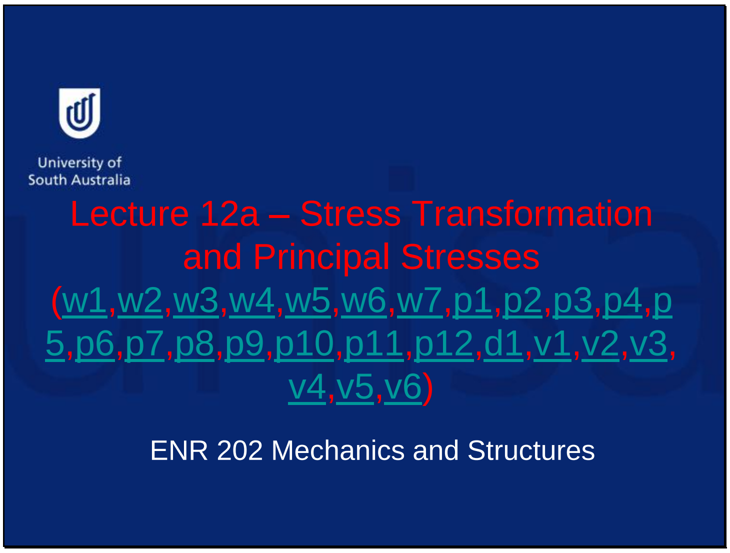
ENR202 Mechanics of Materials Lecture 12A Slides and Notes

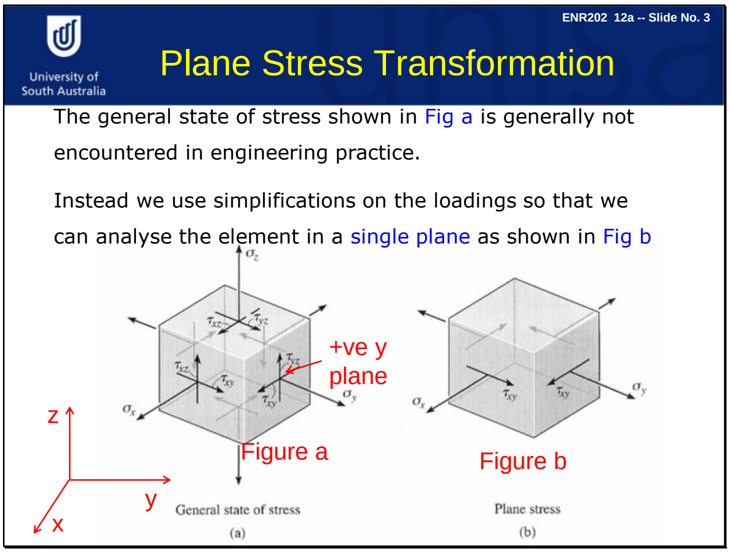
Slide 1

Slide 2



Hello, and welcome to lecture Summary 12a. As usual, use the live links to look at further information on these concepts.

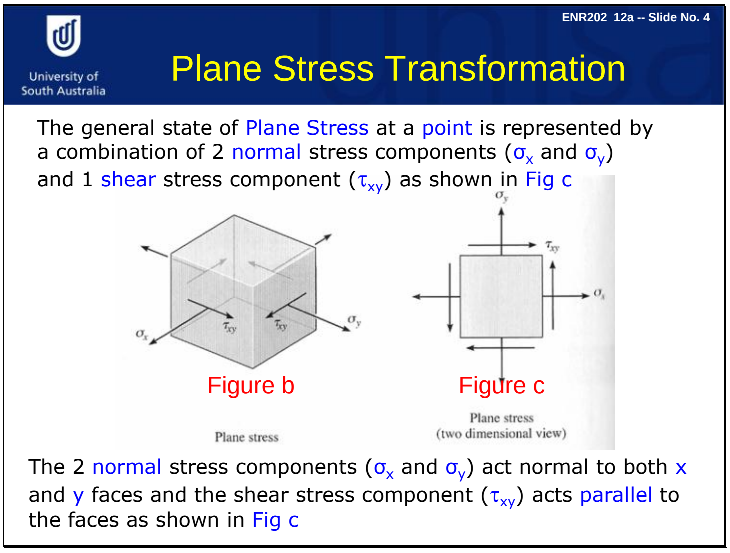
Slide 3



In figure 'a', we have six planes in a block. The 6 planes are the positive and negative x, y and z planes. In each plane, we have one normal stress and two shear stresses. As you know, stress acting perpendicular to a plane is called normal stress, and stress acting parallel to a plane is called shear stress. For example, consider the positive y plane, as shown in the figure. We use sigma for normal stress, tou for shear stress. After the sigma or tou symbol, we have two letters. The first letter indicates the plane and the second letter indicates the direction of the axis. However, we use one letter for normal stress because the plane and axis are the same letter. That means that instead of sigma yy, we use sigma y. Sigma yy means normal stress at the y plane in the y direction, tou yz means shear stress in the y plane in the z direction, and tou yx means shear stress in the y plane in the x direction. However tou xy is equal to tou yx, so we use tou xy instead of tou yx in the positive y plane. This is called the general state of stress, as shown in figure a. However, it is not usually encountered in general engineering practice.

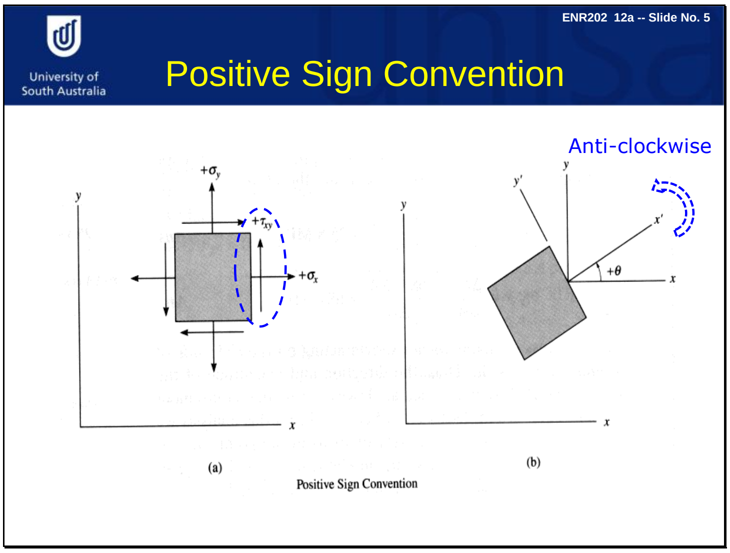
This general state of stress is quite complex. However, we use simplifications on the loads so that we can analyse the element in a single plane, as shown in figure b. As you can see in figure b, we have simplified down from a 3 dimensional state of stress to a 2 dimensional state of stress, by leaving out the z axis. The 2 dimensional state of stress is called the single plane of stress. Whatever stresses are shared with z are left out. For example, consider the same positive y plane. We have sigma y, sigma yz, and sigma xy, as shown in figure a. We have left out sigma yz in figure b. Look at the other planes in the two figures and compare them. In a plane state of stress, we have one normal stress and one shear stress in the x-y plane, as shown in figure b.

Slide 4



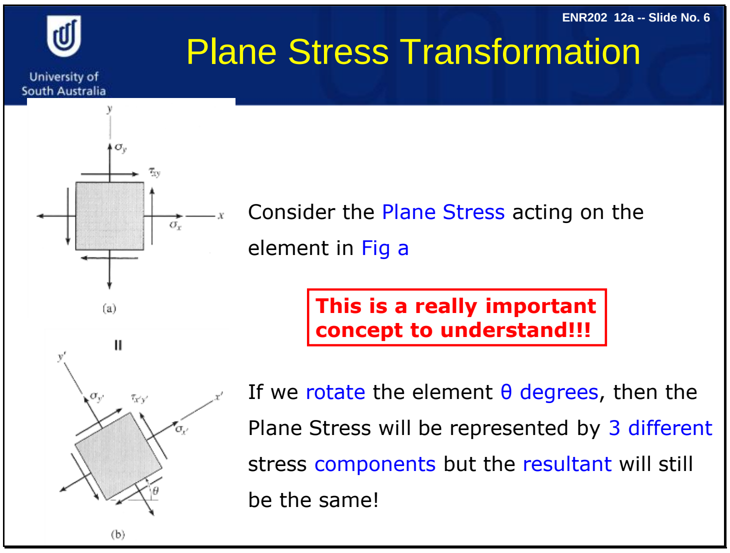
We can talk about a plane state of stress in either the x-y plane, the y-z plane or the x-z plane. For example, consider the x-y plane as shown in figure b. We have 3 stresses acting on the plane stress. They are two normal stresses, sigma x and sigma y, and one shear stress, tou xy. Tou xy is parallel to the face, and sigma x and sigma y act normal for both x and y faces. We simply draw figure c which is a two dimensional view.
If you want to see another example, go back and have a slide 3 of lecture 11b. Assume that you want to know the state of plane stress at point B, as shown in the square hollow cross section for Exercise 1. We have one normal stress and one shear stress. That means that we have sigma x equal to a compressive normal stress of 4.3 Mega Pascals, sigma y equal to zero, and a shear stress of 4 Mega Pascals. You need to work out combined loading to find the normal stresses sigma x and sigma y, and the shear stress tou xy for the plane state of stress. Once you know the plane state of stress, then you can transform it into a plane in such a way that there are no shear stresses on that plane. This the reason we study stress transformation and principal stresses. Our aim is to find a plane for which there are no shear stresses. We call this plane the principal stress plane.

Slide 5



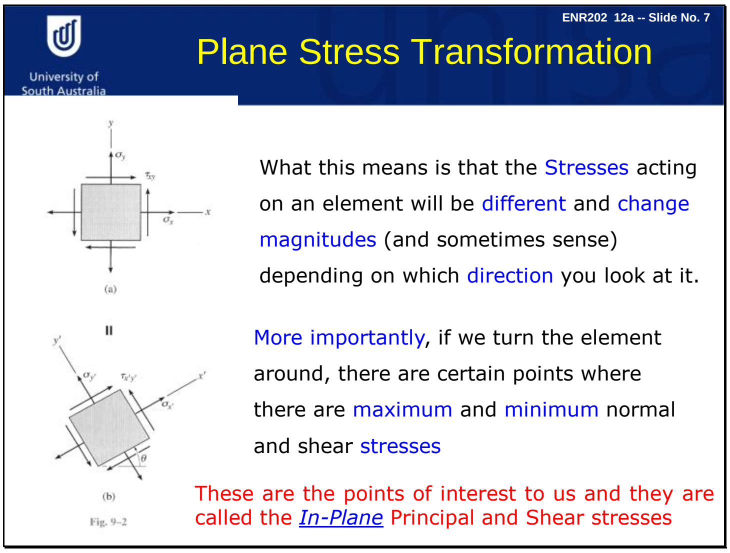
First, you need to follow sign convention to find the principal stress plane. If the normal stress on the plane is acting away from the plane, it is called positive. If the shear stress on the right edge of the element is acting in an upward direction, it is positive, as shown in figure a. If you turn the right edge of the element in an anti-clockwise direction, it is called positive theta, as shown in figure b. As you can see in figure b, we turn the element from the x-y plane into x dash –y dash plane in an anti-clockwise direction. That is called the positive angle.

Slide 6



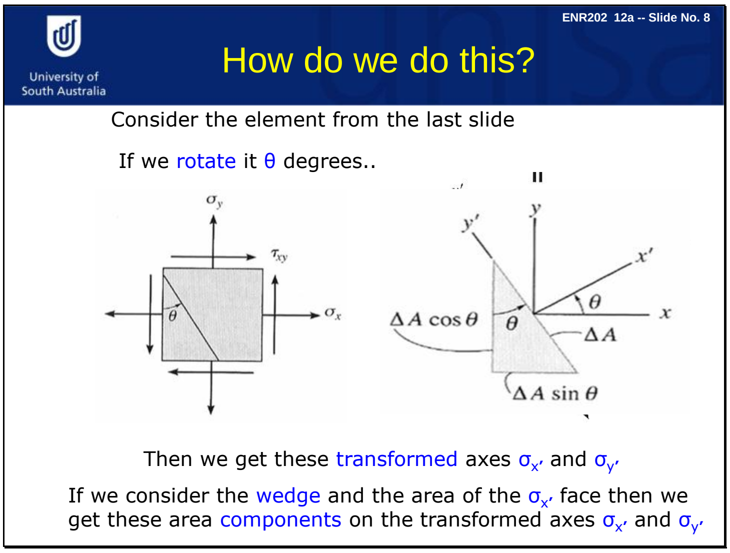
The is a really important concept in this slide. Consider the plane of stress acting on the element in the x-y plane, as shown in figure a. Now we rotate the element theta degree from an x-y plane element to a x dash – y dash plane element as shown in the figure b. However, we have another different set of stress components in the x dash – y dash plane. Even so, the resultant of stress in the x dash – y dash plane is still same. That means the state of stress in the x-y plane is same as the state of stress in the x dash – y dash plane. It is the same because we calculate the stress at a point only.

Slide 7



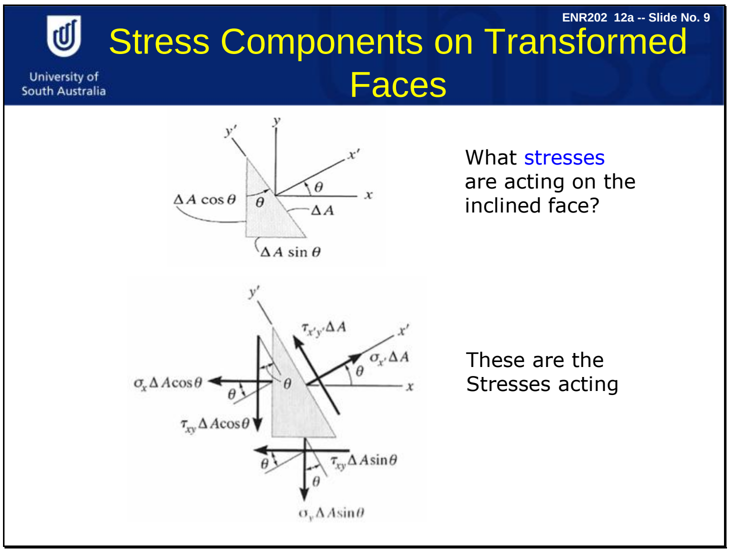
This means that the stresses acting on an element will be different and change in magnitude, depending on the direction you look at it. You can turn the element 360 degrees. We need to know the planes which have the maximum and minimum normal stresses, and the maximum shear stresses acting on the planes. These are the points of interest to us and they are called the *In-Plane* Princ.ipal and Shear stresses

Slide 8



How do we find the in plane principal stresses that are the maximum and minimum normal stresses, and the in plane shear stress that is the maximum shear stress? Consider the element that has two positive normal stresses (sigma x and sigma y) and one positive shear stress (tou x y) in the x-y plane, as shown in the left figure. Now we rotate the x-y plane element into an x dash – y dash element in positive theta degrees, as shown in the right figure. The x dash – y dash wedge area is ‘delta A’ and the area of left wedge is ‘delta A cos theta’ and the bottom wedge is ‘delta A sin theta’, as shown in the right figure.

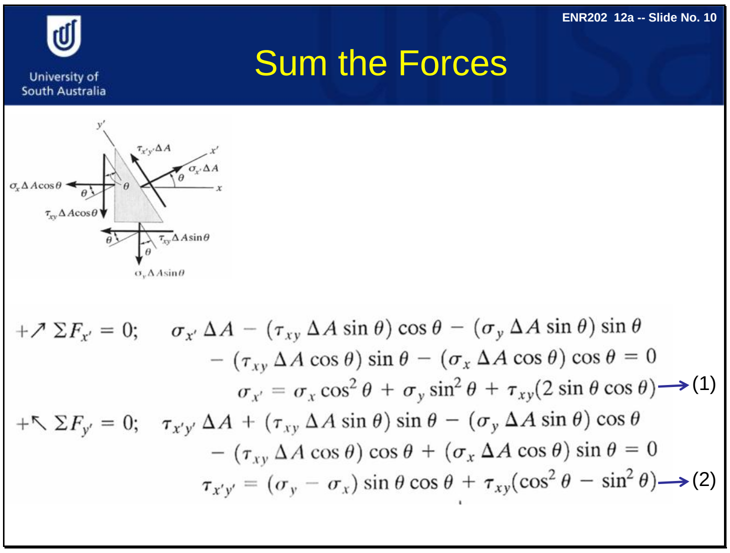
Slide 9



Now we need to find the stresses acting on the inclined face. Suppose the normal stress on the inclined surface is sigma x dash, and the shear stress on the inclined surface is tou xy dash. We need to know these normal and shear stresses on the inclined plane. On the left edge, the normal stress is sigma x, the shear stress is tou xy, and the area is 'delta A cos theta'. On the bottom edge, the normal stress is sigma y, the shear stress is tou xy, and the area is 'delta A sin theta'. You know that the stress is equal to the force divided by the area. Now, we want to know the forces on the left, bottom and inclined edges. We know the stresses and area of each edge, so we can easily calculate the force, which is a product of stress and area. On the left edge, the normal force is equal to 'sigma x' multiplied by 'delta A cos theta'. The shear force is 'tou xy' multiplied by 'delta A cos theta'. In the bottom edge, the normal force is equal to 'sigma y' multiplied by 'delta A sin theta' and the shear force is 'tou xy' multiplied by 'delta A sin theta'. In the inclined edge, the normal force is equal to 'sigma x dash' multiplied by 'delta A' and the shear force is 'tou x dash y dash' multiplied by 'delta A'.

Now, resolve the normal forces and shear forces on the left edge and the bottom edge in the direction of the x dash and the y dash axis (that means the inclined edge axis, as shown in the bottom figure).

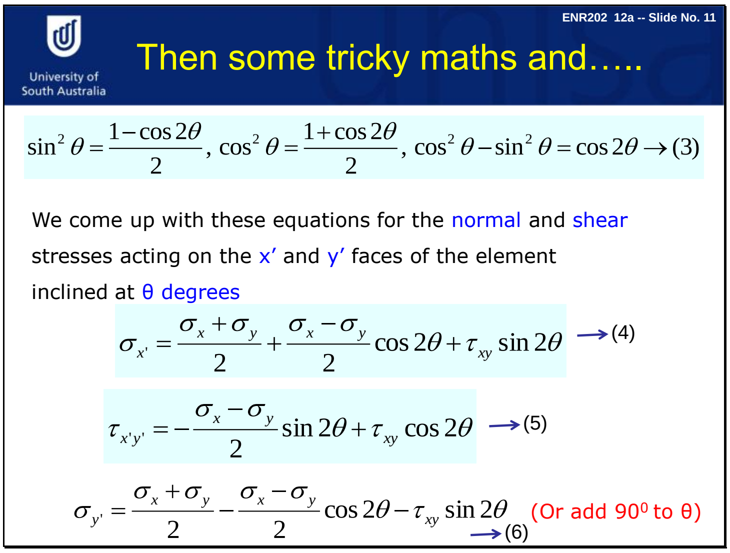
Slide 10



Now, we resolve the normal forces and shear forces on the left edge and the bottom edge in the direction of the x dash and the y dash axis. Then, we apply the equilibrium equation which states that all forces in the x dash axis direction are equal to zero. On the inclined edge, the normal force sigma x dash times delta A in the x dash axis direction is positive. On the bottom edge, resolve the shear force 'tou xy times delta A sin theta' in the x dash direction; that is, 'tou xy times delta A sin theta times cos theta' is negative. Resolve the normal force 'sigma y times delta A sin theta' in the x dash direction; that is, 'sigma y times delta A sin theta times sin theta' is negative. On the left edge, resolve the shear force 'tou xy times delta A cos theta' in the x dash direction; that is, 'tou xy times delta A cos theta times sin theta' is negative. Resolve the normal force 'sigma x times delta A cos theta' in the x dash direction; that is, 'sigma x times delta A cos theta times cos theta' is negative. Finally, you will get the normal stress on the inclined edge; that is, sigma x dash in terms of sigma x, sigma y and tou xy and direction theta as shown in equation 1.

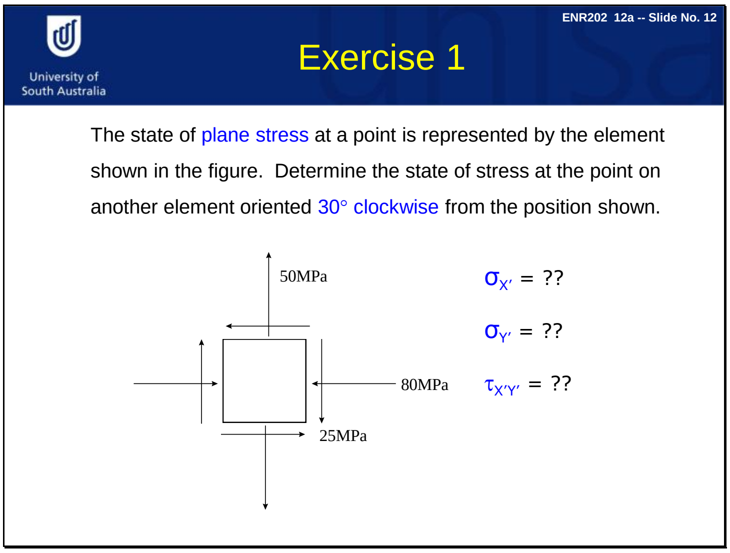
Now, we resolve the normal forces and shear forces on left edge and bottom edge in the direction of y dash axis. Then, we apply the equilibrium equation that all forces in the y dash axis direction are equal to zero. On the inclined edge, the shear force tou xy dash times delta A in the y dash axis direction is positive. On the bottom edge, resolve the shear force 'tou xy times delta A sin theta' in the y dash direction; that is, 'tou xy times delta A sin theta times sin theta' is negative. Resolve the normal force 'sigma y times delta A sin theta' in the y dash direction; that is, 'sigma y times delta A sin theta times cos theta' is negative. On the left edge, resolve the shear force 'tou xy times delta A cos theta' in the y dash direction; that is, 'tou xy times delta A cos theta times cos theta' is negative. Resolve the normal force 'sigma x times delta A cos theta' in the y dash direction; that is, 'sigma x times delta A cos theta times sin theta' is negative. Finally you will get the shear stress on inclined edge: that is, tou x dash y dash in terms of sigma x, sigma y and tou xy and direction theta as shown in equation 2.

Slide 11



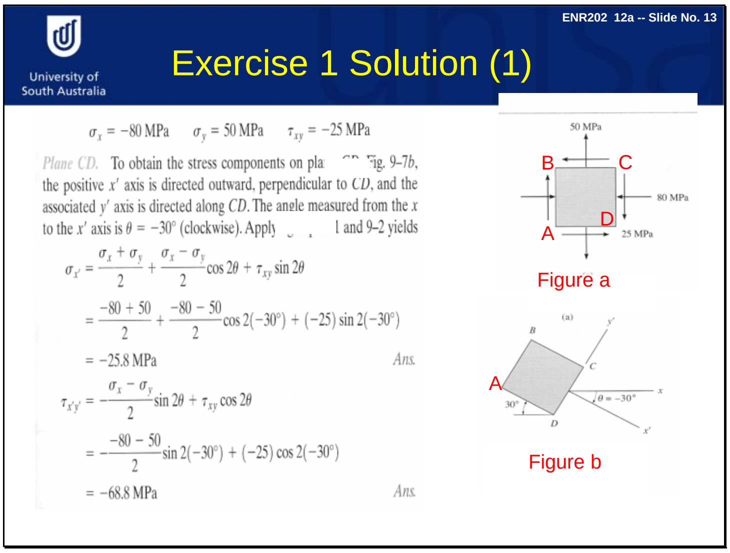
By using trigonometric equations as shown in equation 3, we come up with equations 4 and 5 to calculate normal stress 'sigma x dash' and shear stress 'tou x dash y dash' in the x dash y dash plane. If you want to calculate another normal stress 'sigma y dash', simply add 90 degrees to theta in equation 6. Now you have two normal stresses 'sigma x dash' and 'sigma y dash', and shear stress 'tou x dash y dash' in the x dash – y dash plane.

Slide 12



This exercise is based on equations 4,5 and 6 from the previous slide. The state of plane stress at a point has two normal stresses and one shear stress, as shown in the figure. The compressive normal stress in the x direction is 80 Mega Pascals, the tensile normal stress in the y direction is 50 Mega Pascals, and the shear stress on the right edge of the plane is 25 Mega Pascals in a downward direction, as shown in the figure. You need to calculate the state of stress at the same point on another plane with 30 degrees in a clockwise direction from the x-y plane. That means you need to calculate two normal stresses, 'sigma x dash' and 'sigma y dash', and shear stress 'tou x dash y dash' in the x dash – y dash plane which is 30 degrees in a clockwise direction. The solution to this problem is on the next slide.

Slide 13



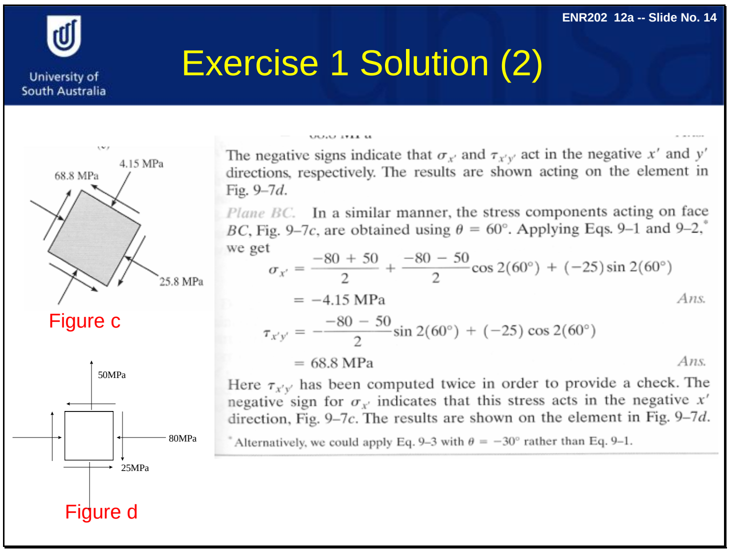
First, work out the sign convention (as explained in slide 5). Tensile normal stress is positive. If the shear stress on the right edge of the element is acting in an upward direction it is positive, if you rotate an element in an anti-clockwise direction, it is positive.

Now, compare the sign convention in slide 5 and in this example. In example 1, we have compressive normal stress in the x plane. That means that sigma x is equal to minus 80 Mega Pascals. We have tensile normal stress in the y plane. That means that sigma y is equal to positive 50 Mega Pascals. We have shear stress on the right edge of the element acting in a downward direction. That means that 'tou xy' is equal to minus 25 Mega Pascals, as shown in figure a. Now we rotate the A-B-C-D element 30 degrees in a clockwise direction, as shown in figure b. That means that theta is minus 30 degrees as per our sign convention definition (slide 5).

Now we need to know the state of stress for the figure b element. We need to calculate normal stress and shear stress on the CD plane and the BC plane, as shown in figure b.

Second, we calculate the normal stress and the shear stress for the CD plane. The normal stress in the CD plane is calculated by using equation 4 in slide 11. From this, we can calculate normal stress in the x dash as being equal to minus 25.8 Mega Pascals on the CD plane. The shear stress in the CD plane is calculated by using equation 5 in slide 11. From this, we can calculate the shear stress 'tou x dash y dash' as being equal to minus 68.8 Mega Pascals on the CD plane.

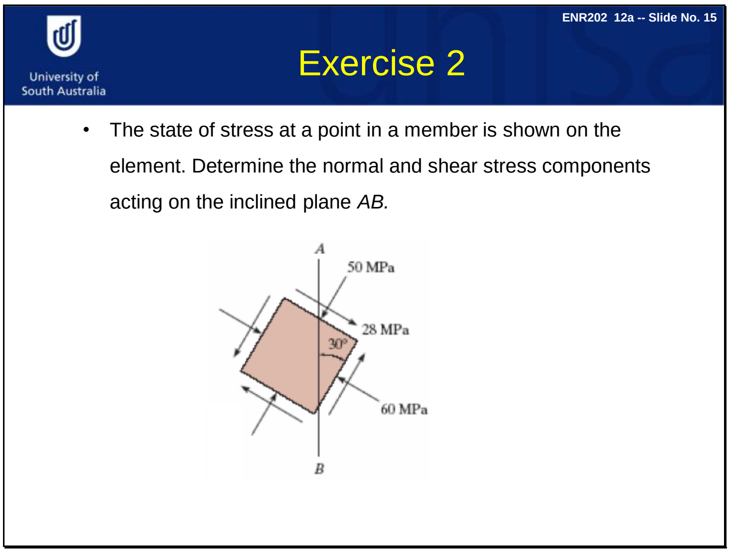
Slide 14



Now we calculate the normal stress and shear stress for the BC plane. The normal stress in the BC plane is calculated using equation 4 in slide 11. However, you need to add 90 degrees to theta to calculate the normal and shear stress in the BC plane because the angle between the CD plane and the BC plane is 90 degrees. Theta is minus 30 degrees plus 90 degrees, which is equal to positive 69 degrees. Finally, we calculate normal stress in the y dash place as equal to minus 4.15 Mega Pascals on the CD plane. The shear stress in the CD plane is calculated by using equation 5 in slide 11. From this, we calculate the shear stress 'tou x dash y dash' as being equal to minus 68.8 Mega Pascals on the CD plane.

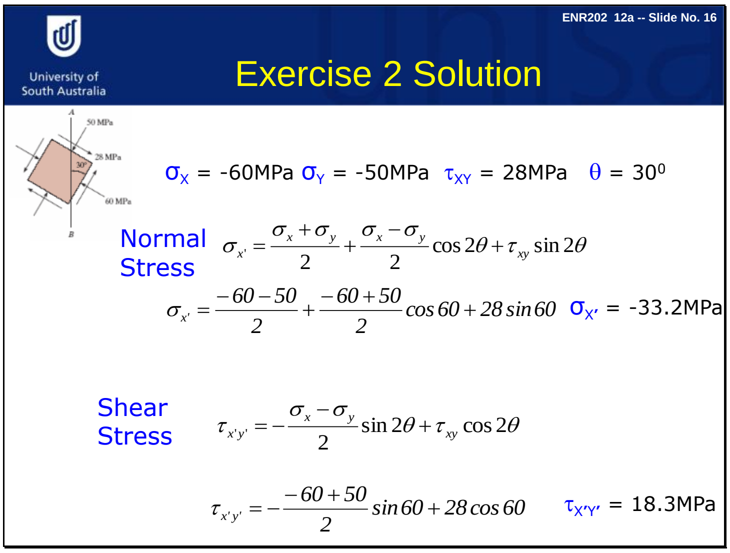
Note the important point that shear stress in the BC plane and the CD plane have the same magnitude and the same sign. Normal stress in the x dash plane is negative 25. 8 Mega Pascals, which means compressive. Normal stress in the y dash plane is negative 4.15 Mega Pascals, which also means compressive.

Slide 15



Now, pause the presentation and try to calculate the normal stress and shear stress for the AB plane for the given state of stress, as shown in the figure. A couple of hints ….we have compressive normal stresses in both the x and y planes of 60 Mega Pascals and 50 Mega Pascals. If you need to revise sign convention, look back at slide 7 in this lecture. Note that we rotate this element 30 degrees in an anti-clockwise direction (that is the AB plane). The solution is on the next slide.

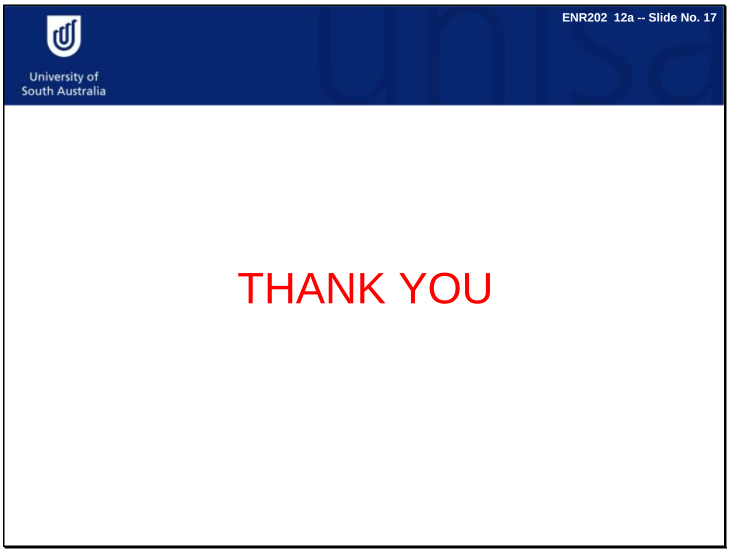
Slide 16



We have compressive normal stress in one plane. That means that sigma x is equal to minus 60 Mega Pascals. Another compressive normal stress in the y dash plane means that sigma y is equal to minus 50 Mega Pascals. The shear stress in the right edge plane is acting in an upward direction. That means that ‘tou xy’ is equal to positive 28 Mega Pascals. We rotate the right edge plane 30 degrees in an anti-clockwise direction. That means theta is equal to 30 degrees.

Now, we can calculate the normal stress and shear stress in the AB plane. The normal stress in the AB plane is calculated by using equation 4 in slide 11. Finally, we calculate the normal stress in the AB plane as being equal to minus 33.2 Mega Pascals. That means compressive normal stress on the AB plane. The shear stress in the AB plane is calculated by equation 5 in slide 11. From this, we can calculate that shear stress ‘tou x dash y dash’ is equal to 18.3 Mega Pascals. That means that the shear stress on AB plane is acting in an upward direction.

Slide 17



In the next lecture summary, we will continue with the same topic, but using the graphical method to find principal stresses and maximum shear stresses. Thank you for your attention.