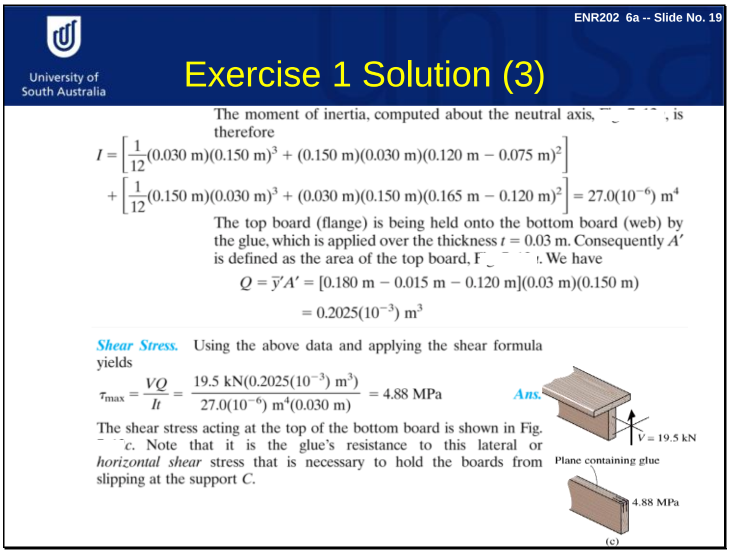
Slide 19



t is the width of the beam where to calculate shear stress. The t parameter varies from point to point. If you want to calculate the shear stress at the junction of the two boards (that is, the junction of the flange and the web of the T-shape cross section), then ‘t’ will be 30 millimeters. If you want to calculate the shear stress anywhere in the top flange, the thickness will be 150 millimeters. If you want to calculate the shear stress at anywhere in the web, t is 30 millimeters. That means that the shear stress is much larger in the web than in the flange, because t is in the denominator.

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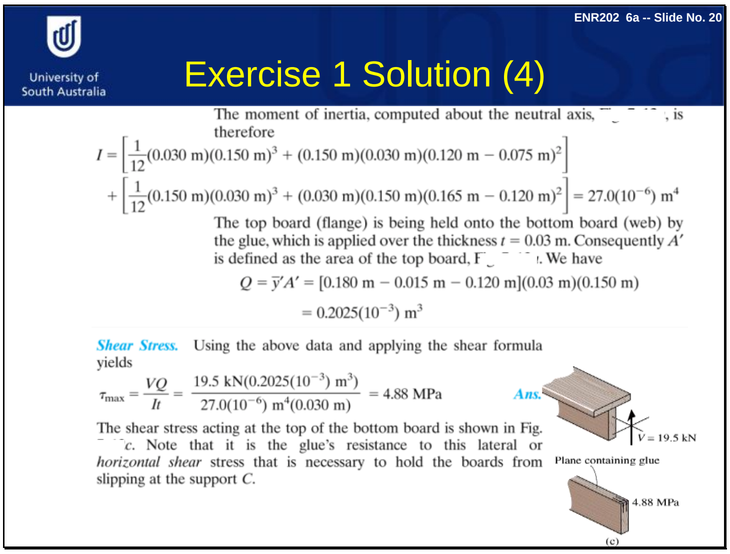

We also need to know Q, the first moment of the cross section. If we want to calculate the shear stress at the junction of two rectangles (that means the junction of two boards), then the shaded area is the top rectangle (which is 150 millimeters times 30 millimeters). The y bar of the shaded area will be the distance from the neutral axis or centroid of whole cross section to the centroid of the shaded area. This distance is 180 millimeters minus 120 millimeters minus 15 millimeters, as shown in the calculation. This is because the 180 millimeters is the total height of cross section, and the centroid distance from bottom is 120 millimeters. The centroid of the shaded area is 30 millimeters divided by 2, which is 15 millimeters. Then finally we have the first moment of the shaded area of the cross section, which is 0.2025 times 10 to the power of minus 3 cube meters. The unit of the first moment of area is cubic meters, and the unit of the second moment of inertia is meters to the power of 4.

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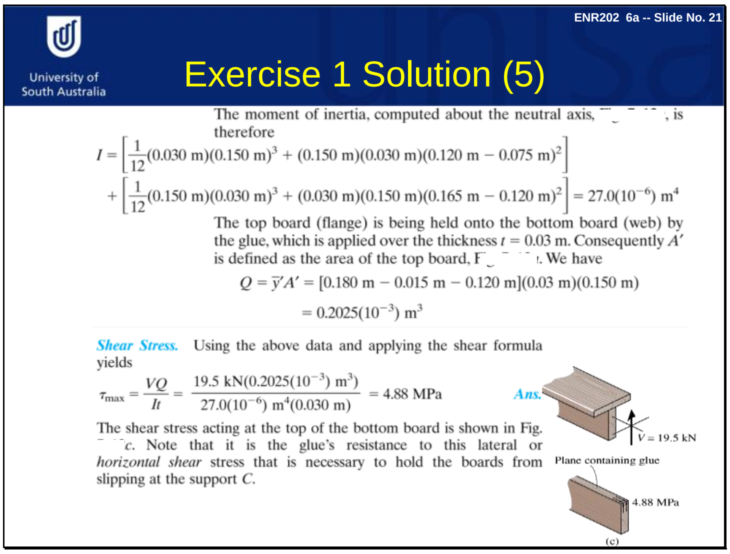

If we input all values into the shear formula, we work out that the shear stress at the junction of two boards is 4.88 Mega Pascals or Newtons per square millimeter. Remember to be careful with the units – it is easy to lose marks in a assessment by putting incorrect units.

This shear stress occurs at the right side support cross section at the junction of the flange and the web - the junction point of two boards.

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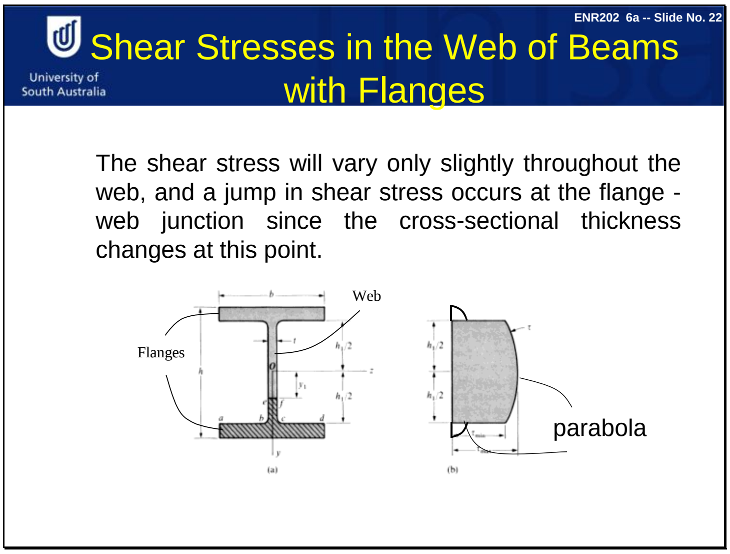

Let's have a look at shear stresses in the web of beams with two flanges; that is, an I shape cross section as shown in the figure. We have shear stress distribution as shown in right figure, and a jump in shear stress at the joints of flanges and web, as shown in the right figure.

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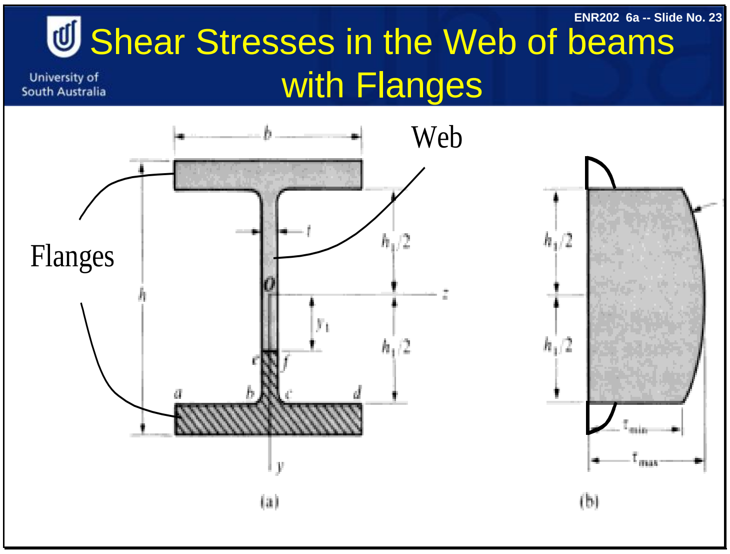

This is based on the formula tou is equal to shear force V times Q divided by It. At the junction of flange and web, we have the same V, same Q and same I, but at the junction of web and flange, we have two options for the width of the cross section. One is the flange width, and the other is the web width at the joint. We have a jump in shear stress because of the jump in the width of the cross section at the joint. Maximum shear stress occurs at the neutral axis of the whole cross section. In the flange, we have low shear stress – the shear stress is much higher in the web.

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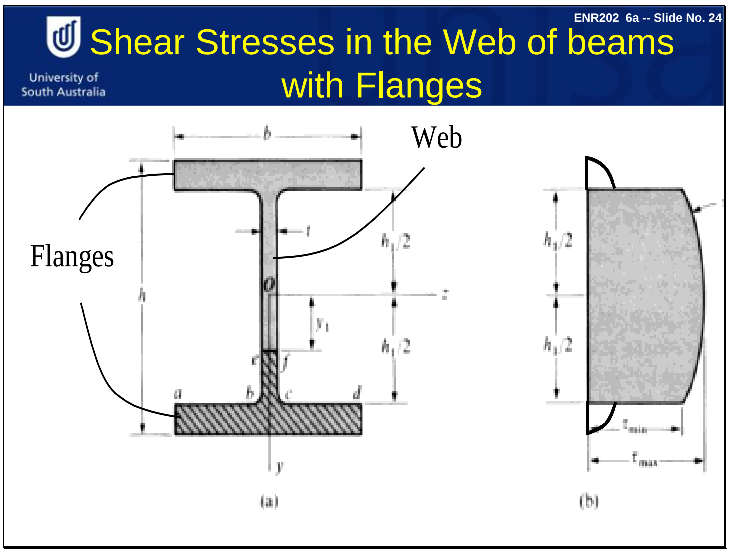

Here are the important point on how to calculate Q.
If we have an I shape cross section, then first calculate the centroid axis of the cross section.
If you want to know the shear stress at some certain point in the cross section, you have to calculate the area of the bottom part or top part of the cross section below /or/ above that point.
Q is equal to integration y times dA . Separate the shape into rectangles to calculate the centroid of the shaded area. Calculate the distance between the centroid of the whole cross section to the individual rectangle centroids. Calculate the individual rectangle area. You will get the first moment of the shaded area as being Q equal to y_1 times A_1 plus y_2 times A_2 , as shown in the figure. You firstly have to know how to calculate Q and secondly how to use the values of t, the width of the cross section.

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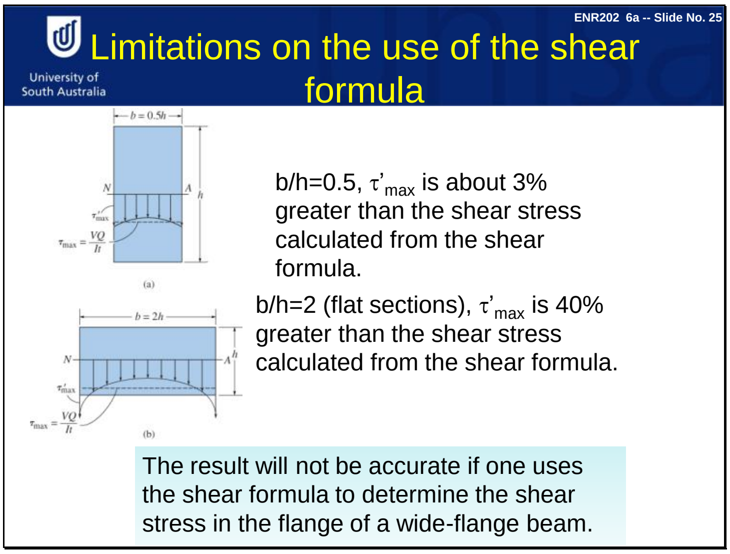

You need remember the limitations on the use of the shear formula. As I mentioned before, the shear stress is uniform throughout the width of the beam at some point on the cross section. Based on width to height ratio, the calculated real maximum shear stress value varies with the shear formula value.
How much can the value vary? If the width to height ratio is 0.5, the maximum shear stress is about 3 percentage greater than the shear stress calculated by the shear formula, as shown in the top figure. You can see real shear stress distribution and shear formula stress distribution in the top figure. If the width to height ratio is 2 (flat sections), the maximum shear stress is about 40 percentage greater than the shear stress calculated by shear formula as shown in the bottom figure. You can see the real shear stress distribution and shear formula stress distribution in the bottom figure.

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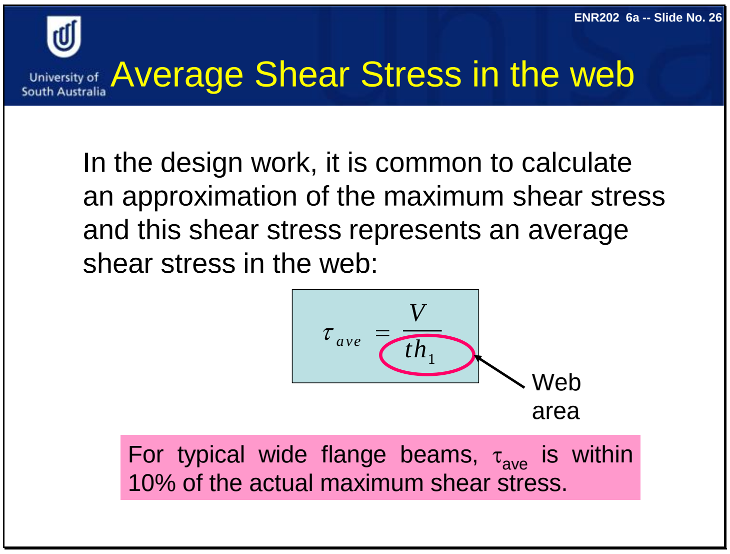

In design work, it is common to calculate an approximation of the maximum shear stress. The approximate shear stress represents an average shear stress in the web equal to the shear force divided by the web area. For typical wide flange beams, the average shear stress is within 10 percentage of the actual maximum shear stress.

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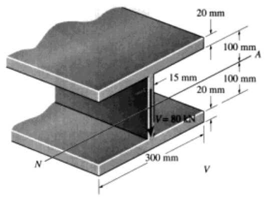
Exercise 2

Example 2: A steel wide-flange beam has the dimensions shown in the figure. If it is subjected to a shear of V=80kN, (a). plot the shear-stress distribution acting over the beams cross-sectional area, and (b) determine the shear force resisted by the web.



Here we have another example - a steel wide flange beam which has the dimensions shown in the figure. If it is subjected to a shear force of 80 kilo Newton, plot the shear stress distribution over the wide flange beam cross section. Then determine the shear force resisted by the web. Pause the presentation now and try to do this problem. The answer is on the following slides.

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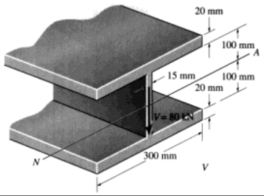
Exercise 2 Solution (1)

SOLUTION

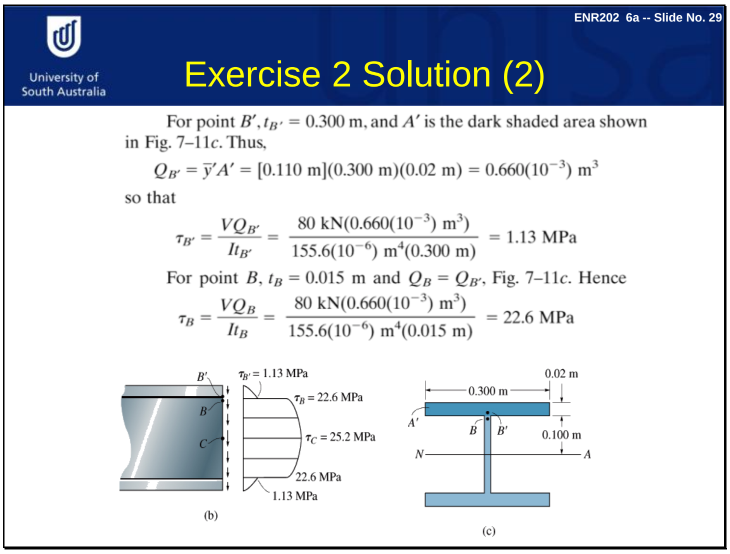
Part (a). The shear-stress distribution will be parabolic and varies in the manner shown in Fig. [illegible]1*b*. Due to symmetry, only the shear stresses at points *B′*, *B*, and *C* have to be computed. To show how these values are obtained, we must first determine the moment of inertia of the cross-sectional area about the neutral axis. Working in meters, we have

$$I = \left[\frac{1}{12}(0.015\text{ m})(0.200\text{ m})^3\right] + 2\left[\frac{1}{12}(0.300\text{ m})(0.02\text{ m})^3 + (0.300\text{ m})(0.02\text{ m})(0.110\text{ m})^2\right] = 155.6(10^{-6})\text{ m}^4$$



Lets start by working out the shear stress distribution over the wide flange beam cross section. Think back to the formula we use to calculate shear stress; tou is equal to V times Q divided by It. We know the shear force is 80 kilo Newtons. The thickness of the beam and the first moment ‘Q’ varies from point to point on the cross section. The moment of inertia about the horizontal axis is constant for whole cross section. In this slide, we are calculating the moment of inertia for the wide flange beam I-shaped cross section. This cross section is symmetric in both the horizontal and vertical axes. You can calculate the moment of inertia about the horizontal axis by the negative area principle or the general procedure, explained in lecture 5a. The web dimensions are 15 mm thickness and 200 mm height. The self moment of inertia of the web is 15 mm times 200 mm cubed divided by 12. The wide flange dimensions are 300 mm width and 20 mm thickness. We have two flanges, one at the top and one at the bottom. The self moment of inertia of the flange is 300 mm times 20 mm cubed divided by 12. The area of the flange is 300 mm times 20 mm and the distance between the centroid of the flange to the centroid of the whole cross section is 110 mm (that is, 100 mm plus 20 mm divided by 2). We have two flanges contributing to this cross section. Finally, you could work out that the moment of inertia for this cross section is equal to 155.6 times 10 to the power of 6 mm to the power of 4.

Slide 29



The given I shape cross section is symmetrical on both axes. Therefore, any shear stress at the top surface and the bottom surface are equal to zero. The shear stress at the junction of the top flange and the web is the same as the shear stress at the junction of the bottom flange and the web. Finally you will get a symmetrical shear stress distribution about the horizontal axis. Now you need to calculate the first moment of area about the neutral axis. The width of the beam at B dash point is equal to 300 mm. The B dash point is located on the flange just above the web, as shown in figure c. Point B is located on the web and just below the flange, as shown in figure c. Point C is located in middle of the cross section. The width of the beam at point B is the thickness of the web, which is 15 mm. The Q values are the same for Points B and B dash. The first moment of the top flange area about the neutral axis is ‘QB or QB dash’. The area of the flange is 300 mm times 20 mm. The y dash bar is the distance from the centroid of the shaded top flange portion to the centroid of the whole cross section, which is 110 mm (that is, 100 mm plus 20 mm divided by 2). The Q value is the product of the area of the shaded top flange portion and the y dash bar, which is equal to 660 times 10 to the power of 3 mm to the power of 3. Now you know all the values of Q, the moment of inertia I, and the thickness at Points B and B dash. Therefore, you can calculate the shear stress at point B dash, which is 1.13 Mega Pascals, and the shear stress at point B which is 22.6 Mega Pascals.

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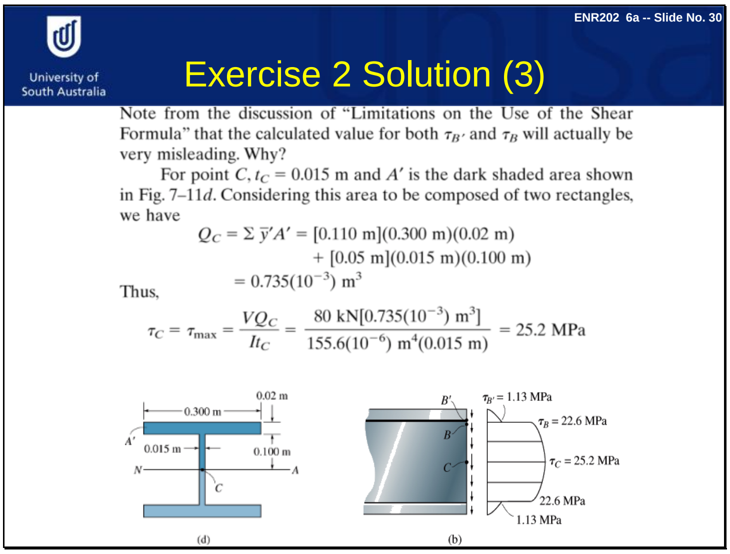

Now, we can calculate the shear stress at point C (the centroid of the cross section, as shown in the figure.) We know that the width of the beam at C is 15 mm. The first moment of the top flange area plus half the web area about the neutral axis is 'QC'. Let's look at calculating the shear stress at any point in more detail. You need to cut the cross section at the point where you want to calculate the shear stress. Then you need to consider the top or bottom of the cutting cross section.

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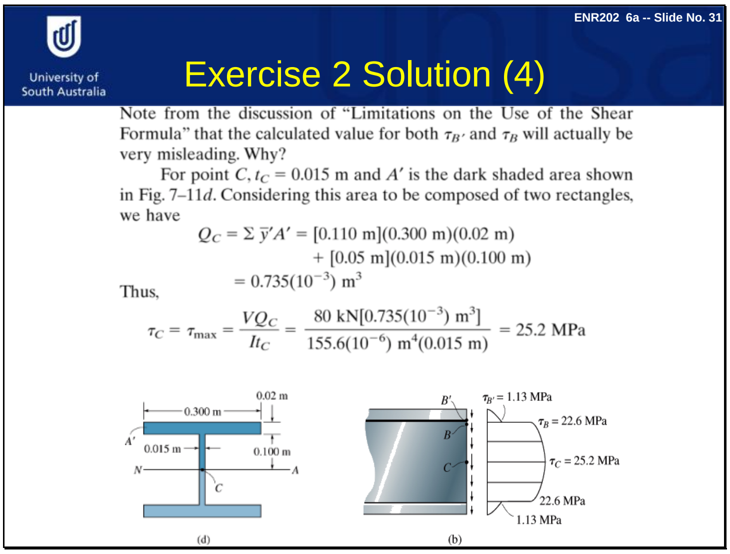

Here we consider the top part of the cross section (that is, the shaded area, as shown in the figure). We calculate the Q value for two different parts. The first is the top flange shaded area and the second is the web shaded area. The contribution of the top flange to the Q value is the product of the flange area and the y bar dash. The area of the flange is 300 mm times 20 mm and the y dash bar is the distance between the centroid of the shaded top flange portion and the centroid of the whole cross section, which is equal to 110 mm (that is, 100 mm plus 20 mm divided by 2). The contribution of the shaded web to the Q value is the product of the shaded web area and the y bar dash. The area of the shaded web is 15 mm times 100 mm and the y dash bar is the distance between the centroid of the shaded web portion and the centroid of the whole cross section, which is equal to 50 mm (that is, 100 mm divided by 2). The sum of both Q values equals 735 times 10 to the power of 3 mm cubed. Now, you know all the values for the shear force: Q, the moment of inertia I, and the thickness at Point C (15 mm). Therefore, you can calculate the shear stress at point C as being equal to 25.2 Mega Pascals. We know the shear stress values at the top surface, the bottom surface, the junction of flange and web, and the middle of the cross section. You know that shear stress distribution is a parabolic variation. Now you can draw the shear stress distribution. However, remember that we have two values at the junction of the flange and web - one if you consider the width of the flange as thickness, and the other if you consider the thickness of the web as thickness. Therefore, there is a jump at the junction of flange and web. You can see all shear stress values in shear stress distribution as shown in the right figure.

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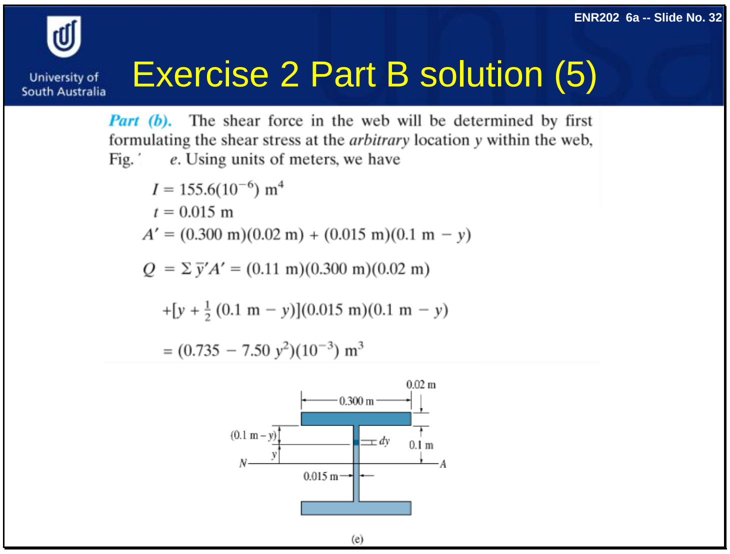

In the second part of the problem, you need to determine the shear force resisted by the web. The shear force in the web will be determined by first formulating the shear stress at the arbitrary location y from the centroid of the I cross section within the web, as shown in the figure. So, we cut the cross section at a distance of ‘y’ from the centroid axis. Now consider the shaded area that is the thick blue color, as shown in the figure. The width of the cutting cross section is 15 mm. Now we need to calculate the Q value for the shaded portion. We have already calculated Q for the top flange portion. Now we come to the web portion. The contribution of shaded web area to the Q values is the product of the area of the shaded web, which is 15 mm times 100 mm minus y, as shown in the figure and the y bar dash. The area of the shaded web is 15 mm times 100 mm minus y. The y dash bar is the distance between the centroid of the shaded web portion to the centroid of the whole cross section, which is equal to y plus half of 100 mm minus y. The sum of both Q values equals 0.735 minus 7.5 times y squared to the power of minus 3 meters cubed.

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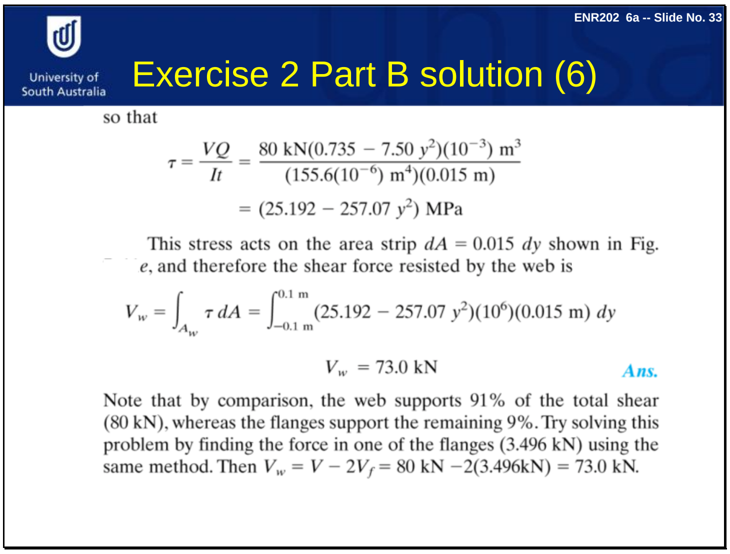

Now, we can calculate the shear stress at a distance of ‘y’ from the centroid axis, but within the web portion which is equal to V times Q divided by It. We know all the values, and can get the shear stress at a distance y from the centroid axis is 25.192 minus 257.07 times y squared Mega Pascals. This stress acts on the area strip dA which is equal to 15 mm times dy, as shown in the previous slides. Now we can determine the shear force resisted by the web. That means the distance ‘y’ changes from minus 100 mm to positive 100 mm. We have to integrate the equation between these two limits. Then we can get the shear force resisted by the web as 73 kilo Newtons. The total shear force is 80 kilo Newtons, which means the shear force resisted by both flanges is equal to 80 kilo Newtons minus 73 kilo Newtons, which is equal to 7 kilo Newtons. By comparison, the web carries 91 percent of the total shear force (that is, 80 kilo Newtons), whereas the flanges resist the remaining 9 percent. The shear force resisted by one flange is 3.5 kilo Newtons. If you would like more practice, try the same problem but considering the point in the flange and calculate the shear force resisted by one flange.

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Thank you for your attention. Next lecture we continue with same topic, but focus on shear stresses in built up beams.