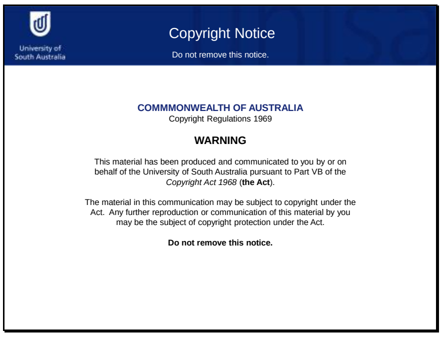
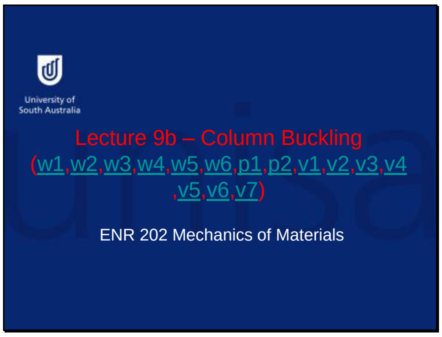
Slide 1

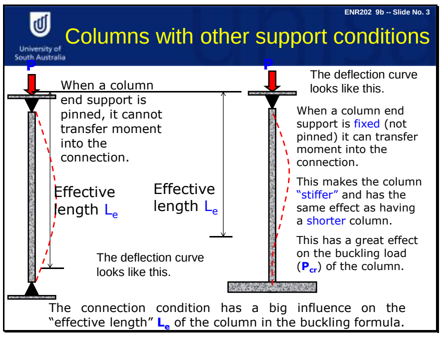


Slide 2



Hello and welcome to Lecture Summary 9b. Let's continue with the same topic as the last lecture: column buckling. Remember that internet links are given in the brackets, in case you have any problems understanding the lectures.

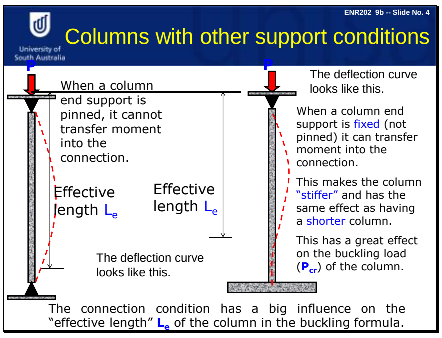
Slide 3



In this lecture, we will look at columns with other support conditions. If a column is subjected to an axial compressive load with both ends pinned, the moment will not be transferred at the pinned ends, because, as you know, pinned ends cannot carry any moment. Therefore, the effective length is equal to the total length of the column, as shown in the left figure.

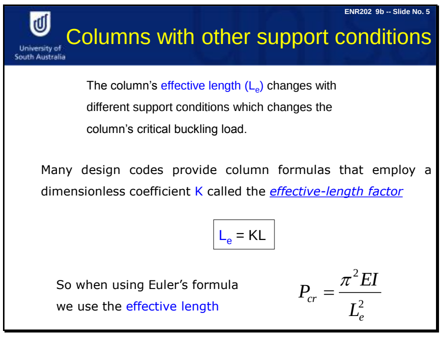
However, suppose that one end is fixed, as shown in the right side figure. You know that the fixed end transfers moment, which means the column has more stiffness in lateral deflection. Therefore, the column on the right has a more critical buckling load than the column on the left. Therefore, the right column is effectively shorter than the left column. In fact, the effective length is the wave length of sine function, as shown in the left figure. The column with both ends pinned has a column wave length which is the full length of the column. The column with one end pinned and other end fixed has a column wave length which is 0.7 times the length of the column. That means the effective length is equal to 0.7 times the length of the column.

Slide 4



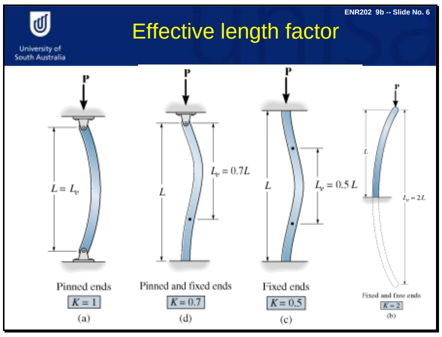
The wave length should be considered from where lateral deflection is zero to another point with zero lateral deflection, as per sine wave length definition. If a column has one end fixed and another end pinned, there is no rotation at the fixed end. In this case, the effective length is not the total length of the column. It is the distance of the sine wave length; that is, the distance from where lateral deflection starts at zero to another point where the lateral deflection is zero, as shown in the right figure. In this case, the effective length is 0.7 times the length of the column. Generally, the column will buckle in the shape of the sine wave length.

Slide 5



Therefore, we use a parameter to indicate columns with different end conditions: the effective length factor is denoted by the symbol 'k' . The effective length is equal to 'k' times the original length of the column 'L'. Now, we use Euler's formula. It becomes pi squared times E times I divided by Le squared.

Slide 6



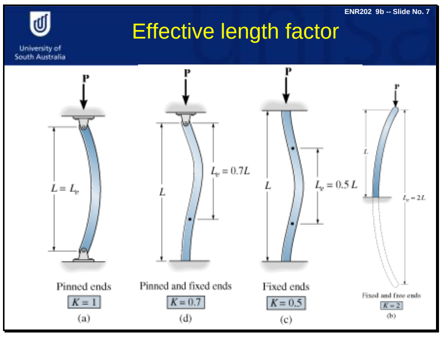
Now, let's look at the effective length factor 'k' for different end conditions. If a column has both ends pinned, the effective length is equal to the total length of the column, as shown in figure a. That means that the effective length factor is 1.

If we have a column with one fixed end and another pinned end, the effective length is 0.7 times the total length of the column, as shown in figure d. That means that the effective length factor is 0.7.

If the column has two fixed ends, the effective length is 0.5 times the total length of the column, as shown in figure c. That means that the effective length factor is 0.5.

If the column has one fixed end and one free end (this is a cantilever column), the effective length is 2 times the total length of the column, as shown in figure b. That means that the effective length factor is 2.

Slide 7

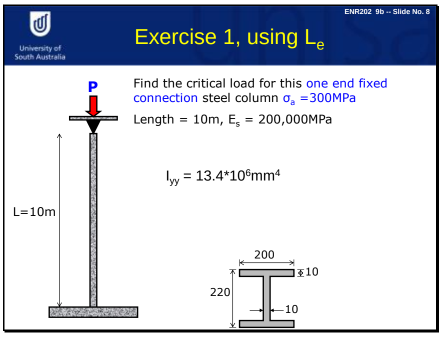


Therefore, the critical buckling load depends on the effective length factor. The effective length is in the denominator in Euler's buckling formula. So if you have less effective length, you will get a more critical buckling load, and vice versa.

Look at the buckle shape of the column in the figures. The columns buckle in the shape of the sine wave length. The sine wave length starts with zero lateral deflection to zero lateral deflection. Indirectly, that means from zero bending moment point to zero bending moment point.

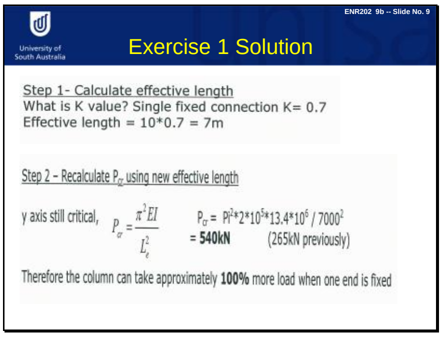
The smallest k value is 0.5, then 0.7, then 1, then 2. So the greatest critical buckling load is in a column in which both ends are fixed; the next is with one pinned end and one fixed end, then with two pinned ends, then with one fixed end and one free end.

Slide 8



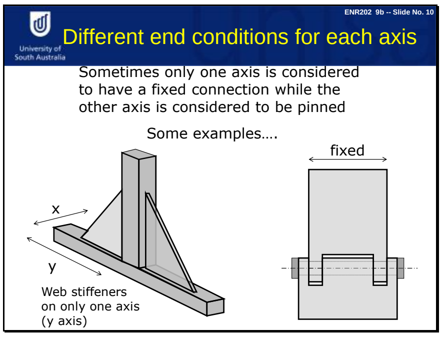
This exercise is the same as the exercise in Slide 19 of the previous lecture, except that the end conditions are different from that example. Here, one end is fixed and the other end is pinned. Go back and look again at the solution to that exercise. Then try to work out this exercise. The solution is on the next slide.

Slide 9



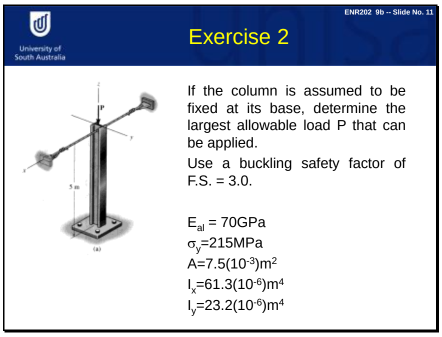
First, you need to calculate the effective length of the column. What is the effective length factor for a column with one fixed end and one pinned end? (Note that you will need to memorise the list of effective length factors for different end conditions.) In the last lecture, for this example, you calculated the moment of inertia about the x-x axis and the y-y axis for the I shape and the y axis. Young's modulus of steel is 200 000 Mega Pascals, and the moment of inertia about the y-y axis is 13.4 times 10 to the power of 6 mm to the power of 4. The effective length is 0.7 times the total length of the column; that is 7 meters. If you substitute all the values into the general buckling formula you get pi squared times E times I divided by Le squared, which is 540 Kilo Newtons. In the previous lecture example, the critical buckling load was 265 kilo Newtons. Therefore, the column with one fixed end and one pinned end can take approximately 100% more load than the column with two pinned ends.

Slide 10



Now we will look at one more concept covering different end conditions for each axis. In the left figure, we consider the x axis to have a pinned connection, while we consider the y axis to be a fixed connection. The column can easily buckle in the direction of the x-axis, but it would be difficult for it to buckle in the direction of the y-axis. We have stiffeners in the y-direction, which stiffen it in the lateral y-axis direction. Therefore, we have a fixed end in the y direction and a pinned end in the x-direction. The moment can transfer in the y-direction, but not in the x-direction. Therefore, the x-axis and the y-axis have different effective length factors. So you need to calculate the critical buckling load for both axes. It is a good idea to make stiffeners on the weaker axis. The critical buckling load in the x-axis is pi squared times E times Ixx divided by Lex squared. In the y-axis, the critical buckling load is pi squared times E times Iyy divided by Ley squared. In the right figure, we have one fixed axis and one pinned axis.

Slide 11

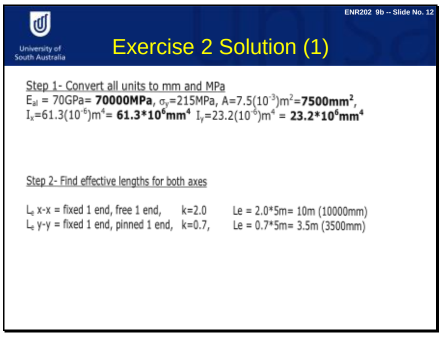


In this exercise, the aluminium column is fixed at the bottom end. It is braced at the top by cables, so as to prevent movement at the top along the x-axis, as shown in image. It is free to move on the y-axis.

The cables give support in the x direction, but there are no supporting cables in the y direction, as you can see in the figure. The total length of the column is 5 meters. You need to calculate maximum load that can apply on this column if the buckling safety factor is 3. The solution is on the next two slides.

Youngs modulus of Aluminium is 70 000 Mega Pascals and yield stress is 215 Mega Pascals. The area of the I-shaped cross section is 7500 square mm. The moment of inertia about the x-x axis is 61.3 times 10 to the power of 6 mm to the power of 4. The moment of inertia about the y-y axis is 23.2 times 10 to the power of 6 mm to the power of 4.

Slide 12

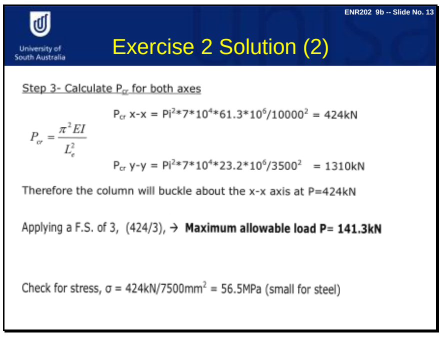


First, you need to convert all units to mms and Mega Pascals. Young's modulus of Aluminum is 70 000 Mega Pascals and the yield stress is 215 Mega Pascals. The moment of inertia about the x-x axis is 61.3 times 10 to the power of 6 mm to the power of 4. The moment of inertia about the xy-y axis is 23.2 times 10 to the power of 6 mm to the power of 4.

Secondly, calculate the effective lengths for both axes. The aluminium column is fixed at bottom end and is braced at its top by cables, to prevent movement at the top along the x-axis, and is free to move on the y-axis. Note an important point here: if the column buckles in the x-direction, you should use the moment of inertia about the y-y axis, and if the column buckles in the y-direction, you should use the moment of inertia about the x-x axis.

So, to apply that rule here: if the column buckles in an x direction, you should use the moment of inertia about the y-y axis (Iy). The effective length will be 0.7 times the original length. If the column buckles in the y direction, you should use the moment of inertia about the x-x axis (Ix). The effective length will be 2 times the original length because there is one fixed end and one free end.

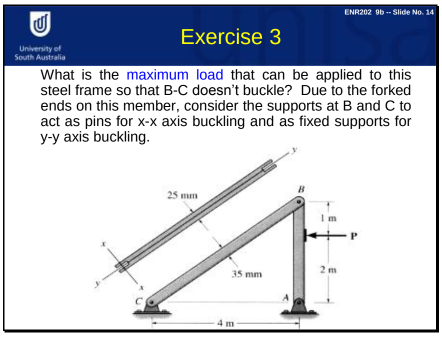
Slide 13



If the column buckles in the y direction, you should use the moment of inertia about x-x axis (Ix). The effective length will be 2 times the original length which is 10 meters (because one end is fixed and the other end is free). The critical buckling load about the x-x axis is equal to pi squared times 70 000 times 61.3 times 10 to the power of 6 divided by 10 000 squared. So, you should get the value for the critical buckling load about the x-x axis as 424 kilo Newtons.

If the column buckles in x direction, you should use moment of inertia about the y-y axis (Iy). The effective length will be 0.7 times the original length (that is, 3.5 meters). The critical buckling load about the y-y axis is equal to pi squared times 70 000 times 23.2 times 10 to the power of 6 divided by 10 000 squared. So, you should get the value for the critical buckling load about the y-y axis as 1310 kilo Newtons. Therefore, the column will buckle about the x-x axis, in the y-direction. Consider a safety factor of 3. The maximum allowable load 'P' is 424 divided 3, which is 141.3 kilo Newtons. To check for stress, the calculation according to compressive failure is 424 000 Newtons divided by 7500 square mms, which is equal to 56.5 Mega Pascals. This is less than 215 Mega Pascals, so the column will fail by buckling.

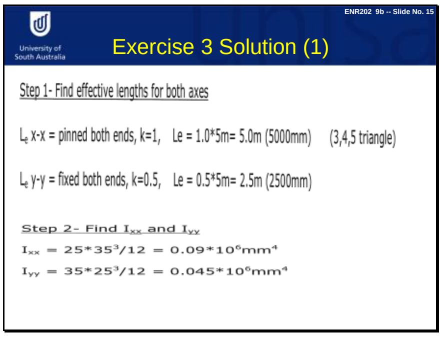
Slide 14



This exercise concerns a truss structure with two members. The BC member has a rectangular cross section 25 mm wide and 35 mm deep. If you apply load 'P' on the AB member at a distance of 2 meters from Point A, as shown in the figure, the BC member will get shorter. Therefore the BC member needs to resist the axial compressive load. You need to check the buckling capacity of the BC member. In fact, the BC member has two types of boundary conditions, due to the forked ends on this member. Member BC acts as a pinned end for the x-x axis and as a fixed support for y-y axis buckling.

You need to calculate the maximum load 'P' that can be applied to this steel frame so that member BC doesn't buckle. This example is interesting because at both ends, there are different end conditions for each axis. Pause this presentation and try to solve this problem. The solution is on the next three slides.

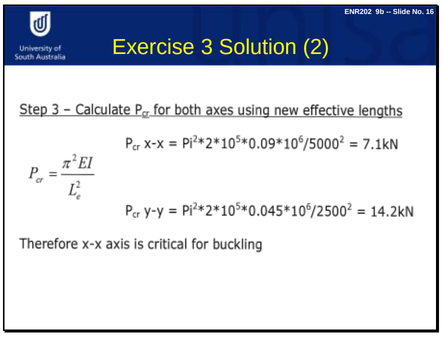
Slide 15



First, calculate the effective lengths for each axis. Suppose member BC buckles about the x-x axis. Due to the forked ends at both joints, both ends are pinned about the x-x axis, so the effective lengths about the x-x axis are equal to 5 meters. That means the effective length factor is 1. The BC member is inclined, with a horizontal distance of 4 meters and a vertical distance of 3 meters. The length of member BC is 5 meters. The effective length about the y-y axis is equal to the effective length factor times the length of the member, which is 2.5 meters. Here, the effective length factor is 0.5 because both ends are fixed, due to the forked ends which mean buckling will be in the x-direction.

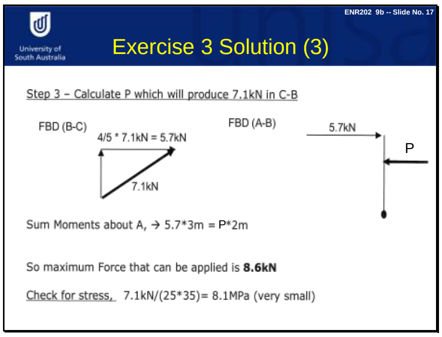
Secondly, you need to calculate the moment of inertia about the x-x axis and the y-y axis. We have a rectangular cross section for the BC member 25 mm wide and 35 mm high. The moment of inertia about x-x is 25 mm times 35 mm to the power of 3 divided by 12, which is equal to 0.09 times 10 to the power of 6 mm to the power of 4. The moment of inertia about y-y is equal to 35 mm times 25 mm to the power of 3 divided by 12, which is equal to 0.045 times 10 to the power of 6 mm to the power of 4.

Slide 16



We know Young's modulus is 200 000 Mega Pascals, and the moment of inertia about the x-x axis is 0.09 times 10 to the power of 6 mm to the power of 4, and the effective length about the x-x axis is 5 meters. You can calculate the critical buckling load about the x-x axis as equal to pi squared times 200 000 times 0.09 times 10 to the power of 6 divided by 5 000 squared, which is equal to 7.1 kilo Newtons. What about the y-y axis? Young's modulus is 200 000 Mega Pascals, the moment of inertia about the y-y axis 0.045 times 10 to the power of 6 mm to the power of 4 and the effective length about y-y axis is 2.5 meters. You can calculate the critical buckling load about the y-y axis as equal to pi squared times 200 000 times 0.045 times 10 to the power of 6 divided by 2.5 thousand squared, which is equal to 14.2 kilo Newtons. Therefore, the x-x axis is critical for buckling.

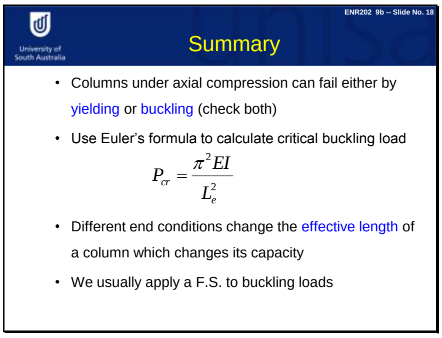
Slide 17



For the third step you need to calculate the maximum load ‘P’ that can be applied on the AB member. The critical buckling load of the BC member is 7.1 kilo Newtons. You need to calculate the horizontal component and the vertical component of the inclined buckling load, as shown in the figure. The horizontal component of the buckling load is equal to the horizontal distance of 4 meters divided by the inclined member BC, which has a distance of 5 meters, multiplied by the buckling load which is equal to 5.7 kilo Newtons. Now apply the equilibrium equation that all forces moments about point A are equal to zero. So 5.7 times the distance of 3 meters in a clock wise direction and P times 2 meters about Point A in an anti-clockwise direction. Finally you will get the value of P equal to 8.6 kilo Newtons.

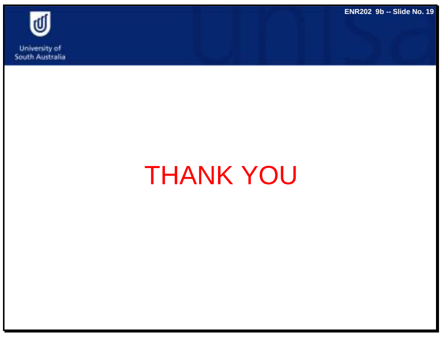
You need to check the maximum compressive stress. The calculation according to compressive failure is 7.1 thousands Newtons divided by 25 mm times 35 mm is equal to 8.1 Mega Pascals, which is a very small value and less than 250 Mega Pascals, so the column will fail by buckling.

Slide 18



Lets summarise the concepts of column buckling. There are two possibilities of failure when axial compression is applied on the column; it may fail by either by yielding or buckling. You need to check both of them. The Euler's buckling load formula is pi squared times EI divided by the effective length squared. The effective length changes with different end conditions and changes its buckling capacity. We usually apply a factor of safety to a buckling load because this failure is sudden and dangerous.

Slide 19



Thanks for your attention. In the next lecture, we will learn about the concepts of torsion that is twisting moment.