Tutorial 1 Solutions

1.1

Question	*Proposition?*	*Negation*
1	Yes	2+5≠19
2	No	
3	Yes	There is no positive integer *n* with 19340=*n*.17.
4	Yes	Audrey Meadows was not the original "Alice" in "The Honeymooners".
5	No, instruction.	
6	Yes	The line "Play it again, Sam" does not occur in the movie "Casablanca".
7	Yes	Not every even integer >4 is the sum of two primes.
8	No, garbage.	

$p = \mathrm{F}, q = \mathrm{T}, r = \mathrm{F}$

17. $\neg(p \vee q) \wedge (\neg p \vee r)$

$\equiv \neg(\mathrm{F} \vee \mathrm{T}) \wedge (\mathrm{T} \vee \mathrm{F})$

$\equiv (\neg\mathrm{T}) \wedge \mathrm{T}$

$\equiv \mathrm{F} \wedge \mathrm{T}$

$\equiv \mathrm{F}$

18. $(p \vee \neg r) \wedge \neg((q \vee r) \vee \neg(r \vee p))$

$\equiv (\mathrm{F} \vee \mathrm{T}) \wedge \neg((\mathrm{T} \vee \mathrm{F}) \vee \neg(\mathrm{F} \vee \mathrm{F}))$

$\equiv \mathrm{T} \wedge \neg(\mathrm{T} \vee \neg\mathrm{F})$

$\equiv \mathrm{T} \wedge \mathrm{F}$

$\equiv \mathrm{F}$

21.

p	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

28. *p*: 5<9, *q*: 9<7, *r*: 5<7
It is not the case that (5<9 and 9<7)

$\neg(p \wedge q) \equiv \neg(\mathrm{T} \wedge \mathrm{F}) \equiv \mathrm{T}$

38. *p*: Today is Monday, *q*: It is raining, *r*: It is hot
$\neg(p \vee q) \wedge r \equiv$ It is not the case that (today is Monday or it is raining) and it is hot $\equiv \neg(\mathrm{T} \vee \mathrm{F}) \wedge \mathrm{T} \equiv \mathrm{F} \wedge \mathrm{T} \equiv \mathrm{F}$

1.2

2. (*q* if *p* form) The *q* part is that follows from the other part, ie Rosa may graduate. Hence:
If Rosa has 160 ¼ hours of credit then she may graduate.

3. (*q* is necessary for *p* form) The necessary part, obtaining $2K, is *q*. Hence:
If Fernando is to buy a new computer then he must obtain $2K.

7. (*p* only if *q*) The only if part, *q*, is being well structured. Hence:
If the program is readable then it is well structured.

33. *p*: Today is Monday, *q*: It is raining, *r*: It is hot
$\neg q \rightarrow (r \wedge p) \equiv$ If it is not raining then (today is hot and it is Monday)

40. $|1| < 3$ if $-3 < 1 < 3$ (*q* if *p* form)
p: -3<1<3, *q*: |1|<3

if -3<1<3 then |1|<3 (conditional)
$p \rightarrow q \equiv \mathrm{T} \rightarrow \mathrm{T} \equiv \mathrm{T}$

if |1|<3 then -3<1<3 (converse)
$q \rightarrow p \equiv \mathrm{T} \rightarrow \mathrm{T} \equiv \mathrm{T}$

if (1≤-3 or 1≥3) then |1|≥3
$\neg q \rightarrow \neg p \equiv \mathrm{T} \rightarrow \mathrm{T} \equiv \mathrm{T}$

44.

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Same
A B

49.

p	q	r	$p \to q$	$q \to r$	$A \wedge B$	$p \to r$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	***F***	***T***
T	F	F	F	T		
F	T	T	T	T		
F	T	F	T	F		
F	F	T	T	T		
F	F	F	T	T		

← False

52.

p	q	p *imp1* q	q *imp1* p	
T	T	T	T	(=T *imp1* T=T)
T	F	F	F	(=F *imp1* T=F)
F	T	F	F	(=T *imp1* F=F)
F	F	T	T	(=F *imp1* F=T)

<u>1.3</u>

37. $\forall x\left(x>1 \to \frac{x}{x^2+1}<\frac{1}{3}\right)$

Now $\frac{x}{x^2+1}<\frac{1}{3}$ can only be true if

$3x < x^2+1$

$0 < x^2-3x+1$

And at $x=1$, $x^2-3x+1=-1$ and so at

$x=1$, $x^2-3x+1=-1.09<0$.

Hence, at $x=1.1$, $\frac{x}{x^2+1}>\frac{1}{3}$

Here I went straight for the boundary, x=1, since that seemed where the claim was most likely to break down.