Test Your Understanding: Week 5

1. Find the following Cartesian products if we have X={1,2,3}, Y={a,b}, Z={x,y,z}.
(a) $X \times Y$ (b) $Y \times Z$

2. Find the power set of A if A={1,2,3}.

3. Test the following sets of ordered pairs to see if they are functions from the set X={1,2,3,4} to Y={a,b,c,d}. For those that are functions, test them to see if they are one to one and onto. For those that are not, explain why they fail to qualify as a function.
(a) f_1={ (1,d), (2,c), (3,a), (4,b), (1,a) }
(b) f_2={ (2,c), (3,c), (1,c), (4,c) }
(c) f_3={ (2,d), (3,c), (4,a) }
(d) f_4={ (1,c), (2,b), (3,d), (4,a) }

4. Show how the following data would be stored in an array of length 13 (indexed from 0 to 12) using the hash function $h(x)=x$ mod 13. Show all working.
19, 43, 56, 78, 64, 129, 47, 55

5. (a) Show that if n is an odd integer then $\lfloor n/2 \rfloor=(n-1)/2$.
(b) Show that if n is an even integer, then $\lfloor (n+1)/2 \rfloor=n/2$.

6. The MOD operator on integers, % in Java, has the form n MOD m = r, where n, m and r are all integers. Is MOD a binary operator or an unary operator?

7. It is desired to test a relation for the properties reflexivity, symmetry, anti-symmetry and transitivity. The set X is all positive integers, and R is defined by $(x,y)\in R$ if $(x+y)$ MOD 2 =0.

(a) What is meant, in plain English, by A MOD 2 = 0, if A is any quantity?
(b) What would it mean for R to be reflexive? Which x values from the set of positive integers possess the property that $(x,x)\in R$?
(c) If $(x,y)\in R$, what can you say about x and y? What does this tell us about x and y? Will (x,y) be in R?
(d) If (x,y) and (y,z) are both in R, what does this tell us about x, y and z? What will be true of x and z? Is $(x,z)\in R$?