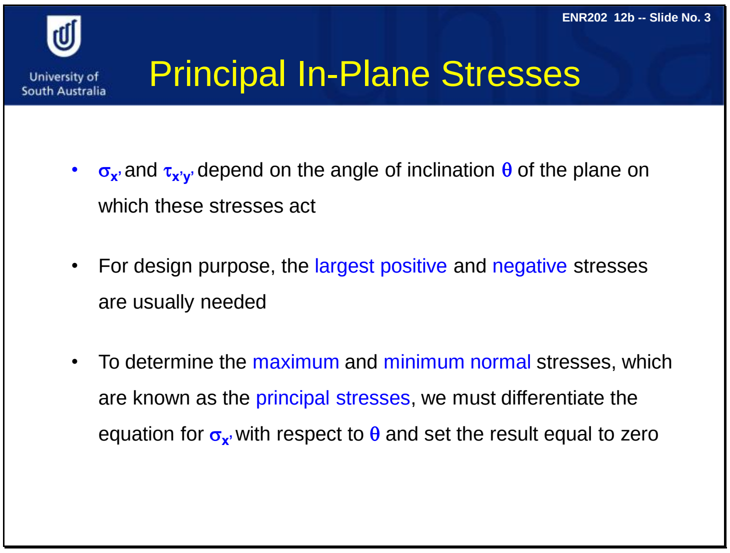
ENR202 Mechanics of Materials Lecture 12B Slides and Notes

Slide 1

Slide 2

Now, you know how to calculate normal stress and shear stress at any plane for given plane state of stress from previous lecture. Now we are interested on which plane we have maximum and minimum normal stress and which plane we have maximum shear stresses acting.

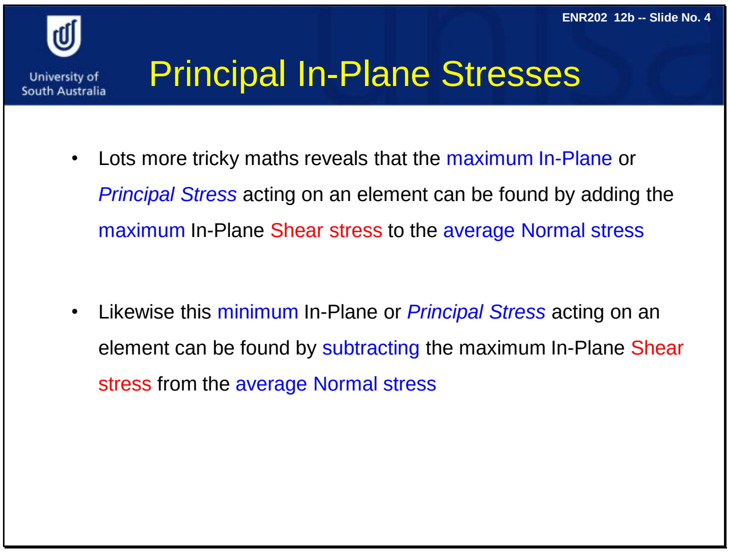
Slide 3



normal stress sigma x dash and shear stress tou x dash y dash depend on the angle of inclination theta of the plane on which we interested on these stresses. In engineering, you need to find largest positive stress and largest negative stress for design purpose.

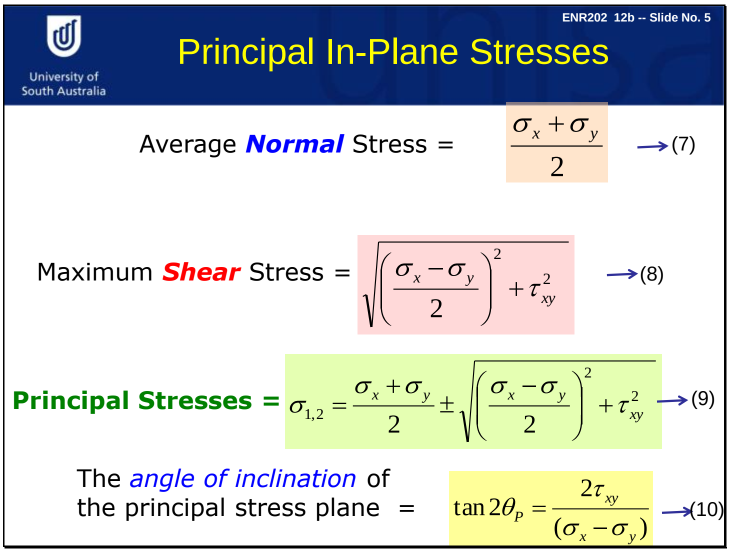
You studied in mathematics that if you want to determine maximum and minimum of any equation, we must differentiate the equation and the result equal to zero. We use same concept here, we are interested to find maximum and minimum normal stresses are known as principal stresses, we differentiate the equation 4 in slide 11 of previous lecture 12a with respect to theta and set the result equal to zero.

Slide 4



Differentiation of equation 4 of previous lecture 12a in slide 11 with respect to theta need so much mathematical back ground. If you are interested on this please have a look a book on Mechanics of Materials, seventh SI edition by author R.C. Hibbeler in the chapter 9 of pages 476 477 and 478 to get more information on differentiation of equation 4.

Slide 5

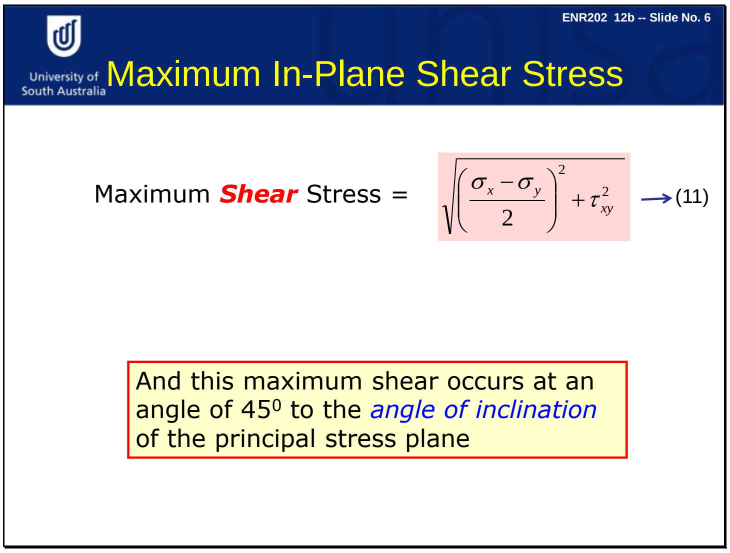


Finally we get average normal stress equal to average of normal stresses sigma x and sigma y of the given element as shown in the equation 7. Maximum shear stress equal to square root of sum of square root of sigma x minus sigma y divided by 2 and square of 'tou xy' as shown in the equation 8. The principal stresses equal to average normal stress plus or minus square root of maximum shear stress as per equation 9. If you use positive sign in equation 9 you will get maximum principal stress, if you use negative sign in equation 9 you will get minimum principal stress. At this stage I will announce that, we wont have any shear stresses in principal stresses planes. However, it is not compulsory that we wont get any normal stresses in maximum shear stress planes. Why?

If you want to know about this, you have to wait for some time. You will get answer for this in coming slides.

The angle of inclination of principal stress plane calculated by using equation 10. theta p is nothing but inclination of principal plane from given plane state of stress at a point.

Slide 6

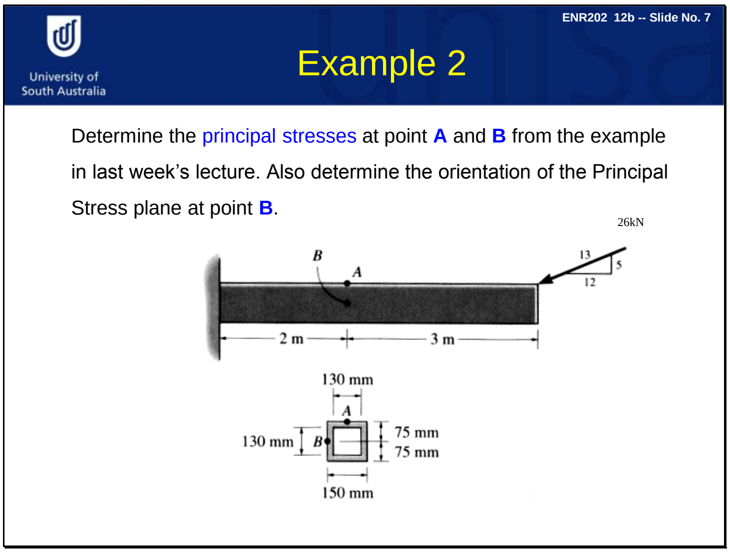


The maximum in plane shear stress equal to square root of sum of square root of sigma x minus sigma y divided by 2 and square of 'tou xy' as shown in the equation 11. If you want to know maximum shear stress plane, you just add 45 degree to the angle of inclination of the principal stress plane that is 'theta p' as explained in previous slide.

You don't need to remember these equations. However you need to know how to use these equations. What is meaning of each equation.

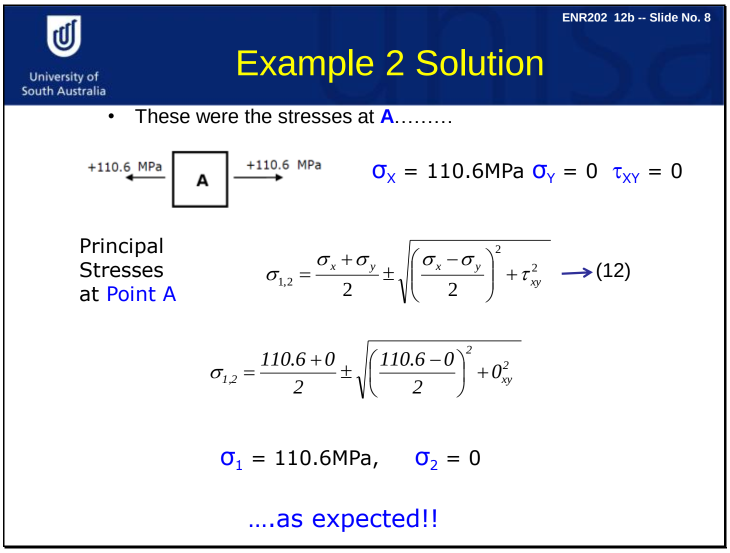
If you solve a problem based on these equations, you can understand that how to use these equations. Let us do one problem on this.

Slide 7



Do you remember this problem? We calculated normal stress and shear stress at points A and B in lecture 11b of example 1. I recommend you that go back to example 1 of lecture 11b and understand very well. We continue same thing from that problem to determine principal stresses at points A and B as shown in the bottom figure. We also need to determine the orientation of principal stress plane at point B.

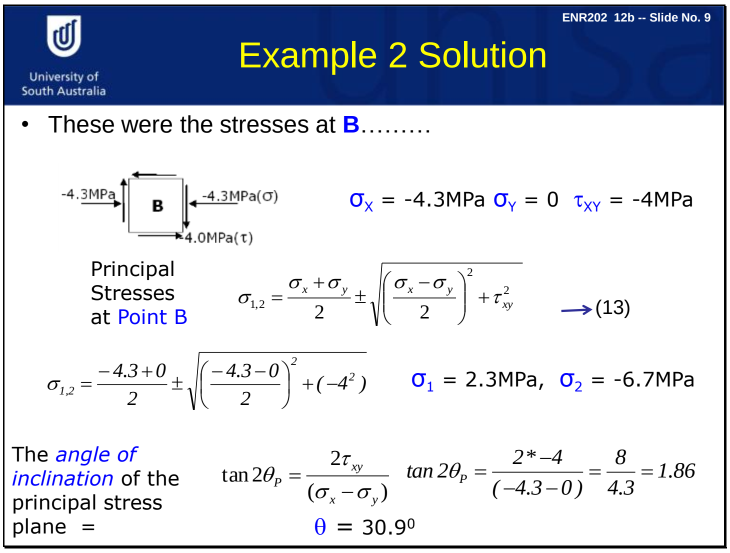
Slide 8



From example 1 of lecture 11b, we will get state of stress at point A is we don't have any shear stresses at point A and we have tensile normal stress is 110.6 Mega Pascals from lecture 11b, slide number 5. go back and have a look on sign convention that is previous lecture 12a in slide 5. we have tensile normal stress at point A that means sigma x equal to positive 110.6 Mega Pascals. We don't have any normal stress in y plane that means sigma y equal to zero and also don't have any shear stresses at A that is 'tou xy' equal to zero. You know the formula that calculation of principal stresses at point A in slide 5 of this lecture or see equation 12 of this slide.

At this stage I remind you one thing that, we don't have any shear stresses in principal stresses. Here, at point A don't have any shear stresses, that means this state of stress is nothing but principal stress plane. However, we need to show this by using the principal stresses formula. We substitute sigma x, sigma y and tou xy of point A into principal stresses formula. Finally we get maximum principal stress sigma 1 equal to 110. 6 Mega Pascals, minimum principal stress sigma 2 equal to zero that means these values are as we expected.

Slide 9



The state of stress at point B as shown in the figure that compressive normal stress in x plane is negative 4.3 Mega Pascals, and we don't have any normal stress in y plane that means sigma y equal to zero, and shear stress at right edge of element 4 Mega Pascals acting downward direction that means 'tou xy' equal to negative 4 Mega Pascals.

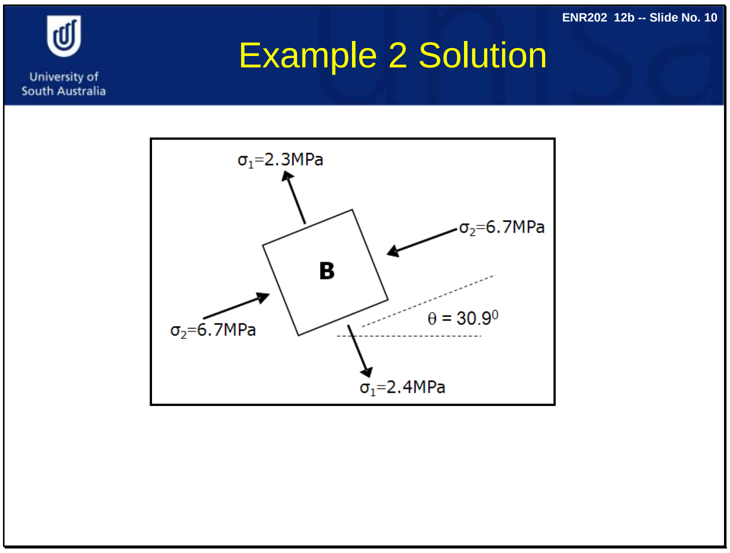
Now, substitute the normal stresses sigma x and sigma y and shear stresses tou xy into principal stresses equation as shown in the equation 13 that same as equation 12.

Finally you will get maximum principal stress sigma 1 equal to 2.3 Mega Pascals that means tensile normal stress on principal plane, minimum principal stress sigma 2 equal to negative 6.7 Mega Pascals that means compressive normal stress on principal plane.

Now, we need to find the angle of inclination of the principal plane. Use the formula 10 in slide 5 of this lecture. Finally you will get theta positive 30.9 degrees that means we need to rotate the state of stress of current position of point B into 30.9 degrees in anti clock wise direction, then you will find the principal stress plane.

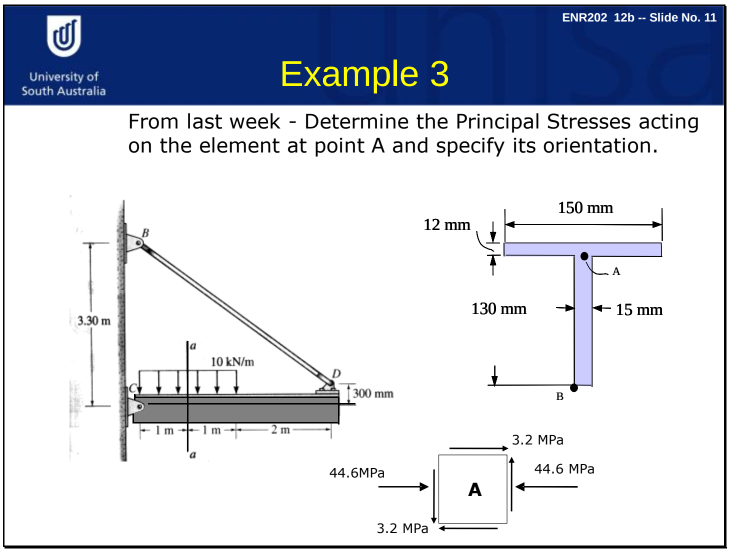
I recommend you at this stage that, you use the formula 5 of lecture 12a in slide 11 to calculate shear stress. Just use this theta and sigma x and sigma y at point B in this formula. Observe that shear stress is zero at this plane.

Slide 10



Can you have a look at this figure. We rotate the state of stress at point B 30.9 degrees in anti clock wise direction to get principal stress plane. And you can see maximum and minimum principal stresses as shown in the figure. As I said, we don't have any shear stresses in principal planes.

Slide 11



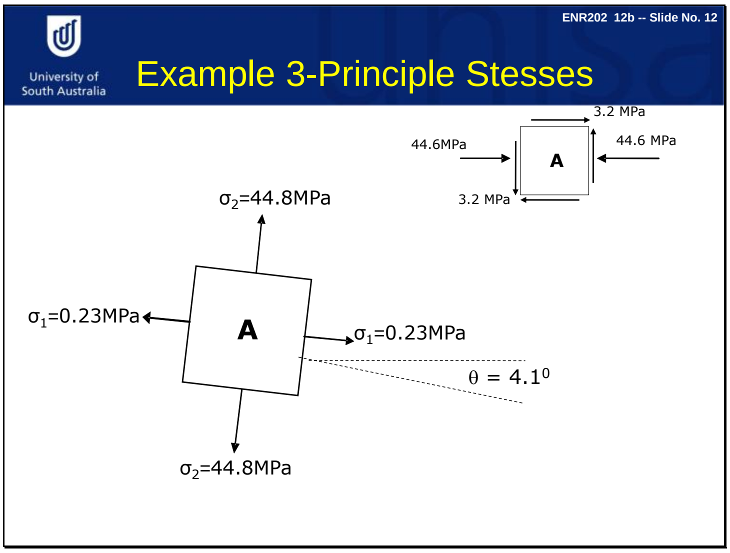
Do you remember this problem? We calculated normal stress and shear stress at points A and B in lecture 11b of example 3. I recommend you that go back to example 3 of lecture 11b and understand very well. We continue same thing from that problem to determine principal stresses at points A and B as shown in the bottom figure. We also need to determine the orientation of principal stress plane at point A.

The given state of stress at point A as shown in the figure came from slide 21 of lecture 11b. Can you remember sign convention.

I recommend you to calculate magnitude and direction of principal planes and maximum principal planes by formulas given in the slides 5 and 6 of this lecture and compare with your results.

Let me push you to get initiation. On right edge of element have compressive normal stress 44.6 Mega Pascals that is sigma x equal to minus 44.6 Mega Pascals and shear stress acting upward direction that means tou xy equal to positive 3.2 Mega Pascals. On top edge of element don't have any normal stress that means sigma y equal to zero. Can you start now? We know sigma x, sigma y and tou xy to calculate magnitude and direction of principal planes and maximum principal planes by formulas given in the slides 5 and 6 of this lecture. Don't worry you can have answer in next slides.

Slide 12



The state of stress at point A as shown in the figure that compressive normal stress in x plane is negative 44.6 Mega Pascals, and we don't have any normal stress in y plane that means sigma y equal to zero, and shear stress at right edge of element 3.2 Mega Pascals acting upward direction that means 'tou xy' equal to positive 3.2 Mega Pascals.

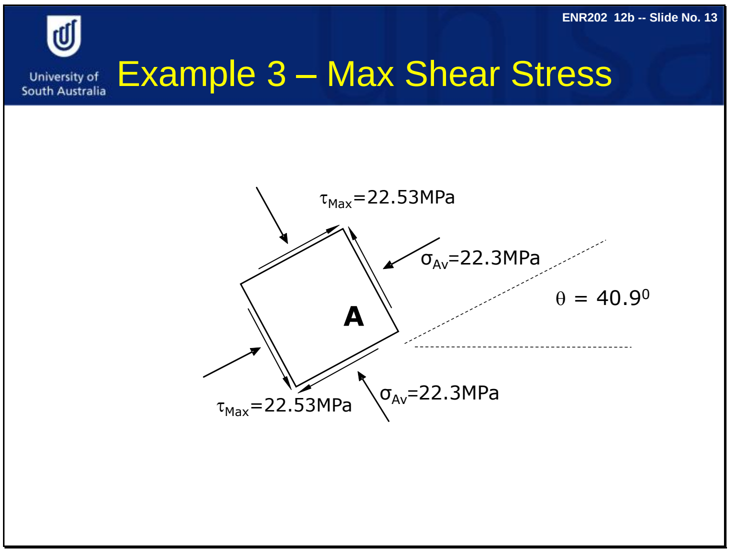
Now, substitute the normal stresses sigma x and sigma y and shear stresses 'tou xy' into principal stresses equation.

Finally you will get maximum principal stress sigma 1 equal to 0.23 Mega Pascals that means tensile normal stress on principal plane, minimum principal stress sigma 2 equal to positive 44.8 Mega Pascals that means tensile normal stress on principal plane.

Now, we need to find the angle of inclination of the principal plane. Use the formula 10 in slide 5 of this lecture. Finally you will get theta negative 4.1 degrees that means we need to rotate the state of stress of current position of point B into 4.1 degrees in clock wise direction. So, we find the principal stress plane.

Can you use the formula 5 of lecture 12a in slide 11 to calculate shear stress. Just use this theta and sigma x and sigma y at point B in this formula. Observe that shear stress is zero at this plane or not.

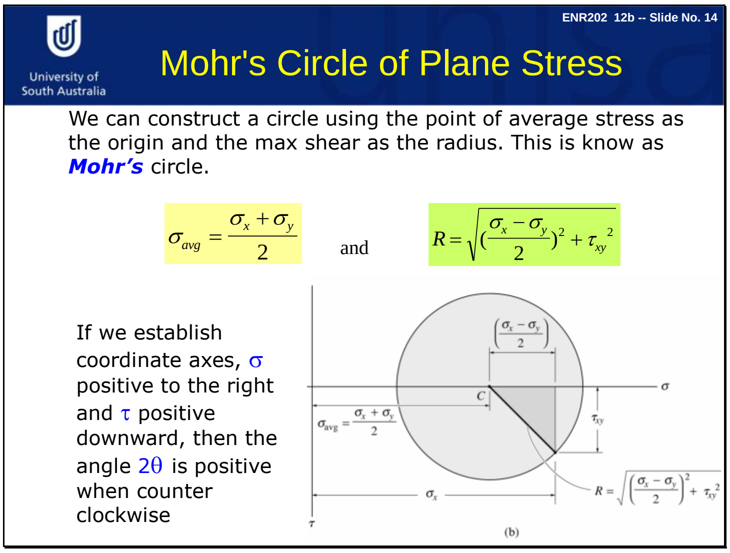
Slide 13



Now we need to find maximum shear stress value and orientation. If you substitute sigma x, sigma y and tou xy of given state of stress of point A in the formula 11 of slide number 6. we will get maximum shear stress equal to 22.53 Mega Pascals. The normal stress on the maximum shear stress plane is average normal stess calculated by the formula 7 of slide 13 of this lecture is compressive normal stress 22. 3 Mega Pascals. The orientation of the maximum shear stress plane, just you add 45 degrees to principal plane orientation that is negative 4.1 degrees plus 45 degrees equal to 40.9 degrees. Do you get same results what we discussed here. If you are unable to get these results. Go back and have a look. Still, you didn't get the same results please contact me with your explanation.

We have complicated equations to calculate magnitude and direction of principal plane and maximum shear stress planes. Is there any easy method for this. We have one easy method that is graphical method is explained in next.

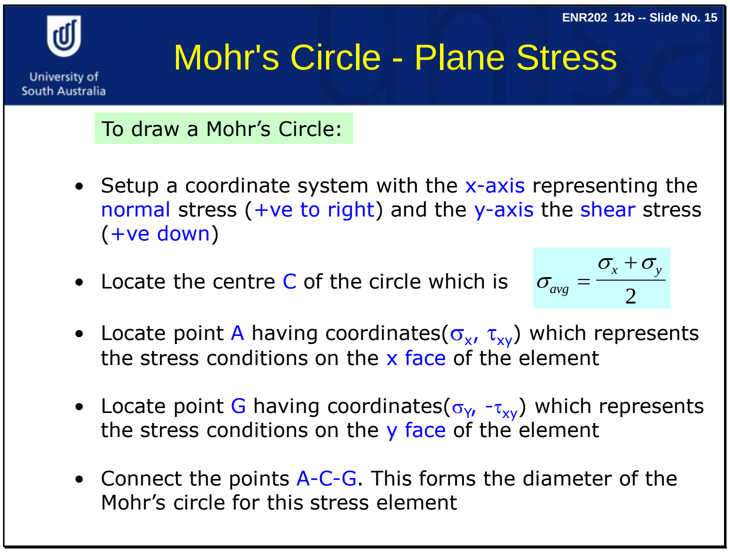
Slide 14



We use Mohr's circle to find magnitude and direction of principal stress and maximum shear stress. We can construct a circle using the point of average normal stress is sigma x plus sigma y divided by 2 as the origin of circle and maximum shear stress as the radius of circle. This circle known as Mohr's circle.

The average normal stress is the point C that centre of circle as shown in the figure. We establish co ordinate axes, if normal stress is positive that is right, if shear stress is positive that is downward that means horizontal axis is normal stress and vertical axis is shear stress.

Slide 15



Now, I explain how to draw Mohr's circle.

First, we need to set up a coordinate system that right x-axis as positive normal stress and down y-axis as positive shear stress.

Second we need to locate the centre of circle that is average normal stress. The average normal stress is sigma x plus sigma y divided by 2.

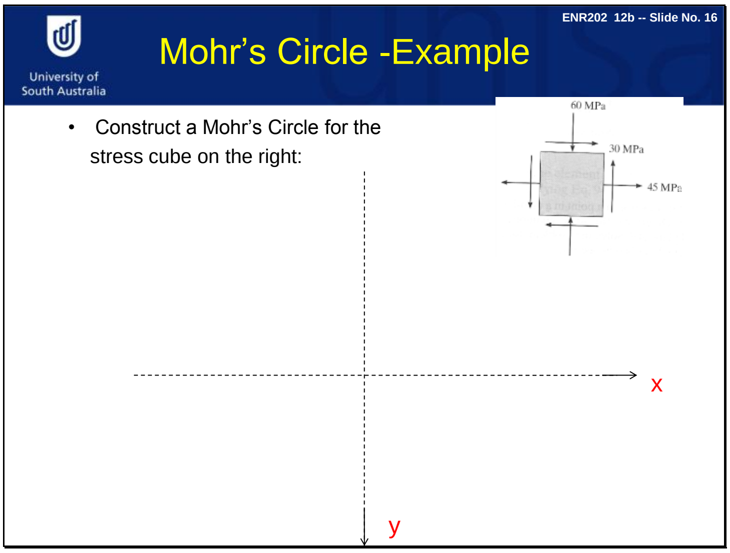
Third, you need to locate point A. the point A have the coordinates sigma x on x-axis and tou xy on y-axis. These sigma x and tou xy as shown in the right edge of the element called x face.

Fourth, you need to locate point G. the point G have the coordinates sigma y on x-axis and tou xy on y-axis. These sigma y and tou xy as shown in the top edge of the element called y face.

Fifth, connect the points A, C and G. This line forms the diameter of the Mohr's circle for this stress element.

Ok, Now we start to draw a Mohr circle for particular state of stress at a point. Can we start now

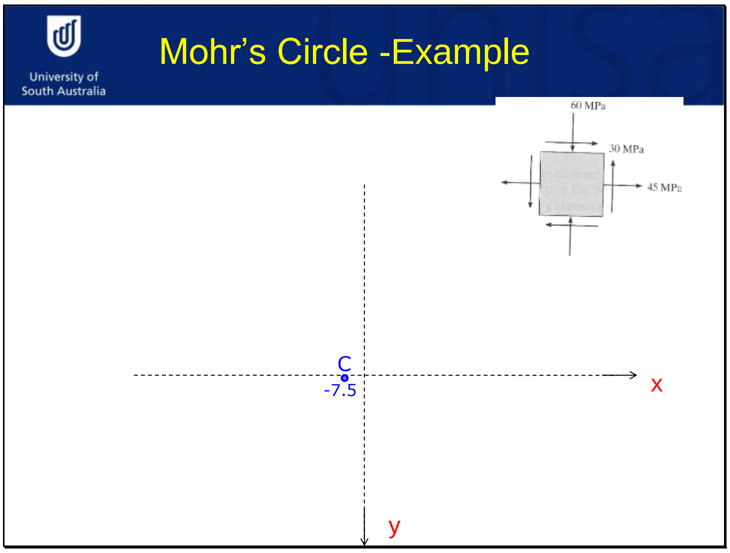
Slide 16



Now, I explain how to draw Mohr's circle for this state of stress at a point. This point have sigma x is positive 45 Mega Pascals, sigma y is minus 60 Mega Pascals and tou xy is positive 30 Mega Pascals.

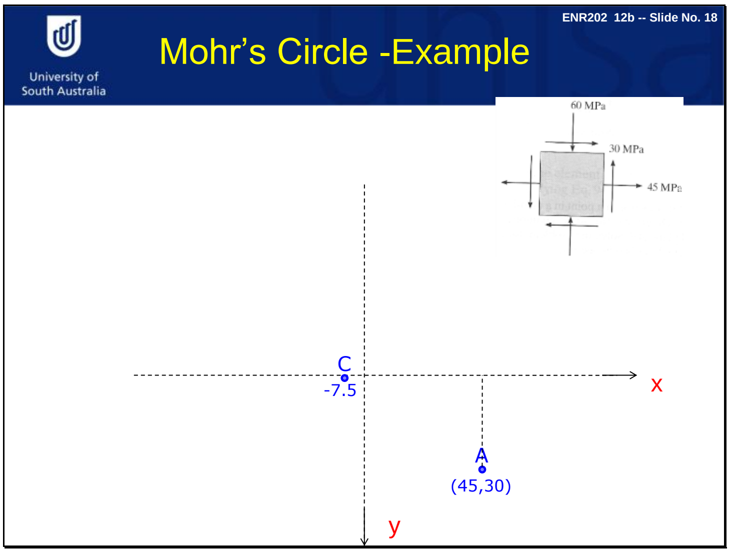
First, we need to set up a coordinate system that right x-axis as positive normal stress and down y-axis as positive shear stress.

Slide 17



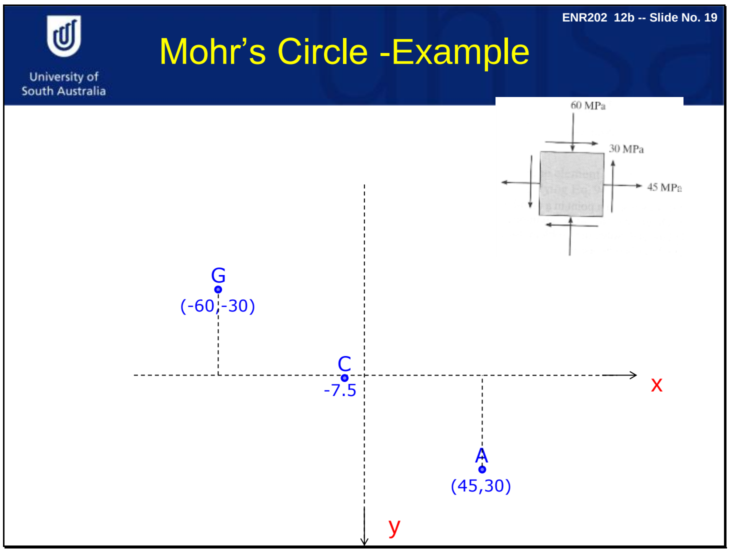
Second we need to locate the centre of circle that is average normal stress. The average normal stress is sigma x plus sigma y divided by 2. sigma x equal 45 Mega Pascals, sigma y equal minus 60 Mega Pascals. The average normal stress equal to plus 45 minus 60 divided by 2 equal to minus 7.5 Mega Pascals. Therefore, the centre point C should be negative side of x axis as shown in the figure.

Slide 18



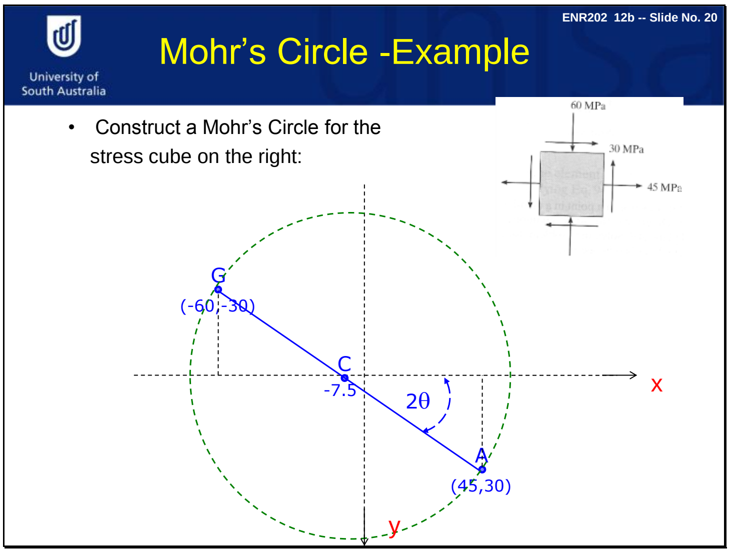
Third, you need to locate point A. the point A have the coordinates sigma x on x-axis and tou xy on y-axis. These sigma x and tou xy as shown in the right edge of the element called x face. On right edge of this element have normal stress is positive 45 Mega Pascals and positive shear stress 30 Mega Pascals. The coordinates of A is 45,30 as shown in the figure.

Slide 19



Fourth, you need to locate point G. the point G have the coordinates sigma y on x-axis and tou xy on y-axis. These sigma y and tou xy as shown in the top edge of the element called y face.
On top edge of the element have sigma y equal to minus 60 Mega Pascals, tou xy equal to minus 30 Mega Pascals. At this state I want to tell one thing that, on top edge if shear stress acting left ward direction is positive, so shear stress on top edge equal to minus 30 Mega Pascals. The coordinates of G point is -60 on x axis -30 on y axis.

Slide 20

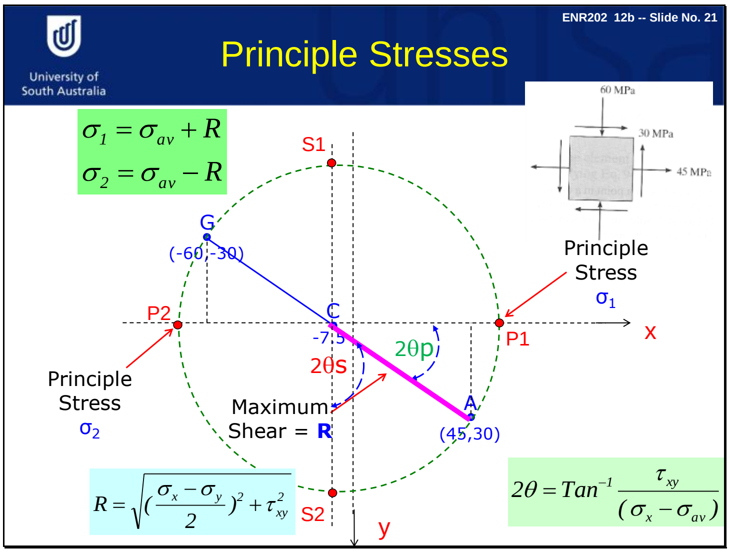


Fifth, connect the points A, C and G. This line forms the diameter of the Mohr's circle for this stress element. Draw Mohr circle with C point as centre of circle and A or G as radius of circle. Finally we have Mohr circle at this state of stress. Why we draw Mohr circle. Do you remember that.

We need to find magnitude and direction of principal stresses and maximum shear stress.

I would like to explain one important concept here that we have 90 degrees from x face to y face that right edge to top edge. The x face point is A point in Mohr circle, the y face point is G point in Mohr circle that means we have 180 degrees from A point to G point that is double to angle of real element angle. So we used 2theta in Mohr circle.

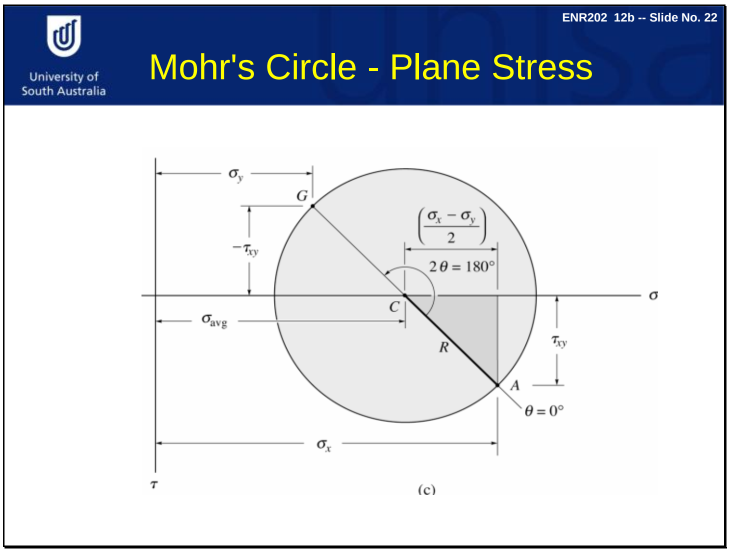
Slide 21



This ACG line represent the plane of given state of stress. We have several lines which join centre of circle. We need to find a line such a way that maximum normal stress and minimum principal stress that is principal stress. As I said before that we don't have shear stresses in principal plane that means x axis is principal stress planes. You know that x-axis represents normal stress. We have maximum principal stress sigma 1 is point P1 as shown in the figure and minimum principal stress sigma 2 is point P2 as shown in the figure. The line P1-C-P2 represents the principal plane. The direction of principal plane is '2 times theta p' as shown in the figure from A-C-G line to P1-C-P2 line is anti clock wise direction. Radius of the circle is nothing but maximum shear stress. S1 and S2 points are maximum shear stresses. Orientation of maximum shear stress is the angle between C-A line to C-S2 line is 2 times 'theta s' in clock wise direction. We used symbol P for principal stress, S for maximum shear stress. P1-C-P2 is principal plane, S1-C-S2 is maximum shear stress plane. Can you observe now. We have normal stress on maximum shear stress plane that is average normal stress that is minus 7.5 Mega Pascals.

Now, I recommend you to draw Mohr circle by on your own, find magnitude and direction of principal planes and maximum principal planes. Can you do now? Try to calculate magnitude and direction of principal planes and maximum principal planes by formulas given in the slides 5 and 6 of this lecture and compare with your results.

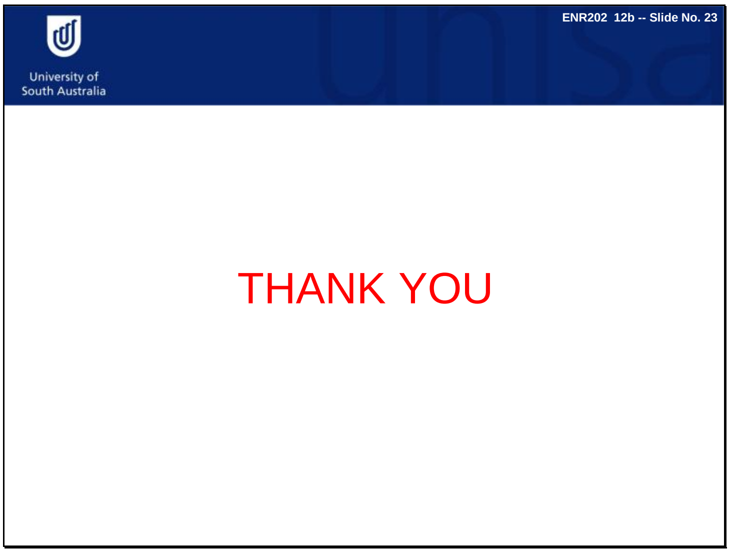
Slide 22



Here, one more Mohr circle that sigma x, sigma y and 'tou xy' as shown in the figure. Average normal stress is centre of circle from origin of coordinates. A point represents the x face that is right edge of given state of stress element, G point represents the y face that top edge of given state of stress element. ACG line represents for given state of stress at a point. X-axis is principal plane. The orientation of angle starts from ACG line. You can find the orientation of any plane from ACG line.

It is two dimensional state of stress. Do you think what will be the Mohr circle for 3-dimensional state of stress. That is shape of 'BALL'.

Slide 23



Thank you for your attention. Now we finished Mechanics and structures. All the best for your final exam.