Chapter 4 Exercises

(Question 22, Exercises section 4.3)

```
for i=1 to n
  for j=1 to n
    for k=1 to i
      x=x+1
```

Complete the following table.

i	*j*	*k*	Ops this *j*	Ops this *i*	Total So Far
1	1	1	1		
	2	1	1		
	…				
	n	1	1	*n*	*n*
2	1	1,2	2		
	2	1,2	2		
	…				
	n	1,2	2	2*n*	*n*+2*n*
3	1	1,2,3	3		
	2	1,2,3	3		
	…				
	n	1,2,3	3	3*n*	*n*+2*n*+3*n*
4				4*n*	*n*+2*n*+3*n*+4*n*
…					
n				*n***n*	$n+2n+\ldots+n^2$

Hence the total number of operations is $n+2n+3n+...+n*n = n(1+2+3+...+n) = \frac{1}{2}n^2(n+1) = \frac{1}{2}(n^3+n^2) = \Theta\left(n^3\right)$.

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(Question 20, Exercises section 4.3)

```
for i=1 to n
  for j=1 to ⌊i/2⌋
    x=x+1
```

i	*j*	Ops This *i*	Total So Far
1	-	0	
2	1	1	1
3	1	1	1+1
4	1,2	2	1+1+2
5	1,2	2	1+1+2+2

6	1,2,3	3	1+1+2+2+3
7	1,2,3	3	1+1+2+2+3+3
8	1,2,3,4	4	1+1+2+3+3+4
…			

So it seems that the number of operations will be of the form $1+1+2+2+3+3+\ldots+\lfloor n/2\rfloor+\lfloor n/2\rfloor$ if n is even, and $1+1+2+2+3+3+\ldots+\lfloor n/2\rfloor$ if n is odd.

Suppose n is even, in which case $\lfloor n/2\rfloor=n/2$. Then the number of operations is $2\left(1+2+\ldots+\frac{n}{2}\right)=2*\frac{1}{2}\frac{n}{2}\left(\frac{n}{2}+1\right)$

$$=\frac{n}{2}\left(\frac{n}{2}+1\right)$$

$$=\frac{n^2}{4}+\frac{n}{2}$$

which is, of course, $\Theta\left(n^2\right)$. If n is odd, then $\left\lfloor\frac{n}{2}\right\rfloor=\frac{n-1}{2}$. Hence the number of operations will be as follows.

$$1+1+2+2+\ldots+\left\lfloor\frac{n}{2}\right\rfloor$$

$$=1+1+2+2+\ldots+\frac{n-1}{2}+\frac{n-1}{2}-\frac{n-1}{2}$$

$$=2\left(1+2+\ldots+\frac{n-1}{2}\right)-\frac{n-1}{2}$$

$$=2*\frac{1}{2}*\frac{n-1}{2}*\left(\frac{n-1}{2}+1\right)-\frac{n-1}{2}$$

$$=\frac{n-1}{2}*\frac{n+1}{2}-\frac{n-1}{2}$$

$$=\frac{1}{4}\left(n^2-1\right)-\frac{1}{2}(n-1)$$

$$=\Theta\left(n^2\right)$$

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