Test Your Understanding: Week 6

1. The relation R_1, from $X=\{\alpha, \beta, \delta\}$ to $Y=\{a, b, c\}$, and relation R_2 from Y to $Z=\{x, y, z\}$ have the matrices below.

$$A_1 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$ (with the above ordering of the sets X, Y and Z).

(a) Multiply the two matrices A_1 and A_2 together.

(b) Use this answer to find the matrix of the composite relation $R_2 o R_1$.

(c) Use this matrix to find $R_2 o R_1$ as a set of ordered pairs.

2. Order the following functions with the fastest growing ones first and the slowest growing ones last. They are currently in random order.

$$n^2 \lg(n), \left(\lg(n)\right)^{10}, n^2, 2^n, n!, n*2^n, n^2\left(\lg(n)\right)^2$$

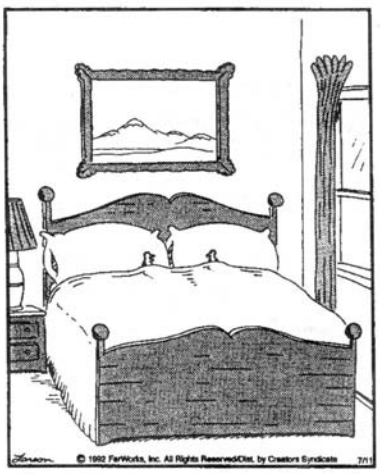
3. The function f and g are known to be $\Theta\left(n^2\right), \Theta\left(n \lg(n)\right)$ respectively.

(a) Write down what these statements mean in terms of the constants c_1, c_2, c_3, c_4, n_1, n_2, n_3 and n_4.

(b) Write down inequalities for $f(n)+g(n)$, and hence show that $f(n)+g(n)=\mathrm{O}\left(n^2\right)$, $f(n)+g(n)=\Omega\left(n^2\right)$.

(c) Draw the appropriate conclusion about the long term behaviour of $f(n)+g(n)$

4. A sequence of length n is to be split into two, at $\lceil n/2 \rceil$. Draw up a table of n, $\lceil n/2 \rceil$ and the members of the upper portion of the sequence. Count them and try to discover a formula for the number of sequence members from $\lceil n/2 \rceil$ to n.



"Well, here we are, my little chickadee."