Test Your Understanding Solutions: Week 2

1. Decide whether the following are true or false. Give reasons.
(a) $\forall x(x > 0)$ False, eg *x*=-1.
(b) $\exists x(x \geq 0)$ True, eg *x*=0.
(c) $\forall x(x^2 \geq 0)$ True, if *x*>0, then x^2>0 also. And if *x*=0, then x^2=0, and if *x*<0, then x^2>0. Hence in all cases, $x^2 \geq 0$.

2. (a) Suppose that *m* and *n* are even integers. Then *m*=2*j* and *n*=2*k*, for some integers *j* and *k*. Now *m***n*=(2*j*)*(2*k*)=2*(2*jk*), and 2*jk* is an integer. Hence *m***n* is also even.
(b) Suppose that *m* and *n* are odd. Then *m*=2*j*+1 and *n*=2*k*+1, for some integers *j* and *k*. Then *m***n*=(2*j*+1)(2*k*+1)=4*jk*+2*j*+2*k*+1=2(2*jk*+*j*+*k*)+1, and of course 2*jk*+*j*+*k* is an integer. Hence *m***n* is odd.

3. Assume that for all integers *n*, 5*n*+2 is divisible by 5. Then for any given value of *n*, 5*n*+2=5k, for some integer *k*. Then

$$5n = 5k - 2$$
$$n = \frac{5k-2}{5}$$
$$= k - \frac{2}{5}$$

And *k* is an integer. Hence *n* itself cannot be an integer, which is a contradiction. Hence 5*n*+2 cannot be divisible by 5 for any *n* value.

4. (a) If the plane has one line running through it, then there are two regions, one on each side of the line. Now $\frac{1^2+1+2}{2} = \frac{4}{2} = 2$, which is the correct number of regions.
Assume: that *n* lines in the plane, no two parallel and no three meeting in a point, for some *n*≥1, will divide the plane into $\frac{n^2+n+2}{2}$ regions.
Try to prove: that *n*+1 lines in the plane, no two parallel and no three meeting in a point, will divide the plane into $\frac{(n+1)^2+(n+1)+2}{2} = \frac{n^2+3n+4}{2}$ regions.

(b) Now $(1+x)^1 = 1+x = 1+1*x$.
Assume that $(1+x)^n \geq 1+nx$, for some *n*≥1. Try to prove that $(1+x)^{n+1} \geq 1+(n+1)x$.

(c) 12c=3*4c, 13c=2*4c+1*5c, 14c=1*4c+2*5c, 15c=3*5c. Hence postage of 12, 13, 14 or 15c can be made up using only 4c and 5c stamps. (Note that the

number of values in my basis step must be equal to the value of my smallest stamp, in this case a 4c stamp.)
Assume that postage of 12, 13, 14, 15,…,nc can be made up using only 4c and 5c stamps, for some $n \geq 15$.
Try to prove that postage of $n+1$c can be made up using only 4c and 5c stamps.