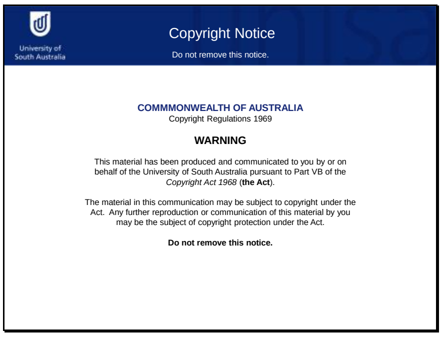
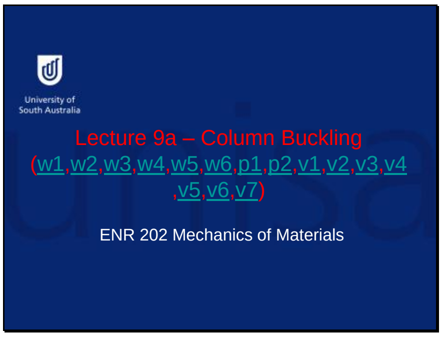
ENR202 Mechanics of Materials Lecture 9A Slides and Notes

Slide 1

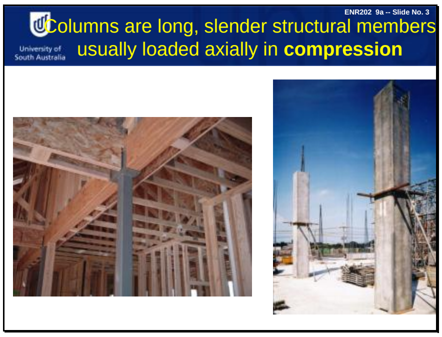


Slide 2



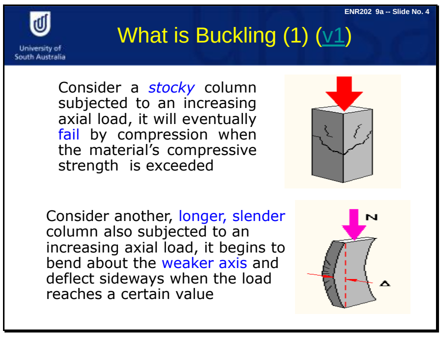
Welcome to lecture summary 9A. In this lecture we will talk about column buckling. Access the live links for further information on the concepts covered.

Slide 3



Columns are important structural members. Columns are long, slender and usually loaded axially in compression. They can be of steel (left figure) and concrete (right figure).

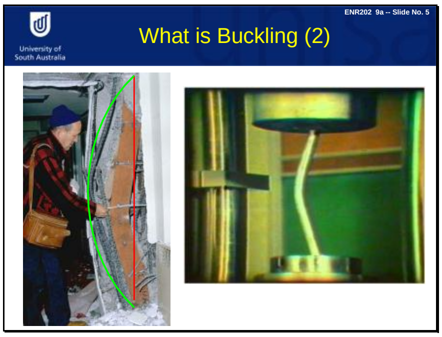
Slide 4



If compressive axial load is acting on the column, there are two possibilities for failure. The first is compressive failure. If a stocky column is subjected to an axial load, this stocky column may undergo compressive failure when the compressive stress exceeds the compressive strength, as shown in the top figure. You can see the cracks all around the column.

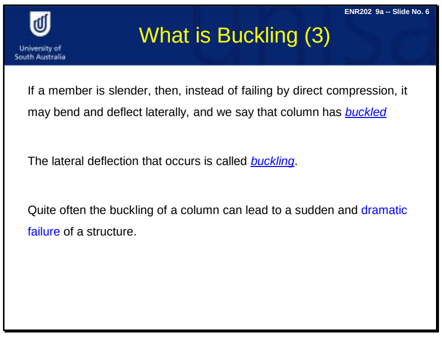
The second is buckling failure. For example, if a long, slender column is subjected to axial load, this column begins to bend about the weaker axis and deflect side ways, in lateral deflection, as shown in the bottom figure. When the load reaches a certain point, the column will fail by buckling. This load is known as the critical load of the column. The buckling failure is sudden failure. You can see the video link above in the slide title with the symbol v1, and this will show you an example of a building failing through buckling.

Slide 5



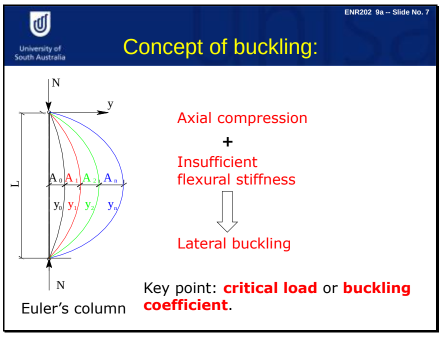
Look at the left photo and you will see that the reinforced concrete column has buckled in a lateral direction from the longitudinal axis. Observe the buckle shape of the rebar in the reinforced concrete column (green colour) and the longitudinal axis of the column (red colour). In the right image, you can see the testing of steel rod subjected to axially loaded compression, with a lateral deflected buckling shape of the column.

Slide 6



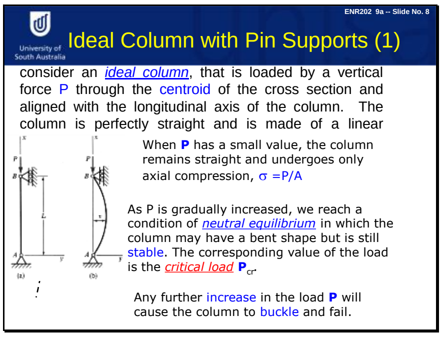
So what does buckling mean? If a member is slender, it may bend and deflect laterally about the weak moment of inertia of the cross section instead of failing by direct compression. We called this phenomenon buckling. Buckling failure is very dangerous and it can lead to sudden failure. Obviously, engineers need to understand the buckling concept very clearly.

Slide 7



The concept of buckling means that if we have an axial compression 'N' acting on the column of length 'L' with insufficient flexural stiffness, it will cause lateral buckling. In the figure, you can see larger and larger lateral deflection due to insufficient flexural stiffness when axial compressive loads are applied. Initially, the column is just slightly imperfect with the shape of yo. If you increase the compressive load, the column will buckle laterally with the y1, y2 and yn shapes as shown in the figure. Finally, at some load, the column will fail by buckling. That load is called critical load. You must calculate that critical load for given conditions of column.

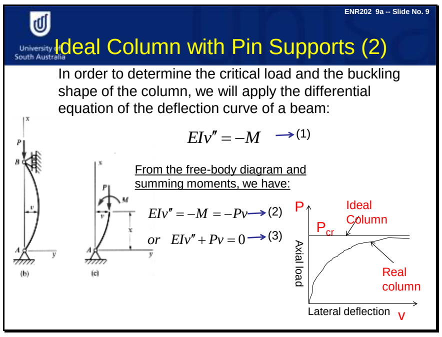
Slide 8



Suppose we have a perfectly straight column, with pinned supports at both ends, and an axial compressive load ‘P’ passes through the centroid of the cross section of the column, as shown in figure a. The column may buckle as shown in figure b. When the axial compressive load ‘P’ has a small value, the column remains straight. The axial compressive stress is equal to the axial compressive load ‘P’ divided by the area of the cross section ‘A’. We have already covered this when we talked about axially loaded members in lecture 2b. Axial compressive stress sigma is equal to P divided by A.

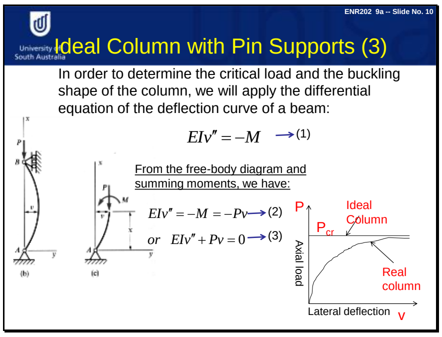
If the axially compressive load ‘P’ gradually increases, we reach a condition of neutral equilibrium, which means that the column may have a bent shape as shown in figure b, but is still stable condition. The corresponding value of the load is called the critical load. You can apply an axial compressive load up to the critical load. If you further increase load ‘P’ after the critical load, it will cause the column to fail by buckling.

Slide 9



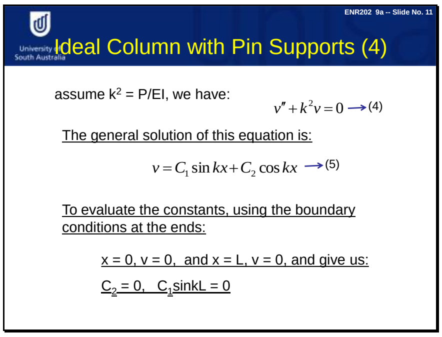
If you apply an axial compressive load ‘P’ on the column with both ends pinned, then the column may also develop a buckle shape, as shown in figure b. The lateral deflection at the centre of the column is ‘v’. If you draw the graph for the ideal column and the real column, with the application of axial compressive load and lateral deflection at centre of the column ‘v’, you can see the result. In the ideal column, the column is straight up to the critical load ‘Pcr’, then it suddenly starts to buckle and fail. In real column, the column starts buckling after some of the load, then finally fails at the critical load. An ideal column will be straight until the critical load. However, all real columns are imperfect - not straight. In reality, the relationship between the compressive load and the lateral deflection at the centre will be a curve. We look at ideal columns in the unit, but they do not really exist – this is only theory.

Slide 10



Suppose you cut the buckled shape of the column (as shown in figure b) at a distance of ‘x’ from point A (as shown in the figure c). You know that we have two internal force effects acting at a cutting cross section. One is an axial compressive load ‘P’ and bending moment ‘M’, as shown in figure c. Remember the differential equation we looked at in Slide 11 of Lecture 7a – it is equation 1 in this slide. Draw the free body diagram of the bottom part of the column (as shown in figure c). Apply equilibrium equations which state that all forces in a vertical direction equal zero, and all forces moments about point A equal zero. Finally, we get the moment 'M' at the cutting cross section equal to the axial compressive load 'P' times lateral deflection 'v', as shown in equation 2. Rearrange the terms and you will get equation 3.

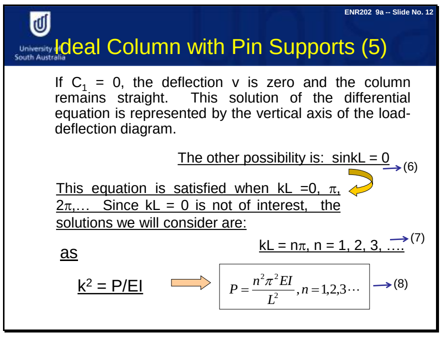
Slide 11



Finally, we have a differential equation in terms of Young's modulus 'E', moment of inertia 'I', axial load 'P', and lateral deflection. K squared equals P divided by EI. Thus, the governing differential equation becomes equation 4. The general solution of equation 4 is that v is equal to one constant of integration sin kx plus another constant of integration cos kx, as shown in the equation 5.

To evaluate the constants of integration, we use boundary conditions at the ends. We have two pinned ends for this ideal column. Point A is at the bottom pinned end, and Point B is at the top pinned end, as shown in the previous slide. The 'x' distance starts from the bottom end as shown in the previous slide. If x is equal to zero, lateral deflection 'v' is equal to zero (that is, the bottom end point A of the column). If x is equal to the length of the column 'L', the lateral deflection 'v' is also zero (that is, top end point B of the column). If you substitute these conditions into equation 5, we get constants of equations C2 equal to zero and C1 sin kL equal to zero.

Slide 12

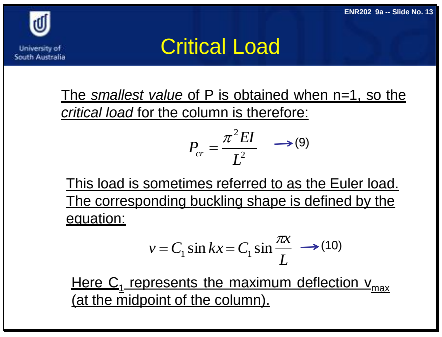


If both constants of integrations C1 and C2 are equal to zero that means the lateral deflection 'v' is also zero, which means the column is straight. However we are not looking for the solution of a straight column because the lateral deflection is throughout the column. We are looking for a lateral buckled shape of the column. Therefore for one condition C2 equals zero, but for the other condition C1 sin kL equals zero. For second condition we are not looking for C1 also being equal to zero, we are looking for sin kL being equal to zero. Let's see what happens if sin kL equals zero.

The equation sin kL being equal to zero is satisfied when kL is equal to zero, pi, 2 times pi and so on. We are not interested in kL being equal to zero. Therefore the solution of equation 6 becomes kL is equal to n times pi, where n equal to 1,2,3,4,5, and so on.

We assume that k squared equals P divided by EI. Substitute this into equation 7 and calculate the P equal to n squared pi squared EI divided by L squared, as shown in equation 8, where n equal to 1,2,3,4,5,6 and so on.

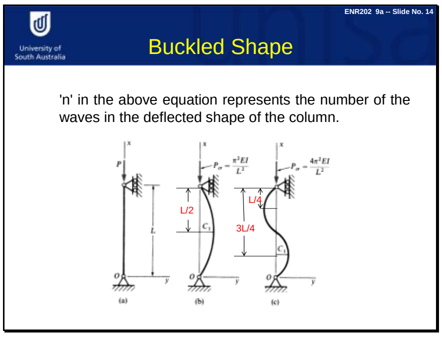
Slide 13



The smallest value of P is calculated if you substitute n equal to 1 into equation 8 in the previous slide. You will get equation 9. This load is the critical buckling load for the column. Equation 9 is very important to remember when you are calculating the buckling load of the column with pinned end conditions.

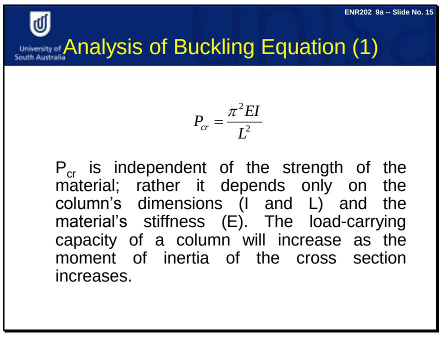
This critical load is sometimes referred to as the Euler load. The corresponding buckling shape is defined by equation 10. Substitute the value of x into the equation, and you will get the lateral deflection at the point where you are interested along the length of the column. For example, if you substitute x equal to zero, you will get v equal to zero. If you substitute x equal to L, then the other end v will be equal to zero. If you substitute x equal to L divided by 2, this will be at the centre of the column. You will get maximum lateral deflection 'vmax'.

Slide 14



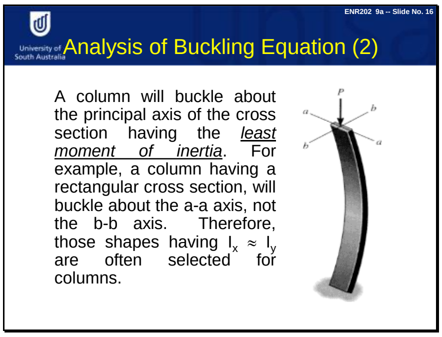
Up to now, we discussed the equation if n equals 1. 'n' in the above equation represents the number of the waves in the deflected shape of the column, if you apply axial compressive load 'P' on the column as shown in figure a. The critical load is equal to pi squared times E times I divided by L squared, required to fail the column by buckling with single wave in the deflected shape of the column as shown in figure b. The maximum deflection occurs at the centre of the column as we expected as shown in figure b. What happens if n equals 2? If you keep maximum deflection at the centre of the column equal to zero as shown in figure c, then we have two waves in the deflected shape of the column. We need four times the critical load with a single wave deflected shape. That means n is equal to 2. We have maximum deflection for the two wave deflected shape at one quarter and three quarter lengths in the column as shown in the figure.

Slide 15



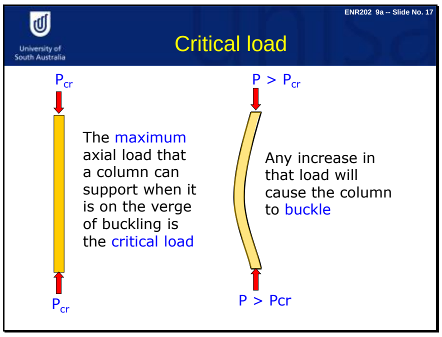
The critical buckling load as shown in the equation is independent of the strength of the material. It depends only on the column dimensions, which are the moment of inertia, the length of the column and Young's modulus of the material. (If you need to, go back and have a look at Slide 15 in lecture 2a.)

Slide 16



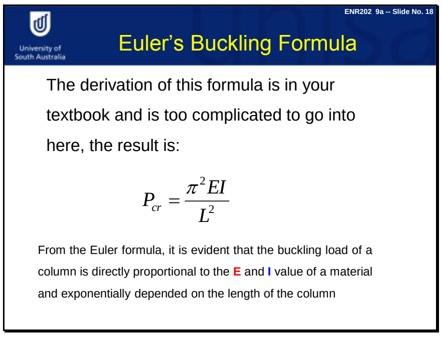
Look at the figure once more. What do you observe? At what moment of inertia does the column buckle. Here, the moment of inertia about a-a is less than the moment of inertia about b-b. Therefore the weaker axis is a-a, as shown in the figure. So, the column buckled at the least moment of inertia. Therefore we usually design a column in such a way that both moments of inertia are almost the same.

Slide 17



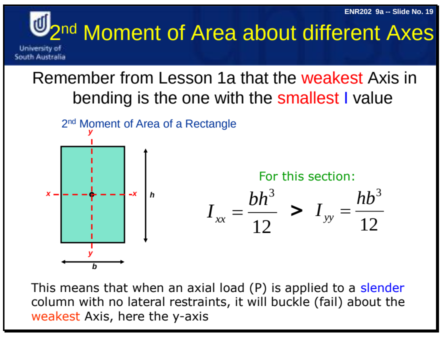
In an ideal column, the maximum axial compressive load that a column can support when it is on the verge of buckling is the critical load. If the applied load is larger than critical buckling load, it will cause the column to buckle, and then collapse.

Slide 18



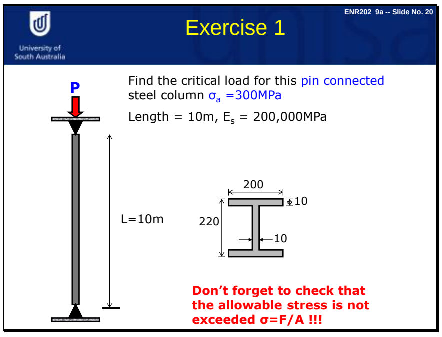
We already looked at derivation of Euler's Buckling formula. (If you are unsure, check the derivation of this formula in the text book Mechanics of Materials, chapter 13, pp 696). However, you need to understand the concept of this formula. The buckling load of a column is directly proportional to the E and I values and exponentially depended on the length of the column.

Slide 19



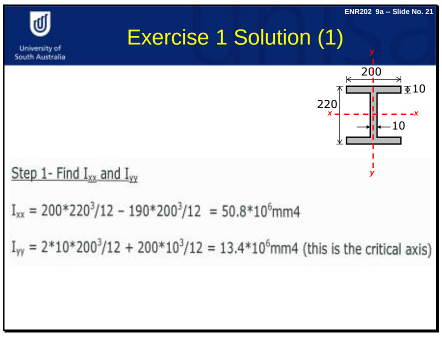
We are looking at the buckling phenomenon about the weakest axis, the one with the smallest moment of inertia, I value. An example is a rectangular cross section, if the height of the rectangle is greater than the breadth of the rectangle as shown in the figure. The moment of inertia about the x-x axis is equal to b times h cubed divided by 12 and the moment of inertia about the y-y axis is equal to h times b cubed divided by 12. Therefore, the moment of inertia about the x-x axis is greater than the moment of inertia about the y-y axis. This means that when an axial compressive load ‘P’ is applied to a slender column with no lateral restraints, the column will fail or buckle about the weakest axis, which here is the y-y axis. So, the y-y axis is the weakest axis.

Slide 20



Here we have an I shaped cross section steel column with both ends pinned, which is subjected to an axial compressive load ‘P’ acting on the column. The length of the column is 10 meters. Youngs modulus of the steel material is 200 thousands Mega Pascals. ‘The allowable stress or yield stress of the material is 300 Mega Pascals. Don’t forget to check that the allowable stress for this column (critical load divided by the area of the I shaped cross section) does not exceed the allowable stress of the material, which is 300 Mega Pascals. Pause the presentation here and try to complete the problem. The solution is on the following three slides.

Slide 21

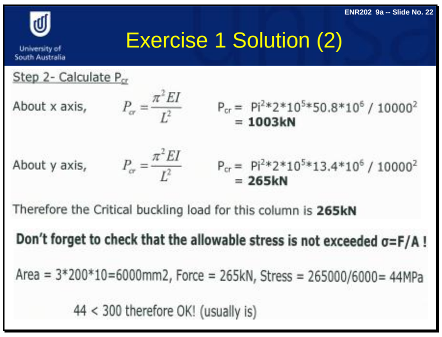


First, find the moment of inertia about the x-x axis and the moment of inertia about the y-y axis. In Lecture 5A, we looked at how to calculate the moment of inertia for an I shape. The I shape in the figure is symmetrical in the x-x and the y-y axis. Therefore, there is no need to use parallel axis theorem to calculate moment of inertia. You can use the negative area method to calculate the moment of inertia about the x-x axis. The dimensions of the large rectangle are 200 mm width and 220 mm depth. There are two small rectangles of 95 mm width and 200 mm depth. The moment of inertia of the large rectangle is 200 times 220 cubed divided by 12 minus 2 times 95 times 200 cubed divided by 12, which is equal to 50.8 times 10 to the power of 6 mm to the power of 4.

Now we can calculate the moment of inertia about the y-y axis. The centre of flanges and web passes through the y-y axis and is symmetrical about the y-y axis. Therefore, the moment of inertia for the is flanges equal to 2 times 10 times 200 cubed divided by 12 plus the moment of inertia about the web, which is 200 times 10 cubed divided by 12. You should get the moment of inertia about the y-y axis as equal to 13.4 times 10 to the power of 6 mm to the power of 4.

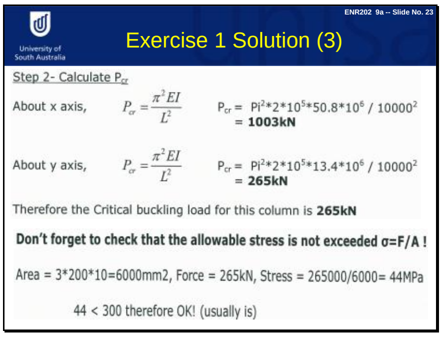
As we discussed earlier, you need to find weakest axis that is the smallest I value. Here the moment of inertia about the y-y axis is smaller than the moment of inertia about the x-x axis. Therefore, the critical axis moment of inertia is 13.4 times 10 to the power of 6 mm to the power of 4.

Slide 22



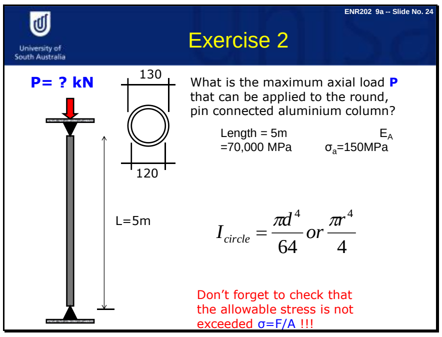
The second step is to find the critical buckling load for both the x-x and y-y axes. First, we calculate the critical buckling load about the x-x axis which is equal to pi squared E times the moment of inertia about the x-x axis divided by L squared. Young''s modulus of steel is 200 thousand Mega Pascals, and the moment of inertia about the x-x axis is 50.8 times 10 to the power of 6 mm to the power of 4. The length of the column is 10 meters. Thus, we get the critical buckling load about the x-x axis as equal to 1003 kilo Newtons. Now we calculate the critical buckling load about the y-y axis, which is equal to pi squared E times the moment of inertia about the y-y axis divided by L squared. Young's modulus of steel is 200 thousands Mega Pascals, the moment of inertia about the y-y axis is 13.4 times 10 to the power of 6 mm to the power of 4. The length of the column is 10 meters. Thus, we get a critical buckling load about the y-y axis of 265 kilo Newtons. Therefore, the smallest of the two critical buckling loads is the critical buckling load for this column which is 265 kilo Newtons.

Slide 23



Don't forget to check that the allowable stress for this column (critical load divided by the area of the I shaped cross section) does not exceed the allowable stress of the material which is 300 Mega Pascals. The critical load is 265 kilo Newtons. The area of the I-shaped column is equal to three rectangles with 200 mm width and 10 mm depth, which is equal to 3 times 200 times 10 which is 6000 square mm. The allowable stress for this column is 265 000 Newtons divided by 6000 square mm, which is equal to 44 Mega Pascals or Newtons per square mm. The allowable stress of the steel is 300 Mega Pascals. Therefore, allowable stress for this column is the critical load divided by the area of the I-shaped cross section, which must not exceed the allowable stress of the material - 300 Mega Pascals. And it doesn't.

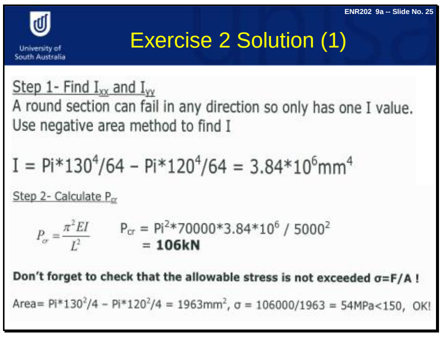
Slide 24



A circular hollow cross section aluminium column with both ends pinned is subjected to an axial compressive load 'P' acting on the column. The length of the column is 5 meters. The Young's modulus of the Aluminium material is 70 000 Mega Pascals. The allowable stress or yield stress of the material is 150 Mega Pascals. (Don't forget to check that the allowable stress for this column does not exceed the allowable stress of the material).

The Moment of inertia of the circle is pi times the diameter of the circle to the power of 4 divided by 64. Pause this presentation and try to solve this problem. The answer is on the next two slides.

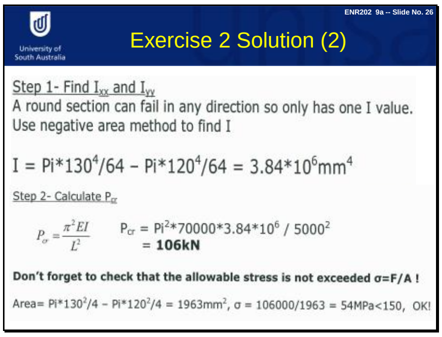
Slide 25



First, you need to find the moment of inertia about the x-x axis and the moment of inertia about the y-y axis. You already know how to calculate the moment of inertia for a circular hollow cross section. The circular hollow cross section has symmetry in both axes. Therefore, there is no need to use the parallel axis theorem to calculate the moment of inertia. You can use the negative area method to calculate the moment of inertia about the x-x axis and the y-y axis. The moment of inertia about the x-x axis is equal to the moment of inertia about the y-y axis. This circular hollow cross section has an outer diameter of 130 mm and an inner diameter of 120 mm. The moment of inertia about the x-x axis is equal to the moment of inertia about the y-y axis which is equal to pi times 130 to the power of 4 divided by 64 minus pi times 120 to the power of

4 divided by 64, which is equal to 3.84 times 10 to the power of 6 mm to the power of 4.

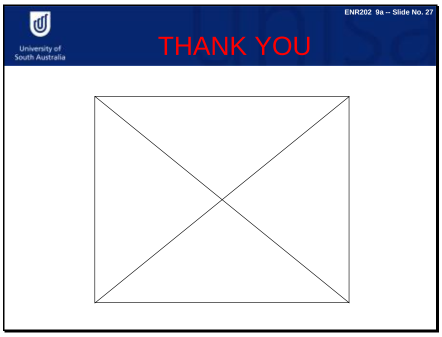
Slide 26



For the second step, you need to find the critical buckling load for this circular hollow cross section column. Now, we calculate the critical buckling load as equal to pi squared times E times the moment of inertia divided by L squared. The Young's modulus of aluminium is 70 000 Mega Pascals. The moment of inertia about the x-x axis and the y-y axis is 3.84 times 10 to the power of 6 mm to the power of 4. The length of the column is 5 meters. Therefore, we get a critical buckling load about the x-x axis and the y-axis of 106 kilo Newtons.

Don't forget to check that the allowable stress for this column (critical load divided by area of circular hollow cross section) does not exceed the allowable stress of the aluminium material, which is 150 Mega Pascals. The critical load is 106 kilo Newtons. The area of the circular hollow column is equal to pi times 130 squared divided by 4 minus pi times 120 squared divided by 4, which is equal to 1963 square mm. The allowable stress for this column is 106 000 Newtons divided by 1963 square mm, which is equal to 54 Mega Pascals or Newtons per square mm. The allowable stress of the aluminium is 150 Mega Pascals. Therefore, the allowable stress for this column does not exceed allowable stress of the material.

Slide 27



Next lecture, we will continue with column buckling, and look at columns with other support conditions. Thanks for your attention.