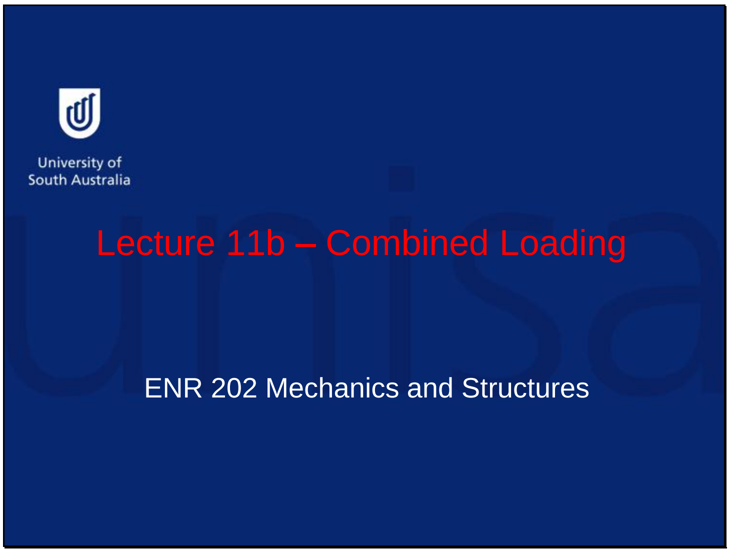
ENR202 Mechanics of Materials Lecture 11B Slides and Notes

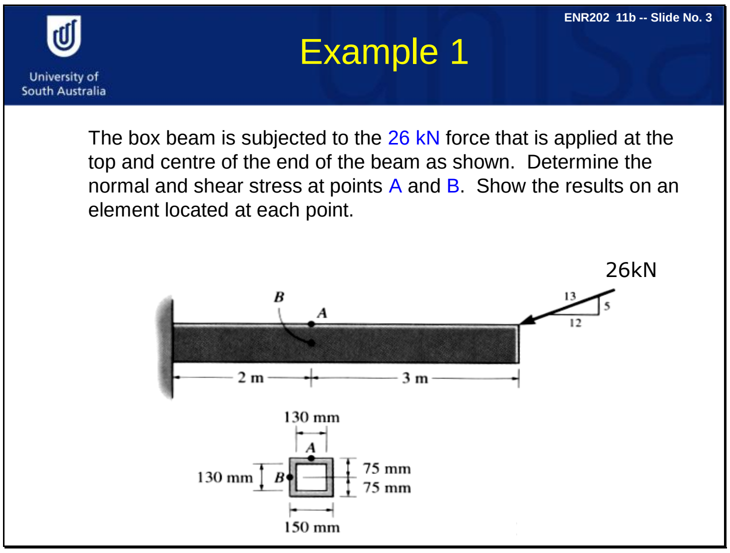
Slide 1

Slide 2



Hello, and welcome to Lecture 11b. In this lecture summary, we will continue with the combined load topic, and we will practice four problems.

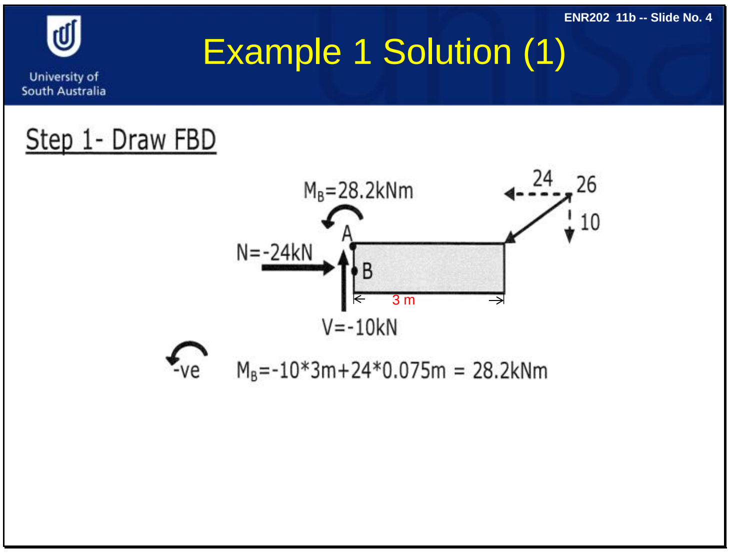
Slide 3



A cantilever beam with square hollow section is subjected to an inclined force 26 kilo Newtons at the top and centre of the end of the beam, as shown in the top figure. The square hollow cross section is 150 mm wide and 10 mm thick, as shown in the bottom figure.

You need to determine the resultant normal stress and resultant shear stress at points A and B. These two points are 3 meters from the free end as shown in the top figure. Point A is located at the centre of the top side of the square hollow section. Point B is located at the centre of the left side of the square hollow section, as shown in the bottom figure.

Slide 4

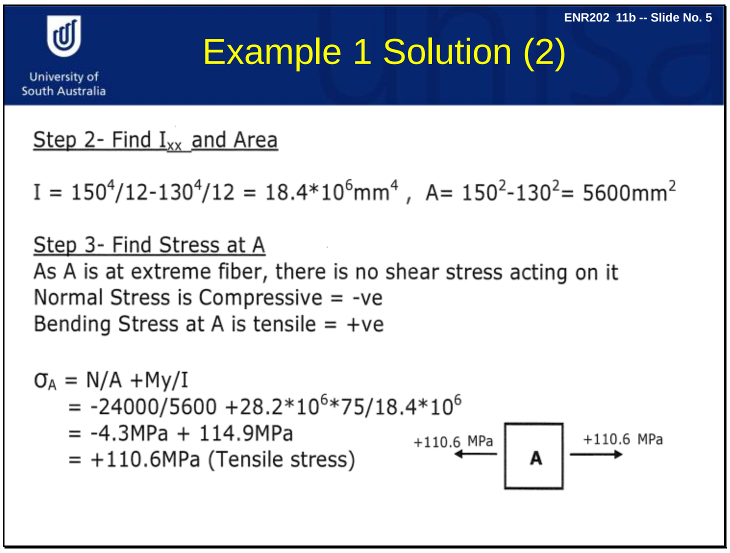


You need to determine the resultant normal stress and resultant shear stress at points A and point B, located at a distance of 3 meters from the free end. Firstly, you have to cut the beam 3 meters from the free end as shown in the figure. You need to show the internal force effects at the cutting cross section. These are normal force N, shear force V and bending moment MB, as shown in the figure.

The inclined load 26 kN may be resolved into the horizontal component and vertical component. The angle of the inclined load is tan inverse 5 divided by 12. The horizontal component will be 26 kilo newtons times 12 divided by 13, which is 24 kilo Newtons, and the vertical component will be 26 kilo Newtons times 5 divided by 13, which is 10 kilo Newtons, as shown in the figure.

Now apply equilibrium equations. If all forces in a vertical direction are equal to zero, the compressive normal force N is equal to 24 kilo Newtons. If all forces in a vertical direction are equal to zero, the shear force V is equal to 10 kilo Newtons. Finally, if the moment of all forces about the centroid of the cross section point is zero, MB is equal to minus 10 kilo Newtons times the distance of 3 meters plus 24 kilo Newtons distance times distance 75 mm, which is 0.075 meters. Thus, the MB is equal to 28.2 kilo Newton meters, causing tension in the top of the beam and compression in the bottom of the beam.

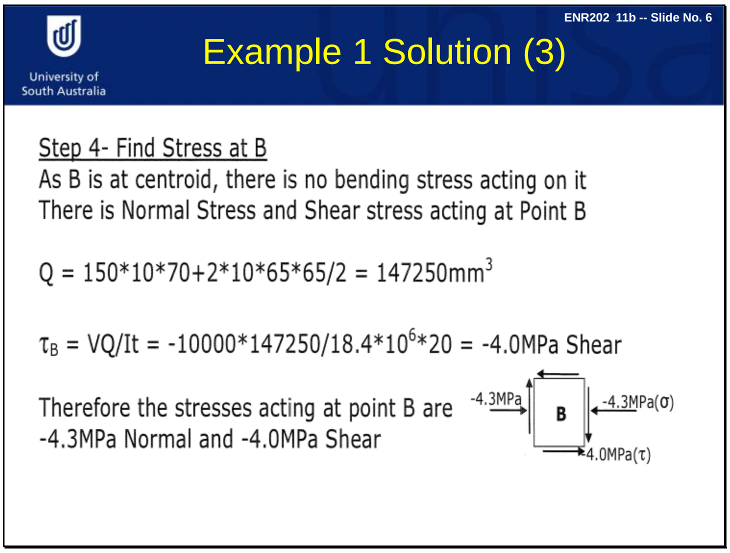
Slide 5



The area of the square hollow section is equal to 150 to the power of 2 minus 130 to the power of 2, which is equal to 5600 square mm. The moment of inertia is 150 to the power of 4 divided by 12 minus 130 to the power of 4 divided by 12, which is equal to 18.4 times 10 to the power of 6 mm to the power of 4. We studied centroid and moment of inertia in lecture 5a. You can go back and have look.

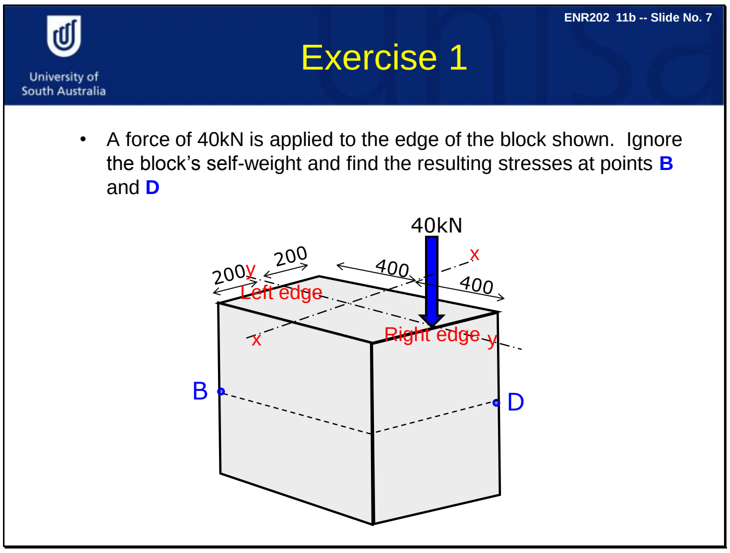
Point A is located at centre of top side of the square hollow section at a distance of 3 meters from the free end. Bending normal stress is positive at point A. There is also compressive normal force acting axially. We studied this in lecture 2b. The compressive normal stress is equal to compressive normal force, which is 24 kilo Newtons divided by the cross sectional area of the cross section (5600 square mm) which is equal to minus 4.3 Mega Pascals. This value is negative because normal force is compressive. The bending normal stress is equal to the bending moment M times y divided by I. The bending moment MB is 28.2 kilo Newton meters. y is the distance from the centroid to point A, which is 75 mm. We know the moment of inertia. Finally, we get the tensile normal stress at point A as plus 114. 9 Mega Pascals. The sum of these two normal stresses are minus 4.3 Mega pascals plus 114.9 Mega pscals, which is equal to 110. 6 Mega Pascals. We don't have any shear stress at point A due to shear force V because the bending shear stress distribution is a parabola, with the maximum at the centre and zero at the bottom and top of the beam. Finally, the stress cube at point A has zero shear stress and 110.6 Mega Pascals normal stress.

Slide 6



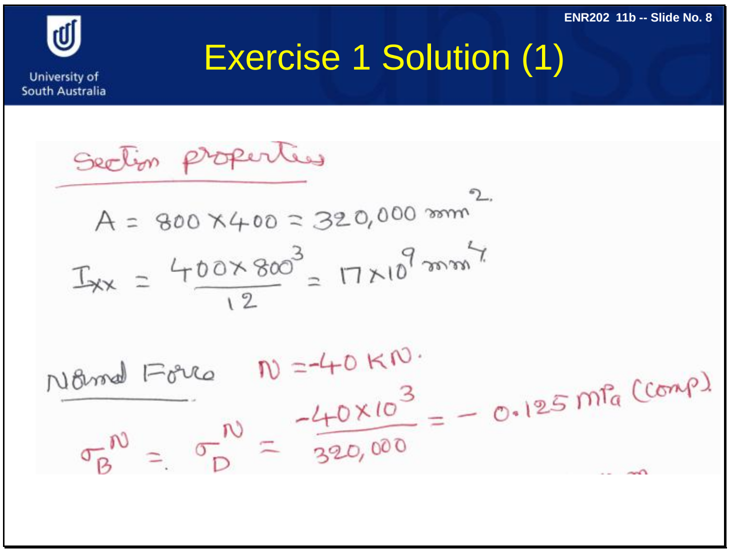
Point B is located at the centre of the left side of the square hollow section, 3 meters from the free end. The bending normal stress at point B is zero, because bending stress distribution is a linear function with zero at the centre and the maximum at the top and bottom. There is compressive normal force acting axially. The compressive normal stress is equal to compressive normal force, which is 24 kilo Newtons divided by the cross sectional area of the cross section (5600 square mm), which is equal to minus 4.3 Mega Pascals. This value is negative because normal force is compressive. There is shear stress at point B due to shear force V. We studied shear stress distribution due to shear force in lecture 6a. Shear stress due to shear force is shear force times the first moment of area Q, divided by It. The shear force V is 10 kilo Newtons. The first moment above point B (that is top portion of B of the cross section about the neutral axis) is 150 times 10 times 70 plus 2 times 10 times 65 times 65 divided by 2, which is equal to 147250 square mm. You know the moment of inertia of the square hollow cross section. The cutting width at point B ‘t’ is 10 plus 10, which is equal to 20 mm. Finally, we get the shear stress due to the shear force as 4 Mega Pascals. The shear force is negative. In the last lecture 11a in slide number 7, we studied sign convention. If the shear force on the right edge going upwards is positive, then the shear stress here is negative. The stress cube at point B is shown in the bottom figure.

Slide 7



Now pause the slide and try to do this problem. A rectangular block 400 mm wide and 800 mm deep is subjected to an eccentric load acting on the centre of the right edge, as shown in the figure. Please ignore the block's self weight. You need to find the resultant normal stress at point B and point D, as shown in the figure. The answer is shown on the next two slides.

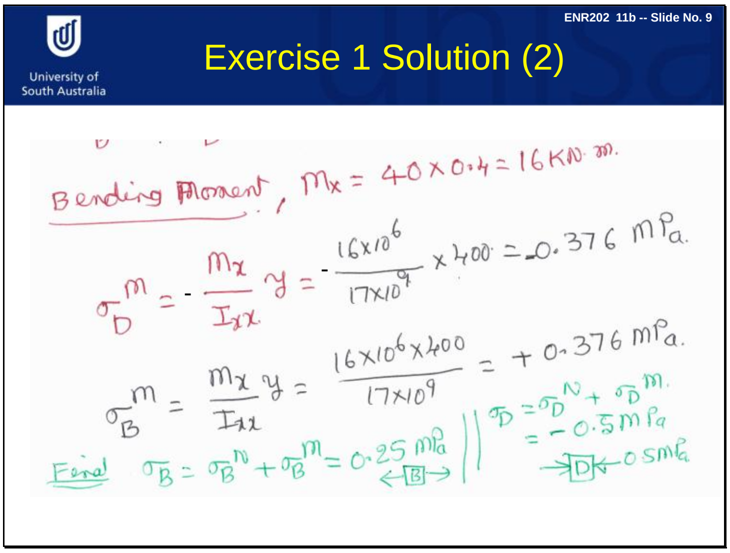
Slide 8



First, calculate the area of the rectangular block and the moment of inertia about x-x. The area of the rectangular block is 800 mm times 400 mm, which is 320 000 square mm. The moment of inertia about the x-x axis is equal to 400 mm times 800 mm to the power of 3 divided by 12, which is equal to 17 times 10 to the power of 9 mm to the power of 4. If you have any questions about the calculation of moment of inertia, look back at slide number 11 in lecture 5a.

Second, calculate the resultant normal stress at points B and D. Cut at that cross section, draw the free body diagram and apply equilibrium equations. If all forces in a vertical direction are equal to zero, the normal force N is equal to minus 40 kilo Newtons. Here the sign is negative because the normal force is compression. The normal stress is uniform throughout the cutting cross section due to the compressive normal force, which is equal to minus 40 000 divided by the area of the cutting cross section (320 000 square mm) which is equal to minus 0.125 Mega Pascals at both points B and D.

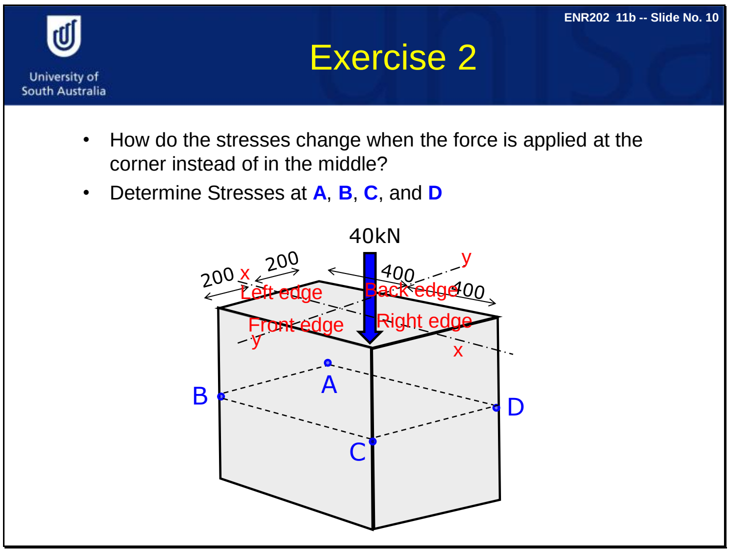
Slide 9



Now taking all forces moments about the centre of the rectangular block as equal to zero, you will get a bending moment about the x-x axis as equal to the eccentric load of 40 kilo Newtons multiplied by the distance between the loading point and the centre of the rectangular block (0.4 meters) which equals 16 Kilo Newton meters. We have compressive stress at the right part of the rectangular block and tensile stress at the left part of the rectangular block due to the bending moment about x-x. Point B is located at the left part of the block, so we have tensile stress at point B. The bending tensile normal stress at point B is equal to the bending moment Mx times y divided by the moment of inertia about the x-x axis, which is positive 0.376 Mega Pascals. The bending compressive normal stress at point D is the bending moment Mx times y divided by the moment of inertia about x-x axis, which is equal to negative 0.376 Mega Pascals.

The resultant normal stress at point B is equal to the sum of the compressive normal stress due to compressive normal force (minus 0.125 Mega Pascals) and the tensile normal stress due to bending moment Mx (positive 0.376 Mega Pascals). This is equal to positive 0.25 Mega Pascals, which means tensile normal stress at point B. You can see the stress cube for point B in this slide.
The resultant normal stress at point D is equal to the sum of the compressive normal stress due to compressive normal force (minus 0.125 Mega Pascals) and the compressive normal stress due to bending moment Mx (negative 0.376 Mega Pascals). The final normal stress at point D is negative 0.5 Mega Pascals, which means compressive normal stress. You can see the stress cube for point D in this slide.

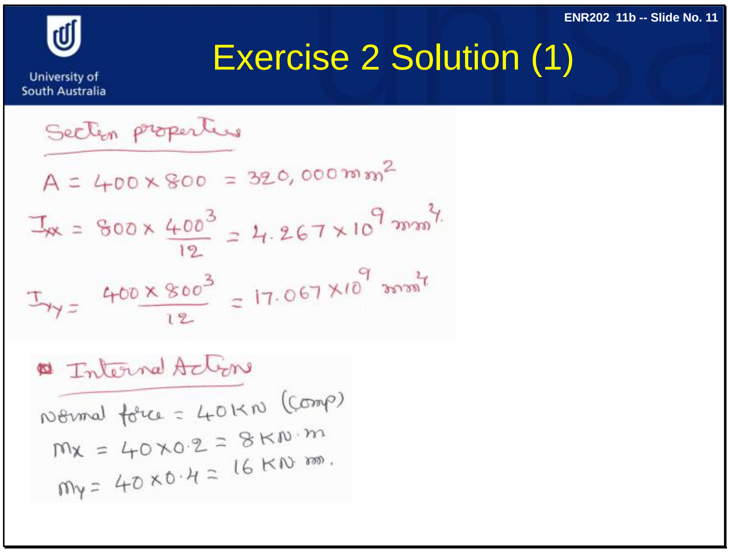
Slide 10



Now let's look at the same rectangular block from Exercise 1, 400 mm wide and 800 mm deep, now subjected to an eccentric load acting on the corner of the right edge and the front edge, as shown in the figure. Again, ignore the block's self weight. The 'y-y' axis is parallel to the left and right edge, and the 'x-x' axis is parallel to the front and back edge, as shown in the figure.

Pause the presentation and work out the resultant normal stress at point A, point B, point C and point D, as shown in the figure. The answer is on the next four slides.

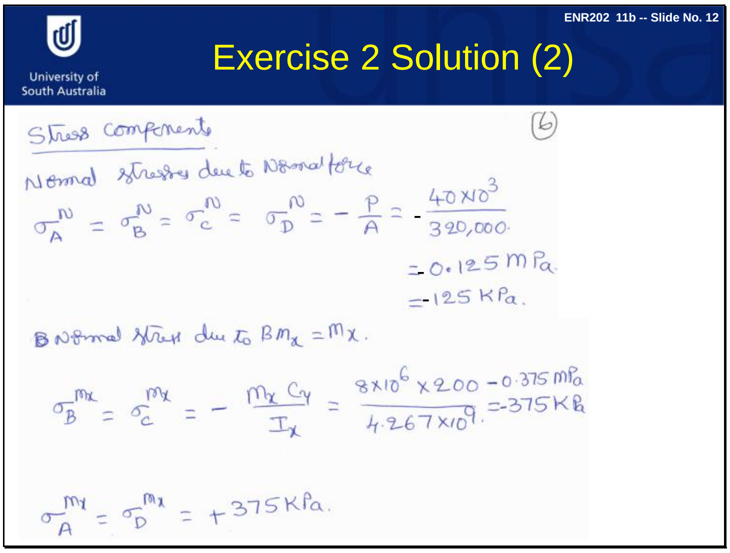
Slide 11



First, calculate the area of the rectangular block and moment of inertia about the x-x axis and the moment of inertia about the y-y axis. The area of the rectangular block is 800 mm times 400 mm, which is 320 000 square mm. The moment of inertia about the x-x axis is equal to 800 mm times 400 mm to the power of 3 divided by 12, which is 4.267 times 10 to the power of 9 mm to the power of 4. The moment of inertia about the y-y axis is equal to 400 mm times 800 mm to the power of 3 divided by 12, which is equal to 17.067 times 10 to the power of 9 mm to the power of 4.

Second, calculate the resultant normal stress at point A, point B, point C and point D. Cut at that cross section, and draw the free body diagram. If all forces in a vertical direction are equal to zero, you will get normal force N equal to minus 40 kilo Newtons. Here the sign is negative because the normal force is compression. Now take all forces moments about the centre of the rectangular block as equal to zero, and you will get bending moment about the x-x axis as equal to the eccentric load of 40 kilo Newtons multiplied by the distance between the eccentric loading point and the x axis, which is 0.2 meters. The bending moment about the x-axis is equal to 8 Kilo Newton meters. Bending moment about the y-y axis equal to the eccentric load of 40 kilo Newtons multiplied by the distance between the loading point and the y axis, which is 0.4 meters. The bending moment about the y-axis is equal to 16 Kilo Newton meters.

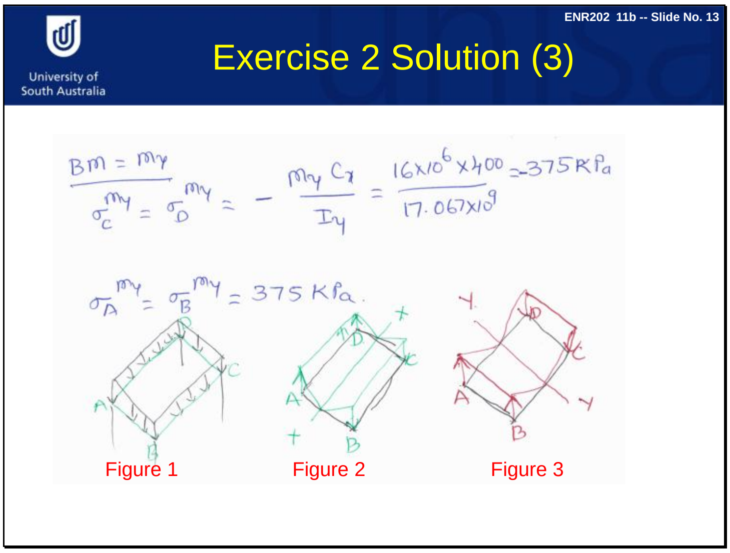
Slide 12



We studied in lecture 2b how to calculate normal stress due to axial normal force. The normal stress is uniform throughout the cutting cross section due to the internal axial compressive normal force. It is equal to minus 40 thousands divided by the cross section area of 320 000 square mm. The answer is minus 0.125 Mega Pascals or 125 kilo Pascals at all points A,B,C and D.

The bending moment about the x-x axis is equal to 8 Kilo Newton meters. There are compressive stress at the front part of the rectangular block and tensile stress at the back part of the rectangular block due to the bending moment about x-x. Point B and point C are located at the front part of the block, so we have compressive stress at point B and point C. The bending compressive normal stress at point B and point C is due to the bending moment about the x-x, which is equal to bending moment Mx, which is 8 kilo newtons times cy (200 mm) divided by the moment of inertia about the x-x axis. This is 4.267 times 10 to the power of 9 mm to the power of 4. The bending stress due to Mx is equal to negative 0.375 Mega Pascals or 375 kilo Pascals. Following the same procedure, we can find the tensile stresses at point A and point D due to bending moment Mx. They are positive 0.375 Mega Pascals or 375 kilo Pascals.

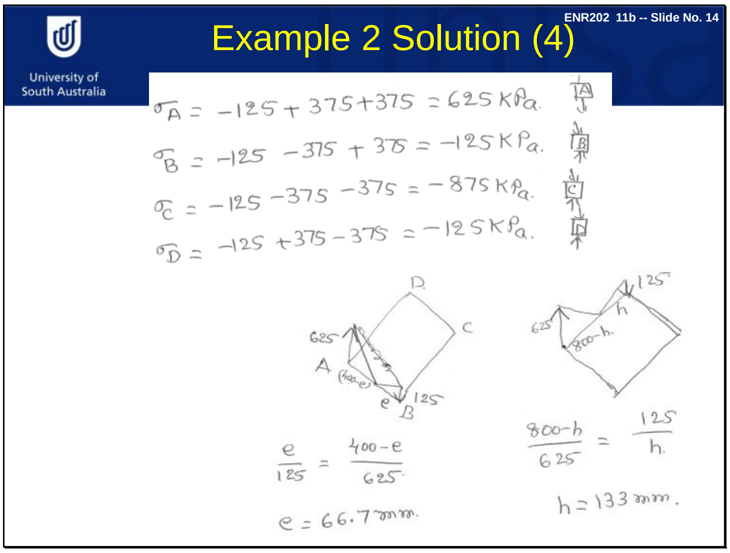
Slide 13



As you know, the bending moment about the y-y axis is equal to 16 Kilo Newton meters. We have compressive stress at the right part of the rectangular block and tensile stress at the left part of the rectangular block due to the bending moment about y-y. Points C and point D are located at the right part of the block. We have compressive stress at point C and point D. The bending compressive normal stress at point C and point D due to bending moment about y-y axis is My (16 kilo newtons) times cx (400 mm) divided by the moment of inertia about the x-x axis, which is 17.067 times 10 to the power of 9 mm to the power of 4, which is equal to negative 0.375 Mega Pascals or 375 kilo Pascals. Following the same procedure, we can get the tensile stresses at point A and point B due to bending moment. They are positive 0.375 Mega Pascals or 375 kilo Pascals.

The normal stress distribution at point A, point B, point C and point D due to compressive normal force is shown in figure 1. The normal stresses due to bending moment about the x-x axis are shown in figure 2. The normal stresses due to bending moment about the y-y axis are shown in figure 3.

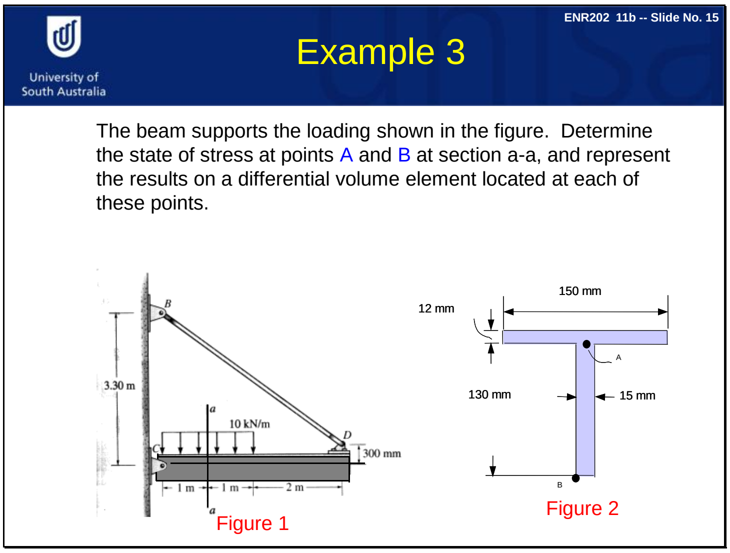
Slide 14



***********************************At point “A” there are normal stress due to normal force, negative 125 kilo Pascals, normal stress due to bending moment about x-x axis, positive 375 kilo Pascals, normal stress due to bending moment about y-y axis, positive 375 kilo Pascals. Summing them together, we have 625 kilo Pascals. At point “B” there are normal stress due to normal force, negative 125 kilo Pascals, normal stress due to bending moment about x-x axis, minus 375 kilo Pascals, normal stress due to bending moment about y-y axis, positive 375 kilo Pascals. Summing them together, we have negative 125 kilo Pascals.
For point C, the total normal stress is negative 875 kilo Pascals. For point D, total normal stress is negative 125 kilo Pascals. You can see the stress cubes for all points A, B,C, and D in this slide.

The resultant normal stress at point A is a tensile normal stress of 625 kilo Pascals and the resultant normal stress at point B is a compressive normal stress of 125 kilo Pascals. The zero normal stress occurs 66.7 mm from right edge. The resultant normal stress at point A is tensile normal stress of 625 kilo Pascals and the resultant normal stress at point D is a compressive normal stress of 125 kilo Pascals. The zero normal stress occurs 133 mm from back edge.

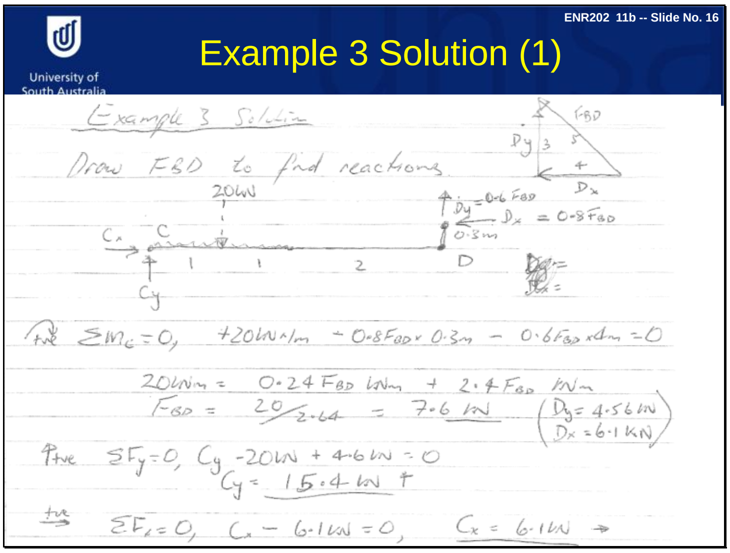
Slide 15



It is a truss system with a beam and rod. A beam supported by a hinged support at C point and with inclined rod at D point as shown in the figure 1. the rod connected with a beam at D point and a wall at B point. The beam have T-cross section as shown in the figure 2. the D point located at 300 mm from centroid of T- cross section as shown in the figure 1. the beam is subjected to uniform distributed load 10 kilo Newtons per meter at left half span of the beam of 2 meters. The total length of the beam is 4 meters. The distance between point B and point C equal to 3.30 meters as shown in the figure 1. The T- cross section with 2 rectangles as shown in the figure 2.

You need to calculate state of stress at points A and B at section a-a. the point A is located at junction of flange and web of T-cross section and point B is located at bottom of web as shown in the figure 2. the cross section a-a is located at a distance 1 meter from C point as shown in the figure 1.

Slide 16

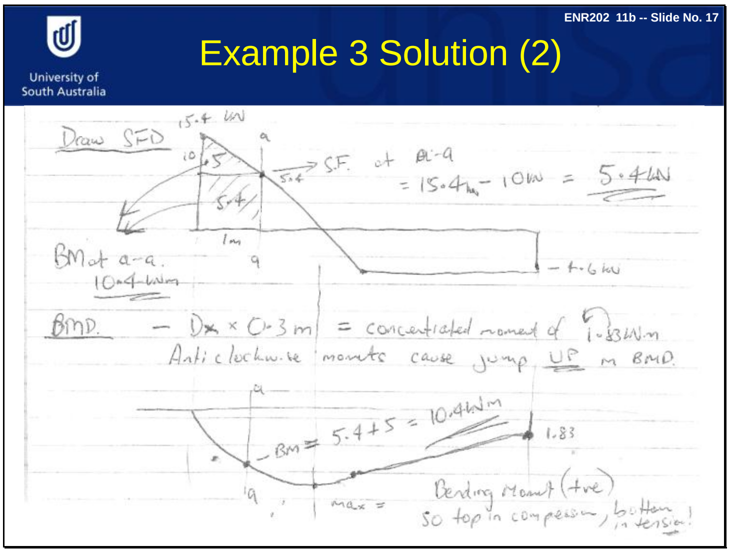


First you need to calculate reaction forces at hinged support at C and another hinged support at D point. Assuming that horizontal reaction force at C point is Cx and vertical reaction force is Cy. At D point horizontal reaction force is Dx and vertical reaction force is Dy. The resultant uniform distributed load is 10 kilo Newtons times 2 meters is 20 kilo Newtons acting at centre of UDL that is 1 meter distance from C point. The inclined rod is connected by hinged support at D point.

The tensile force in the rod can be resolved into horizontal reaction force and vertical reaction force. If the tensile force in rod is FBD, the horizontal component of reaction force at D is Dx equal to 4 divided by 5 times FBD. It is 0.8 times FBD and the vertical component of reaction force at D is Dy due to tensile force in rod equal to 3 divided by 5 times FBD. It is 0.6 times FBD. Apply equilibrium equation that all forces moments about C point equal to zero, resultant UDL 20 kilo Newtons times 1 meter in clock wise direction, the vertical reaction force at D point is 0.8 times FBD times 4 meters in anti clock wise direction, horizontal reaction force at D point is 0.8 times FBD times 0.3 meters in anti clock wise direction. Finally you will get tensile force in the rod FBD equal to 7.6 kilo Newtons. Therefore the vertical reaction force at D is Dy equal to 0.6 times 7.6 kilo Newtons equal to 4.56 kilo Newtons and horizontal reaction force at D is Dx equal to 0.8 times 7.6 kilo Newtons is equal to 6.1 kilo Newtons acting leftward direction at D.

Next, consider all forces in vertical direction equal to zero, the resultant UDL 20 kilo Newtons acting in down ward direction. The vertical reaction force at C is Cy acting in upward direction and vertical reaction force at D is Dy 4.56 kilo Newtons acting in upward direction. Finally we will get vertical reaction force Cy equal to 15. 4 Kilo Newtons. Then, consider that all forces in horizontal direction equal to zero, the horizontal reaction force at C is Cx acting in right ward direction, the reaction force at D is DX is acting in leftward direction with a magnitude of 6.1 kilo newtons. Finally you will get horizontal reaction force Cx at C is equal to 6.1 kilo Newtons.

Slide 17



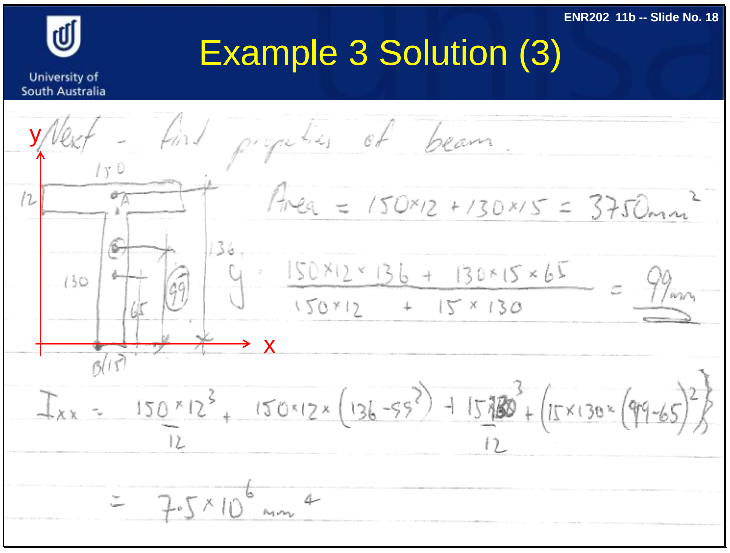
You know very well how to draw SFD and BMD once you calculate reaction forces for the beam. If you forgot, you can go back and have a look lectures 3b, 4a and 4b and try to draw SFD and BMD for this type of beam.

Once you draw SFD and BMD. You can check with above diagrams and compare with your results. Next, you need to find shear force and bending moment at distance 1 meter from C point because you need to calculate state of stress at points A and B at section a-a that is located at a distance 1 meter from C point.

From SFD, Shear force at cross section at a-a equal to shear force at C is 15.4 kilo newtons minus the resultant UDL from C point to a-a cross section is 10 kilo Newtons equal to 5.4 kilo Newton.

From BMD, bending moment at cross section at a-a- equal to area of shear force diagram from C point to a-a cross section is 5.4 times 1 meter plus 10 kilo Newton times 1 meter divided by 2 equal to 10.4 kilo Newton meters. The SFD is positive. So bending moment at a-a cross section also positive bending moment is 10.4 kilo Newton meters. The positive bending moment means top part of the beam in compression and bottom part of the beam is tension.

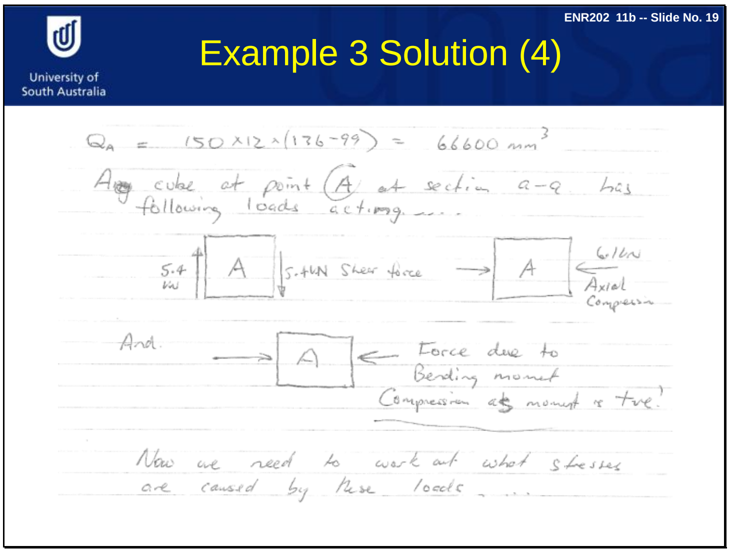
Slide 18



Next, you need to find section properties of the beam. You know that this beam have T-cross section. You need to find centroid and moment of inertia of T-cross section. You studied in lecture 5a that how to calculate centroid and moment of inertia of combined regular shapes of cross section. Here we have two rectangles with 150 mm by 12 mm as a flange and 130 mm by 15 mm as flange. We need to indicate x-axis and y-axis before the calculation of centroid and moment of inertia. Area of flange is 150 times 12 and area of web is 130 times 15. The centroid of flange from x axis 136 mm and centroid of web from x axis is 65 mm. Finally you will get centroid of T cross section equal to 99 mm from x axis as shown in the figure.

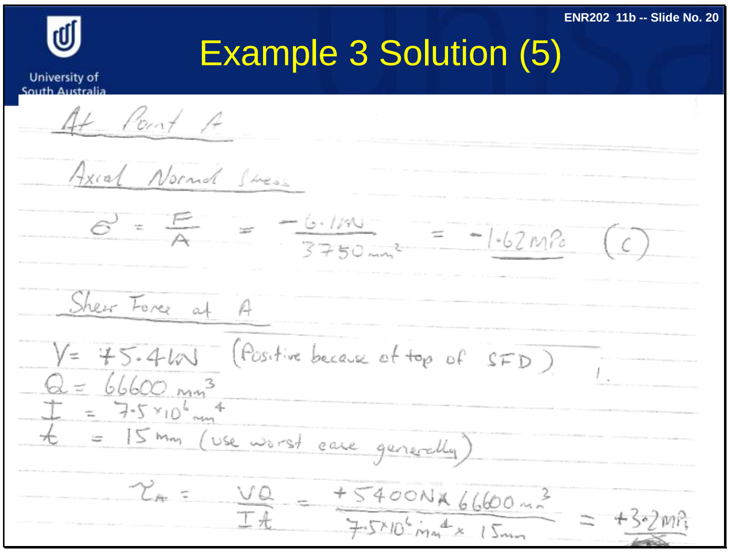
Use self moment of inertia of rectangle and parallel axis theorem to calculate moment of inertia about centroid x-x axis. The self moment of inertia of flange equal to 150 times 12 cube divided by 12 and The self moment of inertia of web equal to 15 times 130 cube divided by 12. you know parallel axis theorem that self moment of inertia of flange plus area of flange times square of the distance between centroid of T-cross section and centroid of flange . The centroid of T-cross section is 99 mm from x axis. The centroid of flange is 136 mm from x axis. So that the distace between centroid of T-cross section and centroid of flange is 136-99. Again use parallel axis theorem that self moment of inertia of web plus area of web times the square of distance between centroid of T-cross section, which is 99 mm minus 65 mm. Finally we can get moment of inertia about centroid x-x axis equal to 7.5 times 10 to the power of 6 mm to the power of 4.

Slide 19



The first moment of area at point A is 66 600 millimetres to the power of 3. (If you need to revise calculation of the first moment of area, look back at lecture summary 6a.) It is very important to calculate the first moment of area on the Q value at A. The shear force at A is 5.4 kilo Newtons, which we calculated from the shear force diagram in slide 17. The axial compression at A is 6.1 kilo Newtons, which we calculated in slide 16. Now we need to work out what stresses are caused by these actions; that is, shear force, axial compressive force, and bending moment.

Slide 20

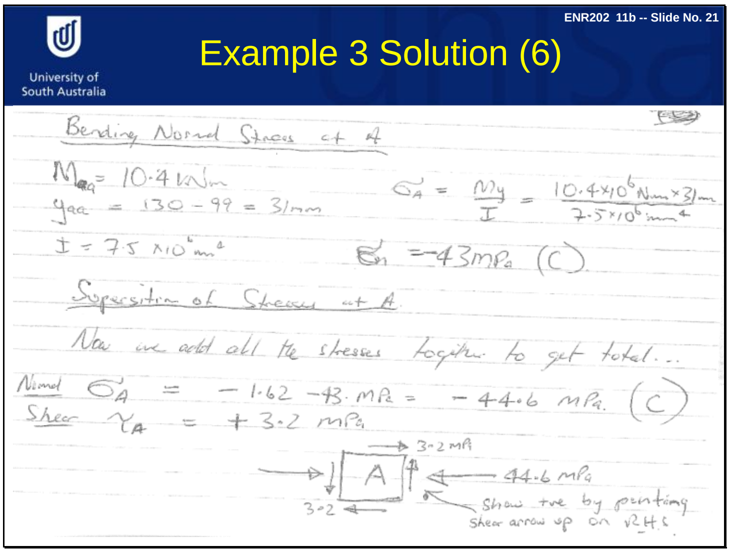


We have already known the shear force and bending moment at a-a cross section and centroid, area, moment of inertia at a-a T- cross section. Now, we can calculate stress at A point and B point, the point A is located at junction of flange and web of T-cross section and point B is located at bottom of web. the cross section a-a is located at a distance 1 meter from C point.

First, calculate stress state at point A. Axial compressive normal force at a-a cross section is horizontal reaction force at D point with a magnitude of 6.1 kilo Newtons. There is a negative normal stress at A point due to the compressive normal force. The uniform compressive normal stress throughout the cross section a-a due to compressive axial normal force is equal to 6.1 kilo Newtons divided by area of T cross section 3750 square mm, which is equal to minus 1.62 Mega Pacals.

Next, calculate shear stress at A point due to shear force at a-a cross section. The basic theory of shear stresses due to shear force has been introduced in lecture 6a. The first moment of area at point A equal to 66600 mm to the power of 3. The shear stress formula is V times Q divided by It. We can get shear stress at A point equal to 3.2 Mega Pascals. Here I would like to mention that shear force V and moment of inertia I is same for whole T-cross section. However calculation of Q and t (cutting width) changes at every point of T cross section. That means Q and t values are different between A point and B point.

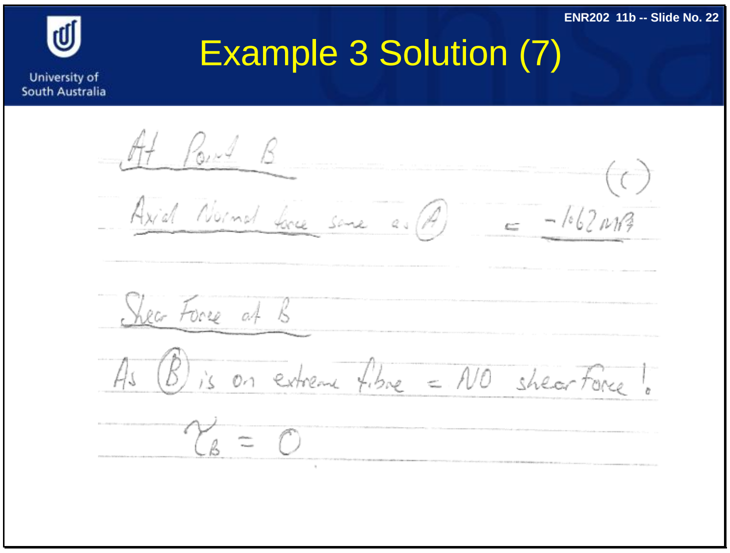
Slide 21



The bending moment at a-a cross section is 10.4 kilo Newton meters. The distance between centroid of T cross section to A point location is 31 mm. Finally you will get bending normal stress at A point equal to 43 Mega Pascal. As mentioned before, this bending moment is positive bending moment at a-a cross section. The top part of beam is compression and bottom part of the beam is tension. The point A located in top part of the beam. Therefore there is a compressive bending normal stress at A point, minus 43 Mega Pascal.

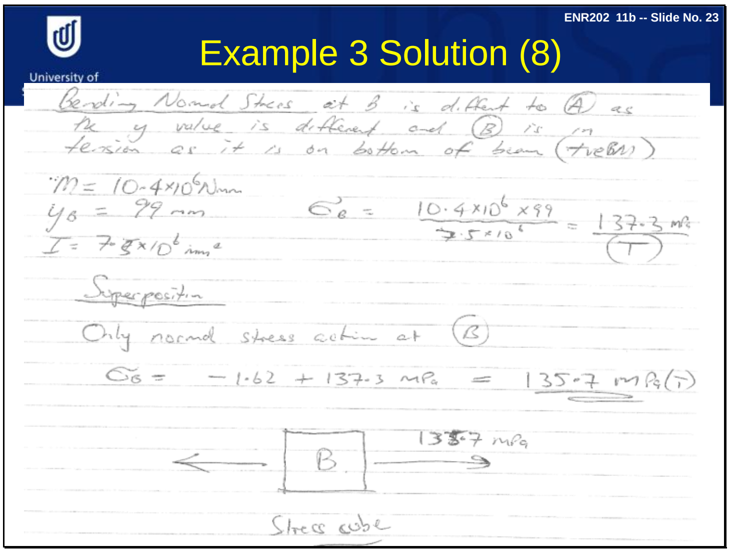
Now, we know axial compressive normal stress due to axial compressive normal force is minus 1.62 Mega Pascal and compressive bending normal stress due to bending moment is minus 43 Mega Pascal and sum of both of them equal to minus 44.6 Mega Pascal. We already calculated shear stress at point A equal to 3.2 Mega Pascal. We can draw the state of stress at point A as shown in the figure. We have positive shear force at a-a cross section. So at right edge of stress cube, the shear stress 3.2 Mega Pascal going upward direction.

Slide 22



Now, we can calculate state of stress at point B. The axial compressive normal stress due to axial compressive normal force uniform throughout the a-a cross section. Therefore, the axial compressive normal stress is same as point A which is equal to minus 1.62 Mega Pascals. Shear stress at point is equal to zero because point B is on extreme fibre means bottom of flange and Q value is zero at point B.

Slide 23



The bending moment at a-a cross section is 10.4 kilo Newton meters. The point B is located at bottom of web. The distance between centroid of T cross section to B point location is 99 mm. Finally you will get bending normal stress at B point equal to 137.3 Mega Pascal. I mentioned before, this bending moment is positive bending moment at a-a cross section. The top part of beam is compression and bottom part of the beam is tension. The point B located in bottom part of the beam. Therefore, we have tensile bending normal stress at B point which is positive 137.3 Mega Pascal. The axial compressive normal stress at point B is equal to minus 1.62 Mega Pascal and sum of both normal stresses are minus 1.62 Mega Pascal plus 137.3 Mega Pascal equal to 135.7 Mega Pascal. You can see state of stress or stress cube at point B as shown in the figure.

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Thank you for your attention. This completes the combined load topic. In the next lecture summaries, we start stress transformation and principal stresses.