# Example of Sketching a Curve using Derivatives

Consider the curve $y = 2x^3 + 4x^2 + 2x$.

(a) Find the $x$–intercepts.
$x$ intercepts when $y = 0$.

$$\begin{aligned} 2x^3 + 4x^2 + 2x &= 0 \\ 2x(x^2 + 2x^2 + x) &= 0 \\ 2x(x+1)^2 &= 0 \end{aligned}$$

So, $x$–intercepts are $(0, 0)$ and $(-1, 0)$ (repeated root - graph will touch the $x$ axis here).

(b) Find the $y$–intercept.
$y$ intercept when $x = 0$.

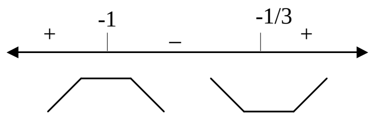
$$\begin{aligned} y(0) &= 0 + 0 + 0 \\ &= 0 \end{aligned}$$

So, $y$–intercept is $(0, 0)$.

(c) Find the first derivative, set it to zero, and solve for $x$.
$y' = 6x^2 + 8x + 2$

$$\begin{aligned} 6x^2 + 8x + 2 &= 0 \\ (x+1)(3x+1) &= 0 \end{aligned}$$

So, $x = -1$ and $x = -1/3$.
$y(-1) = -2(-1+1)^2 = 0$.
$y(-1/3) = -2(1/3)(-(1/3)+1)^2 = -(2/3)(4/9) = -8/27$.

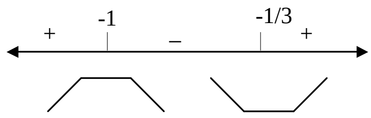



There is a maximum at $(-1, 0)$, and a minimum at $(-1/3, -8/27)$.

(d) Find the second derivative, set it to zero, and solve for $x$.
$y'' = 12x + 8$

$$\begin{aligned} 12x + 8 &= 0 \\ 12x &= -8 \\ x &= -2/3 \end{aligned}$$

So, $x = -2/3$. $y(-2/3) = -2(2/3)(-(2/3) + 1)^2 = -(4/3)(1/9) = -4/27$.




There is a point of inflection at $(-2/3, -4/27)$.