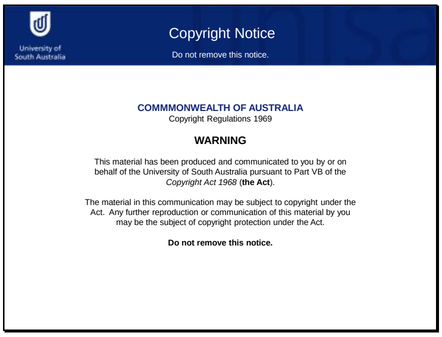
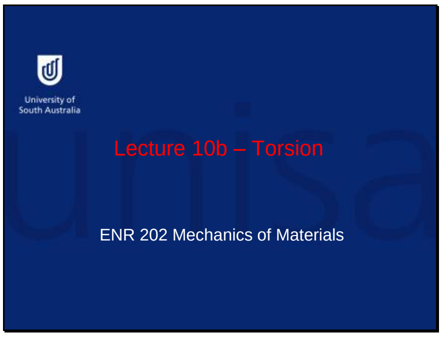
ENR202 Mechanics of Materials Lecture 10B Slides and Notes

Slide 1



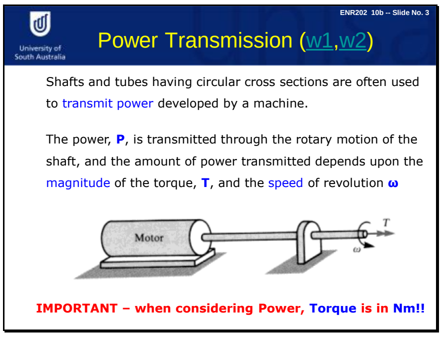
Hello, and welcome to lecture summary 10b. In this lecture, we are going to continue to discuss torsion.

Slide 2



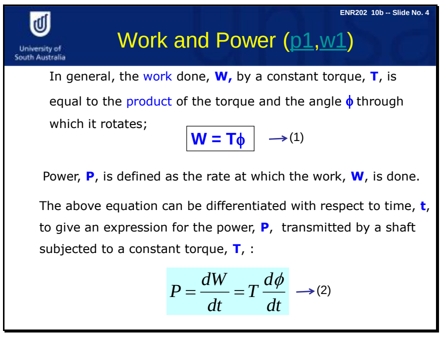
In the last lecture summary, we looked at a number of concepts, including force couples, torque, angle of twist, torsional shear strain, torsional shear stress, the torsion formula, multiple angles of twist and torque sign convention. Now we're going to continue to study torsion. In this lecture summary, we will look at power transmission, and shear flow concepts in thin walled cross section members. Click on the live links for further information.

Slide 3



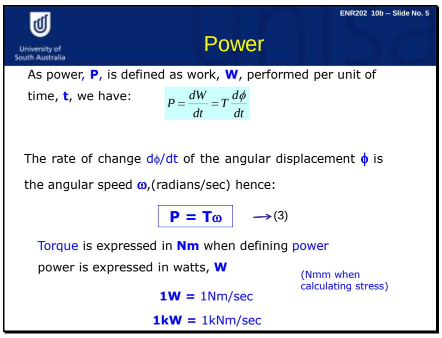
Solid or hollow circular cross section shafts are used to transmit power which is developed by a machine. As illustrated in the diagram, the motor produces power, and this power is transmitted by the solid shaft, through the rotary motion of the shaft. The amount of power which is actually transmitted depends upon the magnitude of the torque and the speed of revolution. We use the symbols P for power, T for torque, and small omega for the speed of revolution, and convert the torque units into Newton meters.

Slide 4



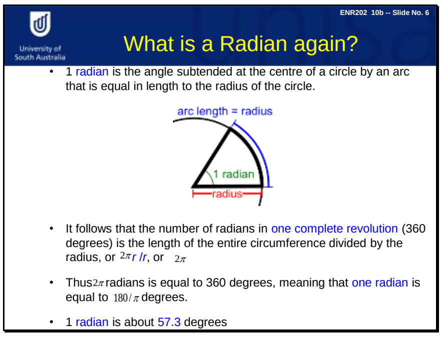
The work done, ‘w’, is defined as product of torque “T” and the angle of twist, ‘FI’, through which it rotates, as per equation 1. Here, the torque should be constant torque. The power, ‘p’, is defined as the rate at which the work is done. You can differentiate the work done with respect to time to get the expression for the power transmitted by a shaft subjected to a constant torque, Power is equal to torque times the differentiation of angle of twist with respect to time as shown in equation 2.

Slide 5



The rate of change of the angle of twist is just angular speed. Remember that the units for angle of twist are radians, and the units for angle of speed are radians per second. Also remember that the units for torque are Newton meters. Therefore, the power units are Newton meters per second, if the power in Watts. The power units are kilo Newton meters per second if the power in kilo Watts. Power is the product of torque and angle of speed, as per equation 3.

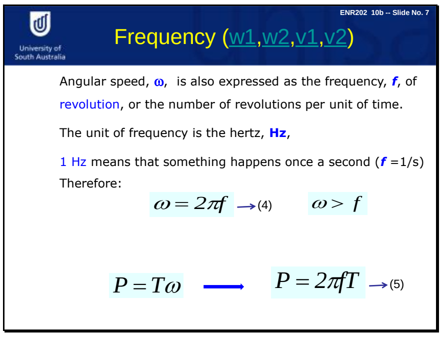
Slide 6



What is a radian? One radian is the angle subtended at the centre of a circle by an arc that is equal in length to the radius of the circle, as shown in the figure. The perimeter of the circle is 2 times pi times the radius of the circle. To work out the number of radians for the perimeter of circle, we use the equation 2 times pi times the radius divided by the radius, which is equal to 2 times pi radians. There are 360 degrees for one complete revolution of a circle, so 360 degrees equal to 2 times pi radians.

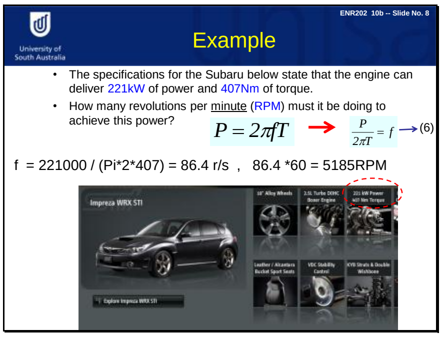
So, one radian is equal to 180 divided by pi degrees. In fact, one radian is equal to 57.3 degrees.

Slide 7



Angular speed, 'omega', is also expressed in the frequency of revolutions or number of revolutions per unit of time. The unit of frequency is hertz. One hertz means that something happens once a second. Therefore, omega is equal to 2 times pi times frequency. That means that omega is always greater than the frequency, as in equation 4. We already know that one revolution is 360 degrees (that is, 2 times pi radians). We know that power is the product of torque and angular speed. If you substitute equation 4, then you can work out that power is equal to 2 times pi times frequency times torque, as in equation 5.

Slide 8

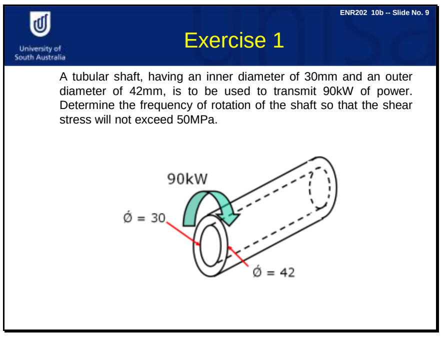


Let's look at a real example. The specifications for the Subaru car engine delive 221 kilo Watts of power and 407 Newton meters of torque. How many revolutions per minute the engine must be doing to achieve this power?

First of all, we need to calculate frequency. We know that power equals 2 times pi times frequency times Torque. Because we already know the power and torque, we can calculate the frequency from equation 6 above. Here the power units are kilo Watts, which we need to convert into Watts, because Watts means Newton meter per second and the given torque unit is Newton meters. If you convert the power into Watts, the power is equal to 221 000 Watts. Substitute power and torque in equation 6, to get a frequency of 86.4 revolutions per second. We need to calculate the revolutions per minute (RPM).

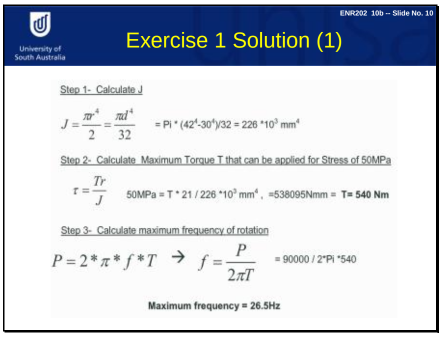
Therefore, the RPM is 86.4 times 60, which is 5185 RPM.

Slide 9



A tubular shaft has an inner diameter of 30 mm and an outer diameter of 42 mm, and needs to transmit 90 kilo Watts of power produced by an engine or motor. Pause the presentation here and find the frequency of rotation of the shaft so that the shear stress will not exceed 50 Mega Pascals. The solution is on the next two slides.

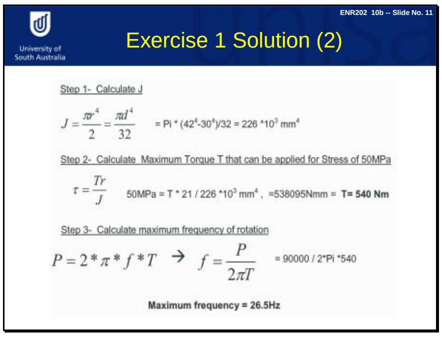
Slide 10



A tubular shaft has an inner diameter of 30 mm and an outer diameter of 42 mm, and needs to transmit 90 kilo Watts of power produced by an engine or motor. The shear stress will not exceed 50 Mega Pascals. You need to find the frequency in terms of Hertz. To solve this problem, you need the torsion formula from Lecture 10a', and the power in terms of frequency formula.

Also remember that you have to be very careful with the units. Decide on the unit before starting to solve the problem. Shear stress is in Mega Pascals, which means Newtons per square mm, so convert all the other units. The radius will be in mm, the polar moment of inertia in mm to the power of 4, and the torque in Newton mm.

Slide 11



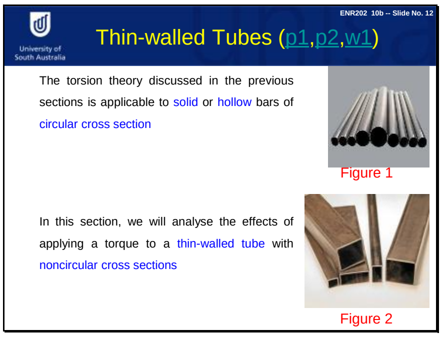
First, you need to calculate the polar moment of inertia for the tubular shaft (42 mm outer diameter and 30 mm inner diameter). The polar moment of inertia is pi times 42 mm to the power of 4 minus 32 mm to the power of 4 divided by 32, which is 226 times 10 to the power of 3 mm to the power of 4.

Secondly, you need to calculate the maximum torque, 'T', that can be applied without exceeding the shear stress of 50 Mega Pascals. The torsion formula states that the maximum shear stress is equal to Torque times the outer radius divided by the polar moment of inertia. We have just calculated the polar moment of inertia, which is 226 times 10 to the power of 3 mm to the power of 4. The outer radius is the outer diameter divided by 2, which is 21 mm, and the given shear stress is 50 Mega Pascals. You can calculate the torque by using the torsion formula, and you should get 538095 Newton mm. Convert this torque into Newton meters and you get 540 Newton meters.

For the third step, you need to

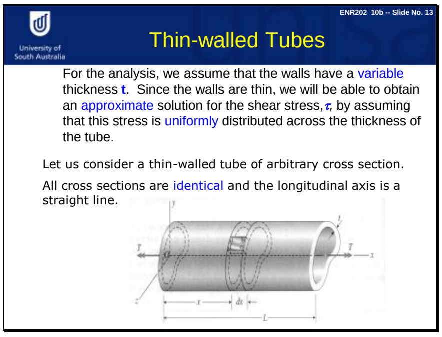
calculate the maximum frequency of rotation, by using the power formula. The given power is 90 kilo Watts, which is 90 Kilo Newton meters per second. Convert the power unit from kilo Watts into Watts, and you should get that the power is 90 000 Watts. 1 Watt is 1 Newton meter per second. The power is equal to 2 times pi times the frequency times the torque. The torque is 540 Newton meters. Now we can calculate the frequency using the power formula. The frequency is the power divided by 2 times pi times the torque. Finally, we can calculate the maximum frequency as 26.5 hertz, which means 26.5 revolutions per second.

Slide 12



So far, you have learnt how torsion applies to solid or hollow circular bars, as shown in figure 1. Now, we will analyze the effect of applying a torque to a thin-walled tube with non-circular cross sections, as in figure 2.

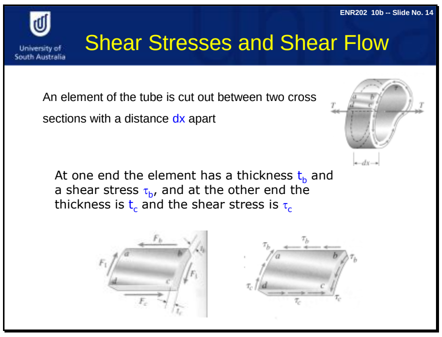
Slide 13



In thin-walled tubes analysis, we assume that the thickness of wall is variable in cross section, and we assume that it is very small. We can obtain an approximate solution for the shear stress because we assume that this shear stress is uniformly distributed across the thickness of the tube.

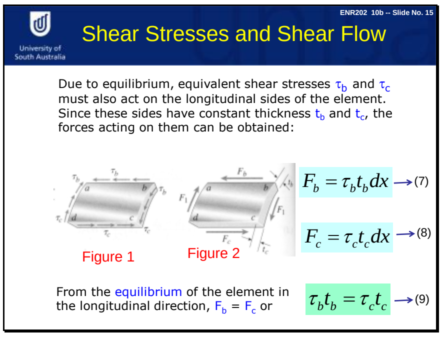
Let us consider a think-walled tube of arbitrary cross section. All the cross sections are identical throughout the longitudinal axis, and this axis is a straight line, as shown in the figure. Let us consider a small segment length ‘dx’ at a distance ‘x’ from the left end. We consider a small strip a-b-c-d. The x-axis lies in a longitudinal axis, the y and z axes lie at the cross section and torque is applied at the cross section.

Slide 14



Suppose you cut the tube between two cross sections with a distance of ‘dx’ as shown in the figure. At one end of the element, ‘abcd’ has a thickness ‘tb’ and a shear stress ‘tou b’. At the other end the thickness is ‘tc’ and the shear stress is ‘tou c’, as shown in the figure.

Slide 15

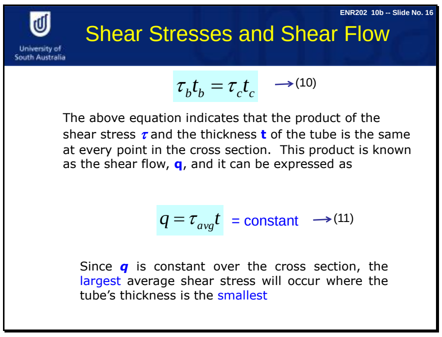


Now consider the small strip a-b-c-d. As you know, if a structure is in equilibrium, all parts of the structure (even joints) are also in equilibrium. Therefore, if the thin-walled tube is in equilibrium, the a-b-c-d part of the thin walled tube is also in equilibrium. This means that you can apply equilibrium equations to this part. As you know, it you apply torque, the structure will deform, and you will get torsional strain and torsional shear stress.

At cutting sides ab-bc-cd-da, we expose the shear stresses as shown in figure 1. We assume that the shear stress is uniform at the thin wall and the ad-bc sides are identical. That means that the thickness at point b is equal to the thickness at point a, and the thickness at point c is equal to the thickness at point d. The thickness at b is not equal to the thickness at c because we assumed variable thickness at the cross section. Therefore, the shear stress at side ab, at point b and at point a are equal to the shear stress ‘b’ as shown in figure 1. The shear stress at side cd, at point c and at point d are equal to shear stress ‘c’, as shown in figure 1. The uniform thickness from point a to point b is ‘tb’ and from point c to point d is ‘tc’.

As you know, stress is equal to force divided by area. Now we can calculate the force ‘Fb’ at side *ab* as being equal to shear stress *b* times thickness *b* times length ‘*dx*’, as shown in equation 7. We can calculate force ‘Fc’ at side *cd* as being equal to shear stress *c* times thickness *c* times length ‘*dx*’, as shown in equation 8. If we apply the equilibrium equation which states that all forces in a horizontal direction are equal to zero, we get Fb minus Fc is equal to zero.   Finally, we get equation 9.

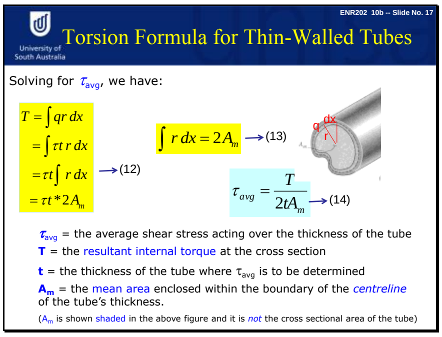
Slide 16



Equation 10 shows that the product of the shear stress and the thickness of the tube is the same at every point in the cross section. This product is known as the shear flow.

We know that shear stress is approximate, because we assume that this shear stress is uniformly distributed across the thickness of the tube. Therefore, the product of approximate shear stress and the thickness of wall is called shear flow and denoted as ‘q’. ‘q’ is constant over the cross section. The largest average shear stress will occur where the tube’s thickness is the smallest and vice versa.

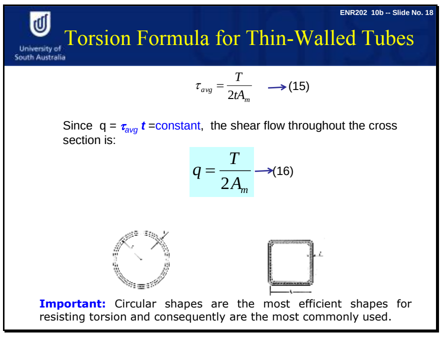
Slide 17



Based on the shear flow concept, we get torque. As we have discussed, torque is just force multiplied by radius. We are calculating the torque at the center of the cross sectional member. Torque is just the integration of the product of shear flow and radius, as per equation 12. Shear flow is just the product of shear stress and thickness, and is constant for a particular cross section. Then integration of radius is 2 times the mean area enclosed within the boundary of the centre line of the tube's thickness, as in equation 13. The area of the triangle as shown in the figure is the radius multiplied by 'dx' divided by 2, which is Am. That means that the integration of rdx is just 2 times Am. 'r' times 'dx' is a rectangular area; however, we have a triangle. R times 'dx' divided by 2 is the triangle area, which is Am. Rearranging the terms of equation 12 givew equation 14.

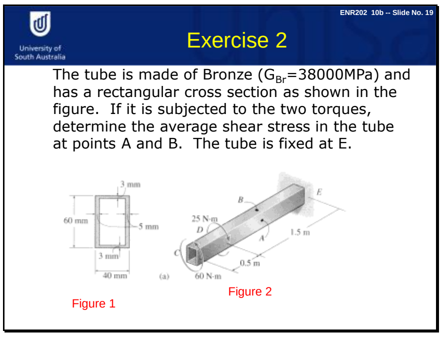
The notations in the final equation 14 are very important. We assume that we are calculating the approximate solution for shear stress in thin-walled tubes, so we use the symbol tou avg. T is the resultant internal torque at the cross section, t is the thickness of the tube where tou avg is to be determined, because the thickness is not constant for the whole cross section. The thickness varies at every point on the cross section. Make sure that you understand what the symbol Am means, because it is easy to calculate incorrectly. Am is the mean area enclosed within the boundary of the centreline of the tube's thickness. It is not the cross sectional area of the tube.

Slide 18



Average shear stress is equal to T divided by 2 times thickness times Am, as in equation 15. The product of average shear stress and thickness is called shear flow and is also constant throughout the cross section. Finally, shear flow q is equal to Torque divided by 2 times Am, as shown in equation 16. Note that circular shapes are the most efficient shapes for resisting torsion, and, consequently, are used more than non-circular cross section members.

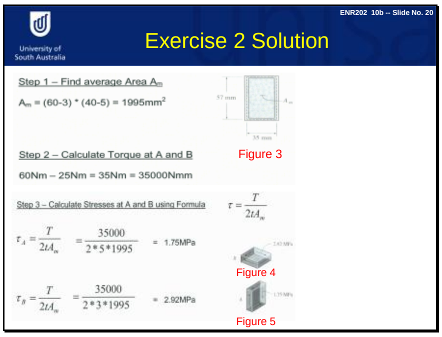
Slide 19



A rectangular cross section bronze tube with shear modulus of 38 000 Mega Pascals is subjected to one torque of 60 Newton meters at cross section C, and one of 25 Newton meters at cross section D. It is fixed at cross section E, as shown in figure 2. You need to calculate the average shear stress in the tube at points A and B, as shown in figure 2. We have different thicknesses for the rectangular cross section: 40 mm wide and 60 mm deep at the cross section, 5 mm thick at left and right and 3 mm thick at top and bottom, as shown in figure 1. The best way to start is to calculate Am, which is the mean area enclosed within the boundary of the *centreline* of the rectangular cross section.

Pause the presentation now and try to do this problem. You can check your answer in the next slide.

Slide 20

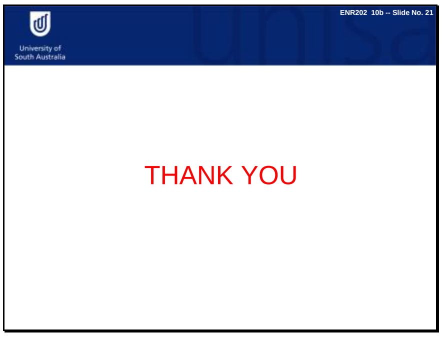


First, calculate Am. The depth of the cross section is 60 mm. From centre line to centre line along the depth is 60 mm minus 3 mm which is 57 mm. From centre line to centre line along the width is 40 mm minus 5 mm, which is 35 mm, as shown in figure 3. Am is 57 multiplied by 35, which is 1995 square mm.

Secondly, you need to calculate the torque at points A and B. You have to cut the tube at the cross section of these points to find the torque. Draw the free body diagram and apply the equilibrium equation. We have one torque of 60 Newton meters in an anti-clockwise direction at cross section C and another torque of 25 newton meters in a clockwise direction at cross section D. Therefore if you apply the equilibrium equation, the torque at the cross section of points A and B is 60 Newton meters minus 25 Newton meters, which is 35 Newton meters in a clockwise direction. Shear flow and shear stress are in same direction of the torque - clockwise. Point A is located on the right side of the rectangular tube and Point B is located on the top side of the rectangular tube.

For the third and final step, calculate the stresses at points A and B using equation 15 from slide 17. The thickness of the tube at point A is 5 mm and at point B is 3 mm. The torque of this cross section is 35 000 Newton meters in a clockwise direction. Am is 1995 square mm. Finally, you should calculate the average torsional shear stress at point A as 35 000 divided by 2 times the thickness at point A (5 mm) times Am (1995), which is 1.75 Mega Pascals, as shown in figure 5. Calculate the average torsional shear stress at point B which is 35 000 divided by 2 times the thickness at point B (3 mm) times Am (1995), which is equal to 2.92 Mega Pascals, as shown in figure 4.

Slide 21



Well, that finishes our work on torsion. In the next lecture summary, we will start looking at combined loadings. Thank you for your attention.