ENR202 Mechanics of Materials Lecture 7A Slides and Notes

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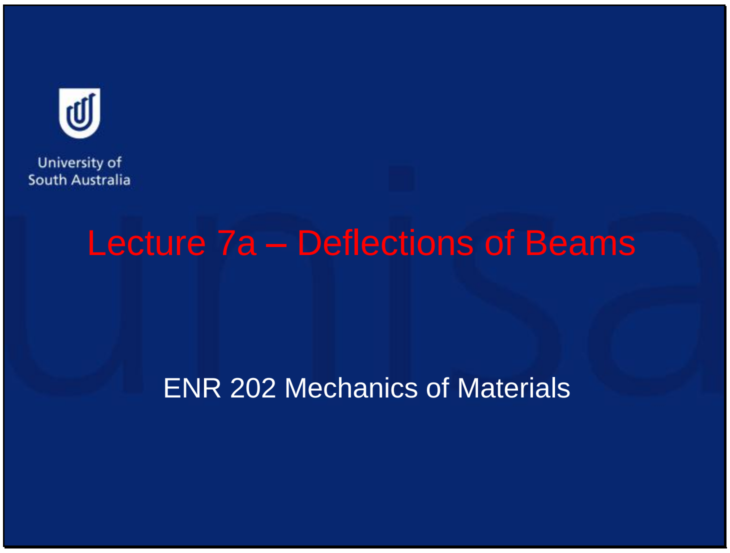

Welcome to lecture summary 7A. In this lecture, we will be looking at the concepts of deflections of beam, which we will then practice in the following lecture summary. Remember to click on the live links throughout the presentations for further information.

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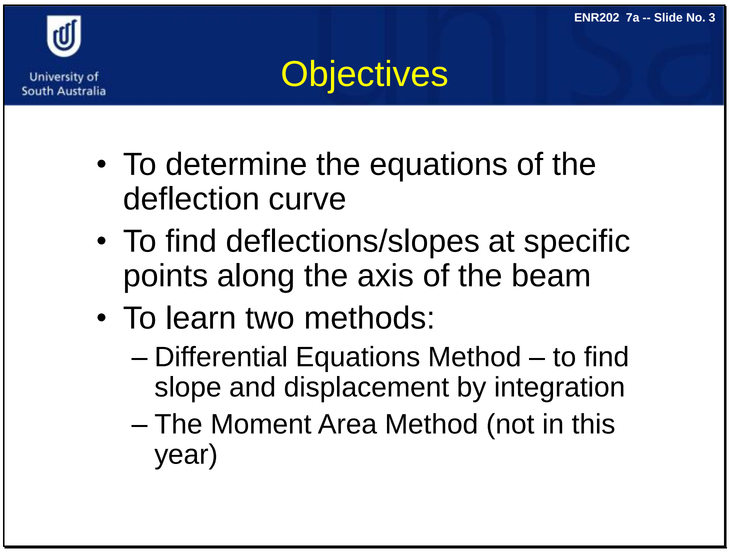

We will cover stiffness and deflection of beams. We will learn how to determine the equations of the deflection curve, and find deflection and slope at specific points along the axis of the beam. There are two methods to find slope and deflection along the axis of the beam: the differential equations method and the moment area method. In this lecture, we will learn the differential equations method.

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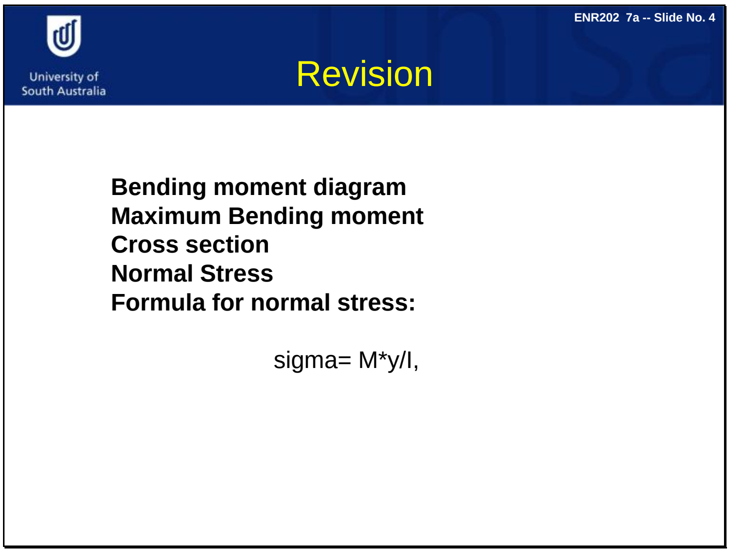

Let's start by looking back at what we have covered so far. We have learnt how to work with a beam subjected to lateral loads. We can draw the bending moment diagram, and based on the bending moment diagram, we can find the maximum bending moment. Consider the cross section where the maximum bending moment occurs. If this cross section is subjected to maximum bending moment, we know that there is compression at the top and tension at the bottom in a simply supported beam if the vertical load is acting downwards. We know how normal stress is distributed in a linear function, we might have a neutral axis at the centroid of the cross section. We have zero normal stress at the neutral axis. We have the top in compressive normal stress, and the bottom in tension normal stress. We can calculate these normal stresses by using the formula that states that sigma equals M times y divided by I. M stands for bending moment, y is the distance from the neutral axis to the point for which we want to know the normal stress, I is moment of inertia. So, we need to know M, the moment from the bending moment diagram, and we need to know I, the moment of inertia of cross section, and we need to know the y distance to calculate normal stress distribution along the cross section. If we want to calculate stress at one point in the cross section, we need to know the distance between the neutral axis and the point where we want to calculate stress. Once we know M, y and I, we can calculate the normal stress distribution, to deal with the safety issue of stress.

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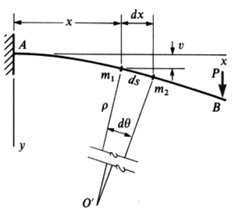
The Deflection Curve



(a)

- When a beam with a straight longitudinal axis is loaded by lateral forces, the axis is deformed into a curve, called the ***deflection curve*** of the beam
- We used the curvature of the bent beam to determine the normal strains and stresses in the beam in lecture 5a.

If we have a straight beam and apply lateral loads on the beam so the straight axis of the beam deforms into a curve as shown in the figure, we have a deflection curve. We have to calculate the deflection function of this curve. First, we calculate the curvature of the bending beam to determine the normal stress, normal strain and deflection of beam.

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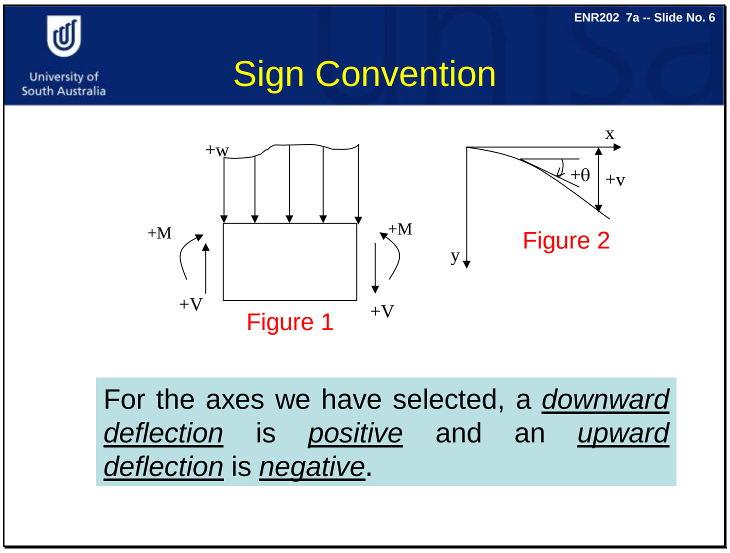

We follow the sign conventions. For example, we have moment causing compression at the top, and the tension at the bottom is positive moment. We have a positive shear force that the beam segment rotates in a clock-wise direction. That means the right side shear force is in a downward direction, and the left side shear force is in an upward direction as shown in figure 1. With regard to deflection, if we apply the load vertical down, the deflection is also downward, so the downward deflection is positive, and of course the upward deflection is negative. Here, we use ‘v’ to denote deflection of beams and theta to denote the positive slope of the beam, as shown in the figure 2.

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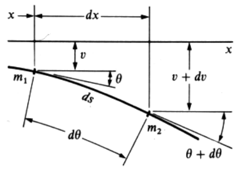
The Deflection



The ***deflection*** **v** of the beam at any point m_1 at distance x from the origin is the translation (or displacement) of that point in the y direction, measured from the x axis to the deflection curve.

Consider the infinitesimal beam length 'dx', which is the distance between Point 'm1' and Point 'm2', as shown in the figure. The deflection 'v' of the beam at Point m_1 at a distance 'x' from the origin is the translational displacement of that point in the y direction. The deflection 'v plus dv' of the beam at Point m2 at a distance of 'x plus dx' from the origin is the translational displacement of that point in the y direction as shown in the figure.

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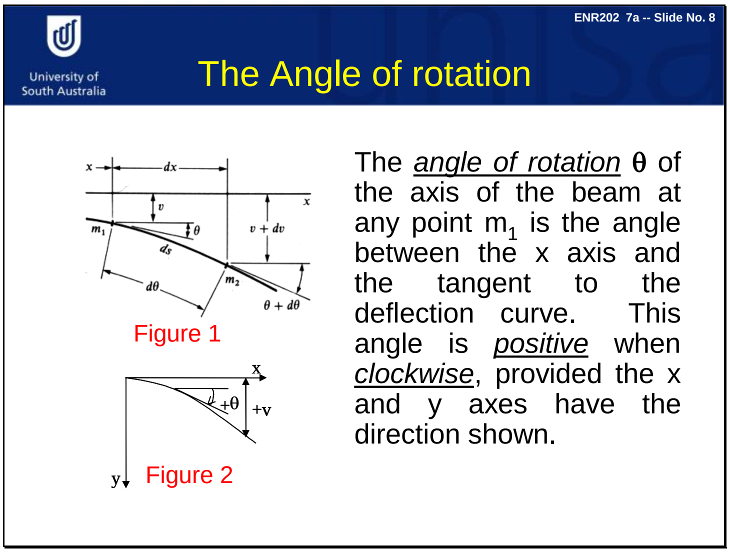

We can also have rotational deflection or slope. For example, at Point ‘m1’ the angle or slope is between the x-axis and the tangent of deflection curve is theta, as shown in the figure 1. At Point ‘m2’, the angle or slope is ‘theta plus dtheta’, as shown in figure 1. Theta is the angle displacement or the angle of rotation or the slope of the beam. So we have two deflections; one is just a vertical displacement, and the other is the slope of the beam which means a change of the angle. Again we can say that this kind of angle is positive. That means that the angle is in a clock-wise direction, as shown in figure 2.

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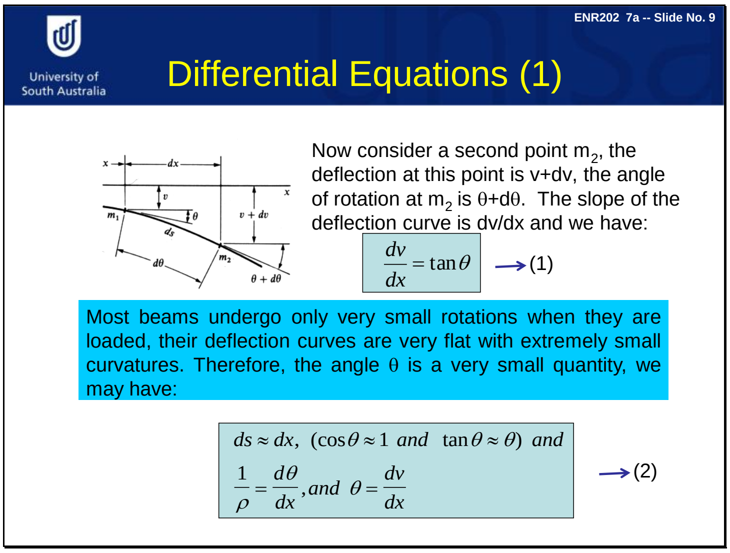

For differential equations, we can say that the angle is theta. To calculate deflection, the relationship between the slope and deflection is ‘dv’ divided by ‘dx’, which is equal to tan theta. Most beams undergo only very small rotations when they are loaded. Their deflection curves are very flat with extremely small curvatures. Therefore, the angle theta is a very small quantity. We know that the curve can be worked out by dx equals ‘rho’ times ‘d theta’. When we look at the normal stress formula, curved distance ds equals ‘rho’ times ‘dtheta’. ‘Rho’ is the radius of curvature. If ds equals dx, 1 divided by ‘rho’ equals ‘d theta’ divided by ‘dx’. Theta is equal to ‘dv’ divided by ‘dx’ .

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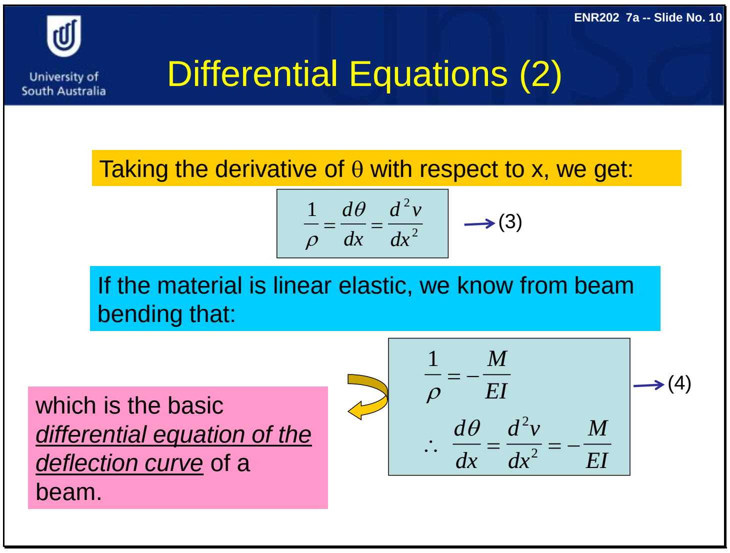

Finally, we can work out the relationship between curvature ‘rho’ , slope ‘theta’, and deflection ‘v’ as shown in Equation 3. We know from bending theory the relationship between curvature and moment M, Modulus of elasticity E, and moment of inertia I, as in Equation 4. So we can calculate the differential equation of deflection curve of the beam.

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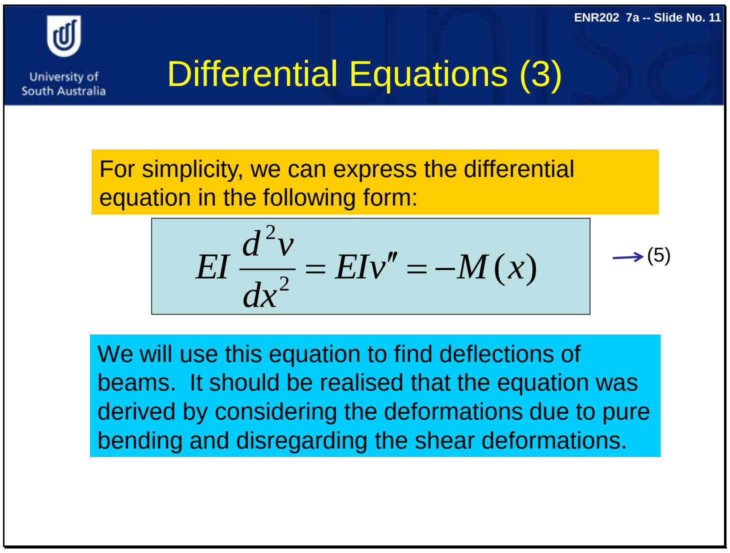

This equation gives us the relationship between the bending moment function and deflection. That means that deflection depends on bending moment. By solving this differential equation, you can find out the deflection function.

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Slope/Displacement by Integration

The equation of the deflection curve in terms of the bending moment may be integrated to obtain the deflection v as a function of x. Since the differential equation is of second order, two integrations are required.

$$EI\frac{d^2v}{dx^2} = EIv'' = -M(x)$$

We can use an integration method to solve this equation. We just double the integrate of the differential equation to get the deflection function 'v'.

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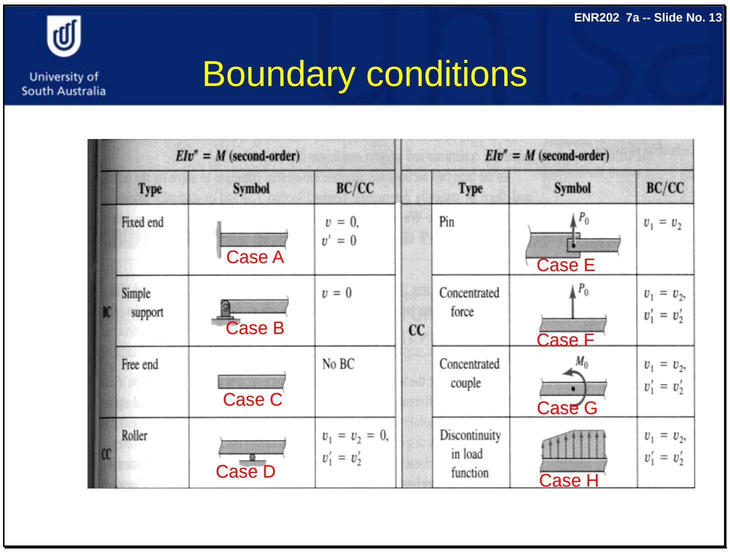

It is important to remember that boundary condition means the displacement condition at the ends of the beam. That means at the supports. Suppose we have a cantilever beam, with zero displacement and zero slope at the fixed end in Case A. If we have a simply supported beam, we have only one boundary condition, which is deflection is zero, as in Case B. If we have a free end, we don't have any boundary condition, as in Case C. If we have an internal roller, the deflection at the left part will be equal to the deflection at the right part which is equal to zero, and the slope at the left part will be equal to the slope at the right part, as in Case D. We have other continuity boundary conditions from Case E to Case H. However, it is the first two types of boundary conditions that are very important to remember: this is, the fixed end and the simply supported end.

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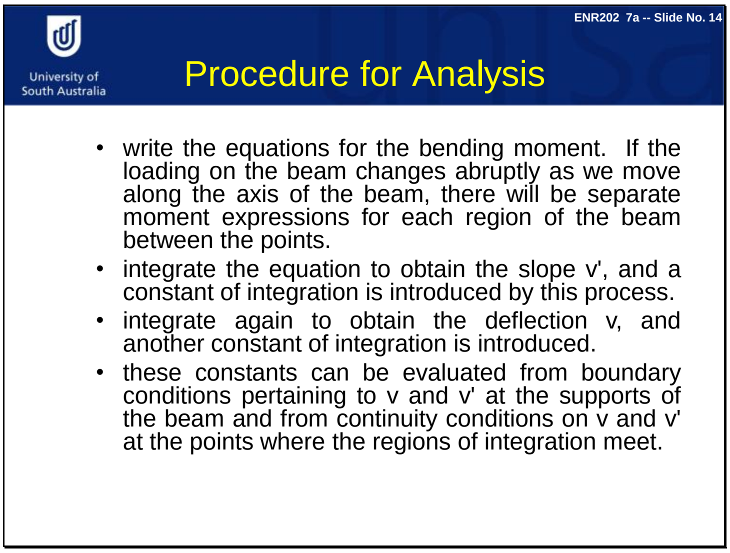

The general procedure for analysis is: firstly, you need to write the equations for the bending moment. Secondly, you have to integrate the bending equation to obtain the slope (that is, the first derivative of deflection and constant of integration). Thirdly, you integrate again, then you get the deflection and one more constant of integration. Fourthly, based on boundary conditions, you can solve the unknowns of constants of integrations. Finally, you get the whole deflection function.

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Thank you for your attention. In the next lecture we will study some examples of deflection of beams.