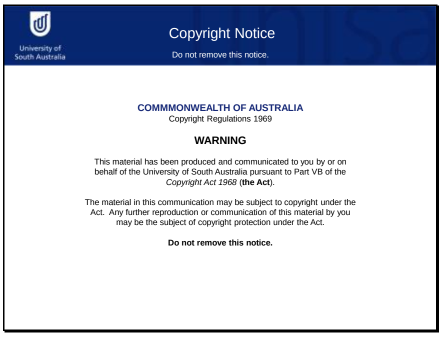
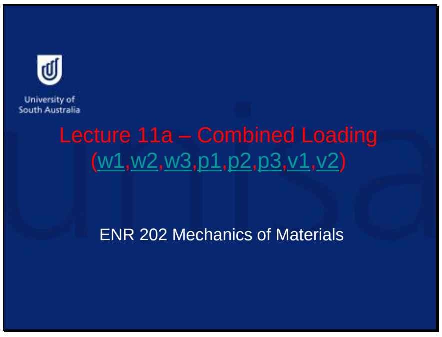
ENR202 Mechanics of Materials Lecture 11A Slides and Notes

Slide 1

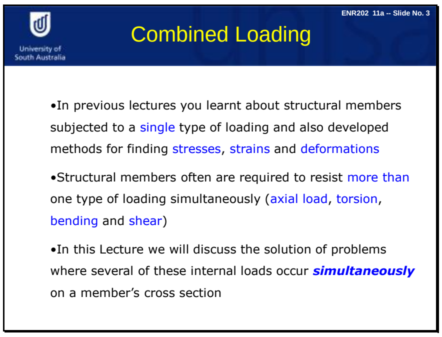


Slide 2



By the end of this lecture, you should be able to find the most dangerous critical cross section in a structural member. You need to work out the shear force and bending moment on that cross section. Based on bending theory and shear stress concepts, you can draw the stress distribution on the critical cross section to get the critical point.

Slide 3



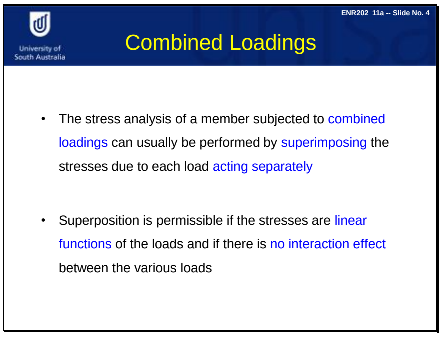
In previous lectures, you learnt about a structural member subjected to a single type of loading. You learnt how to find stresses, strains and deformations based on this load.
For example, let's say we have a simply supported beam subjected to a uniform distributed load. You know how to draw the SFD and BMD for this beam (look back at slide 17 of lecture 4a for revision). You know that the shear force is highest at the supports, but the bending moment is zero. On the other hand, the shear force is zero in the middle, but that is where the bending moment is highest. However, even if all points at the supports and the middle of the

structural member are safe, that is no guarantee that the whole structural member is safe. We also need to consider other points in the structural member, where you will get reasonable (even if it is less than maximum) shear force value, and reasonable bending moment value (even if it is less than maximum). This is the concept of combined loading, the combination of shear force and bending moment

Generally speaking, structural members are required to resist more than one single type of loading simultaneously: these loads are axial loads, shear force loads, bending moment and torsion moment.

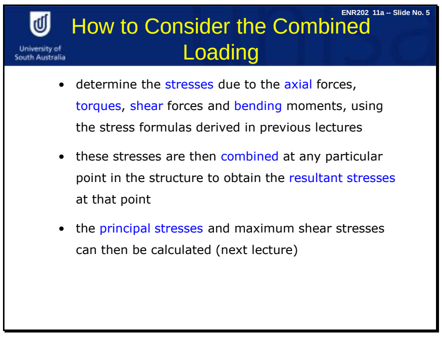
In this lecture, we will discuss how to deal with situations in which several internal loads occur simultaneously on the cross section of a member.

Slide 4



You need to follow the *superimposition method* to calculate stress distribution for different internal load effects. You draw the normal stress distribution for axial force, the normal stress distribution for bending moment, the shear stress distribution for shear force, and the shear stress distribution for twisting moment. Then you add up the both normal stress distribution for axial force and bending moment and the sum of both shear stress distributions for shear force and twisting moment. We'll talk in more detail about this in the next slide. Note that there is a condition for use of the superposition method: the stresses are linear functions of the loads and there is no interaction effect between the various loads.

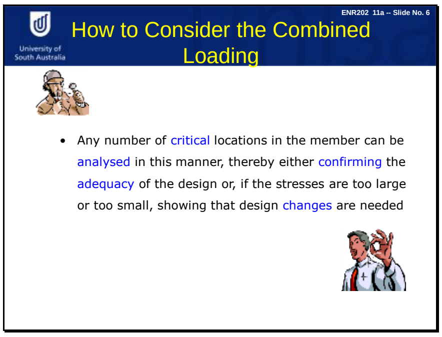
Slide 5



The internal force effects in the structure are the axial force, the bending moment, the shear force and the torque. If you know how to draw AFDs, SFDs, BMDs, and TMDs, then you can draw stress distribution for all internal force effects. Normal stress is due to axial force and bending moment, and shear stress is due to shear force and torque. You can draw normal stress distribution for axial force and bending moment separately, then sum both distributions, which will give you combined normal stress distribution for axial force and bending moment. You can draw the shear stress distribution for shear force and twisting moment separately, then sum both distributions. Eventually you will get combined shear stress distribution for shear force and twisting moment.

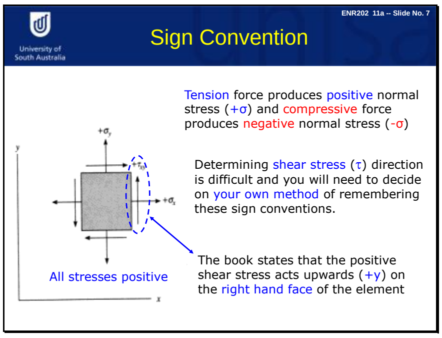
Based on combined stress distribution, you can find combined normal stress and combined shear stress at any point of interest on the particular cross section. Once you know the combined normal stress and combined shear stress, you will know principal stresses and maximum shear stress based on stress transformation concept, which we will cover in the next lecture summary.

Slide 6



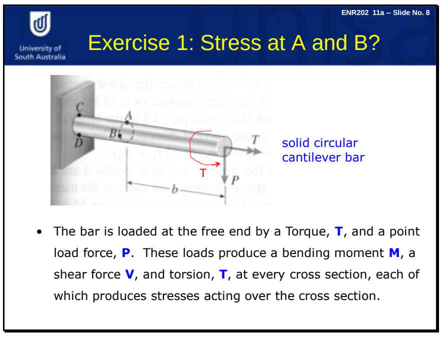
The number of critical cross section locations in the member can be analysed based on either confirming the adequacy of the design or, if the stresses are too large or too small, showing that design changes are needed.

Slide 7



The sign convention here is that tension force produces positive normal stress and compressive force produces negative normal stress. Finding the sign for shear stress is a bit trickier. However, we follow the right edge of the stress cube as shown in the figure. If shear stress acts upwards on the right hand face of the element, it is positive shear stress.

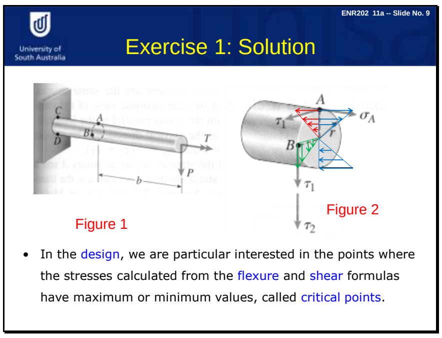
Slide 8



In this example, a solid circular cantilever bar is subjected to torque and a concentrated load at the free end as shown in the figure. Use the right hand thumb rule (just hold your thumb in the direction of torque arrow as shown in the figure and look at the direction of the other fingers) and you can see that the torque moment direction is positive. These loads produce an internal effect bending moment M, a shear force V and torsion T at every cross section. Therefore, the bending moment produces normal stress distribution, and the shear force produces shear stress distribution and torsion produces shear stress distribution acting over every cross section.

You need to calculate the normal stress and shear stress distribution at point A and B points at a distance ‘b’ from the free end as shown in the figure. If you want to try the exercise yourself, pause the presentation and do so (the solution is on the next slide).

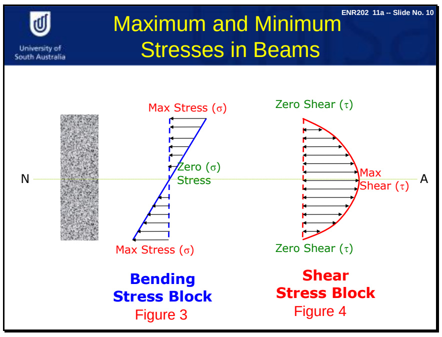
Slide 9



We are interested in calculating two stresses at Points A and B: the normal stress due to the bending moment, and the shear stress due to the shear force and torque. Points A and B are located at a distance ‘b’ from the free end as shown in figure 1. First, draw the normal stress distribution at the cross section which is distance ‘b’ from the free end. We know that if there is a concentrated load acting on the cantilever beam, the tension occurs at the top and compressive stress occurs at the bottom, as shown in figure 2. Maximum stress occurs at the top and bottom, and there is zero normal stress at the centre. We are interested in calculating the normal stress at Point A and B. Based on normal stress distribution, the normal stress at A is sigma A and the normal stress at B is zero, as in figure 2 (the blue lines show the stress distribution).

Now, we concentrate on torsion shear stress distribution. The torsional shear stress distribution at point A is shown by the red lines in figure 2. Based on this distribution, the torsional shear stress at A is 'tou 1'. The shear stress is acting in a horizontal direction as shown in figure 2. The torsional shear stress distribution at point B is shown in figure 2 with green lines. Based on this distribution, the torsional shear stress at B point is also 'tou 1', but direction of this shear stress is acting vertically downward direction.

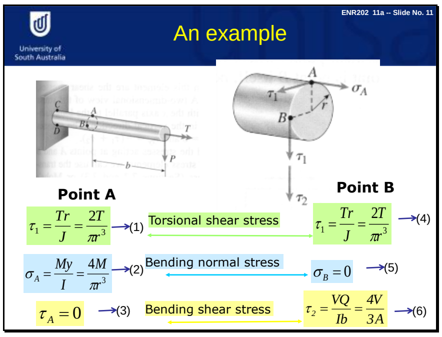
Slide 10



Now, we'll look at shear stress distribution due to shear force. You already studied shear stresses in beams in lecture 6a. Shear stress distribution is shown in figure 4. Based on this distribution, the shear stress at point A is zero and the shear stress at point B is 'tou 2'. This shear stress is acting in a vertically downward direction.

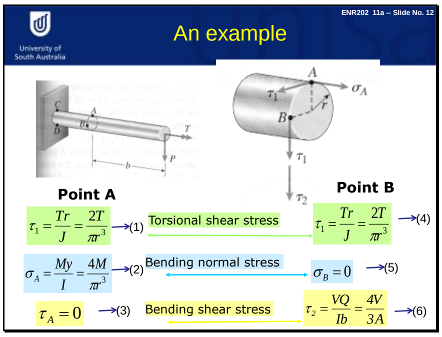
So we have two shear stresses at point B. One is due to torsion 'tou 1' and one is due to shear force 'tou 2'. We have one shear stress at point A because the shear stress due to shear force is zero and the shear stress due to torsion is 'tou 1'. The normal stress at point A is sigma and the normal stress at point B is zero, as per figure 3.

Slide 11



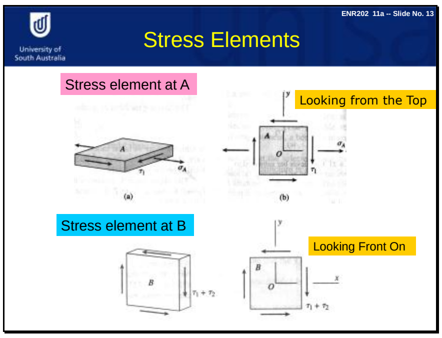
Let's look at an example. First, look at point A. We can calculate the value of the torsional shear stress based on the torsion formula. The torsional shear stress is equal to the torque multiplied by the radius of the solid circular shaft divided by the polar moment of inertia. The polar moment of inertia of the solid circle is pi times the radius to the power of 4 divided by 2. From this, we get the torsional shear stress as 2 times the torque divided by pi times the radius to the power of 3, as shown in equation 1. Normal stress is based on the bending formula. The normal tensile stress is equal to the moment 'M' times 'y' divided by the moment of inertia, as per equation 2. The bending shear stress is zero because we have zero at point A in shear stress distribution.

Slide 12



Now we'll look at point B. We can calculate the value of torsional shear stress based on the torsion formula. The torsional shear stress is worked out in the same way as point A: the torque multiplied by the radius of the solid circular shaft divided by the polar moment of inertia. The polar moment of inertia of the solid circle is pi times the radius to the power of 4 divided by 2. From this we get the torsional shear stress as 2 times torque divided by pi times the radius to the power of 3, as shown in equation 4. However, this torsional shear stress at point B is acting in a vertical downward direction, as shown in the figure. The normal stress is based on the bending formula due to bending moment. The normal tensile stress is equal to zero, as per equation 5. The bending shear stress is equal to the shear force V times the first moment of area 'Q' divided by 'Ib', as per equation 6. Here, I is the moment of inertia, and b is the width of the solid shaft at point B (that is, the diameter of shaft).

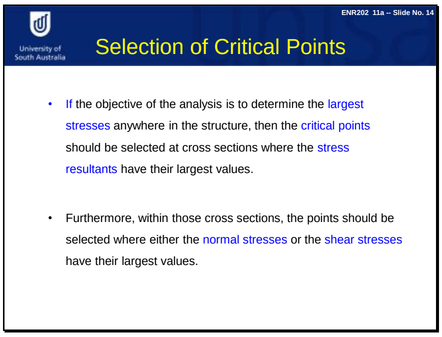
Slide 13



We can draw the stress cube at point A. We have one tensile normal stress and one shear stress at point A, as shown in top left figure. In the stress cube at B, we don't have any normal stress, and we have two shear stresses due to torque 'tou 1' and shear force 'tou 2', as shown in the bottom left figure. Combining these two shear stresses will give the final shear stress at point B, as shown in the bottom left figure. Point A is located on the top of the solid circular shaft, so we have to draw the top view, as shown in the top left figure. Point B is located on the front of the solid circular shaft, so we have to draw the front view for the stress cube, as shown in the bottom left figure.

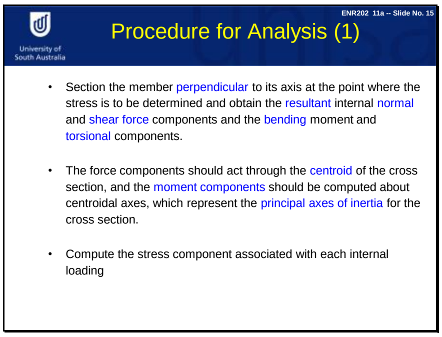
At any point in the structure, we have to combine normal stresses together and shear stresses together, to get the stress cube at that point.

Slide 14



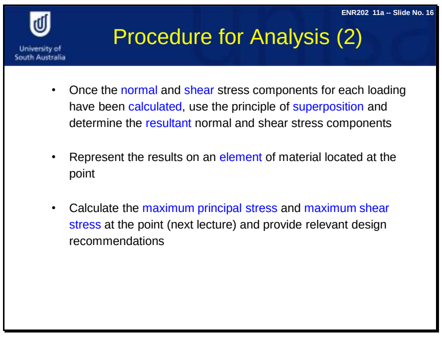
The objective of combined loading analysis is to determine the largest stresses anywhere in the structure. The critical points should be selected at the cross sections where the stress resultants have their largest values. Furthermore, within those cross sections, the points should be selected where either the normal stresses or the shear stresses have their largest values.

Slide 15



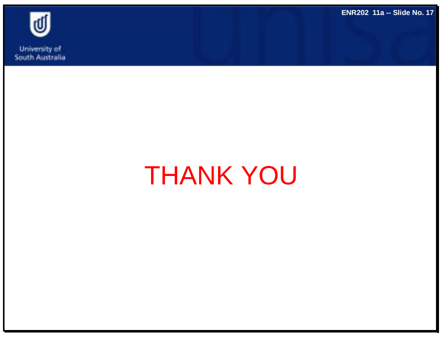
So we can summarise the procedure for analysis on the next two slides. First, you need to find the critical cross section or particular cross section for which you want to know the resultant internal normal force, shear force, bending moment and torsional moment components. Second, the force components should act through the centroid of the cross section and the moment components should be computed about the centroid axes which represent the principal axes of inertia for the cross section. (We will look at the principal axes of inertia in the next lecture summary.) Third, compute the stresses associated with each internal loading at that point.

Slide 16



Fourth, once you find normal and shear stress components for each loading, use the principle of superposition and determine the resultant normal stress and resultant shear stress components. This means combining all normal stresses and combining all shear stresses at that point. Fifth, draw the stress cube for that point. Finally, calculate the maximum principal stress and the maximum shear stress at that point.

Slide 17



Thank you for your attention. In the next lecture, we will practice some examples of combined loading.