Tutorial 3 Solutions

Preparation

Section 2.1

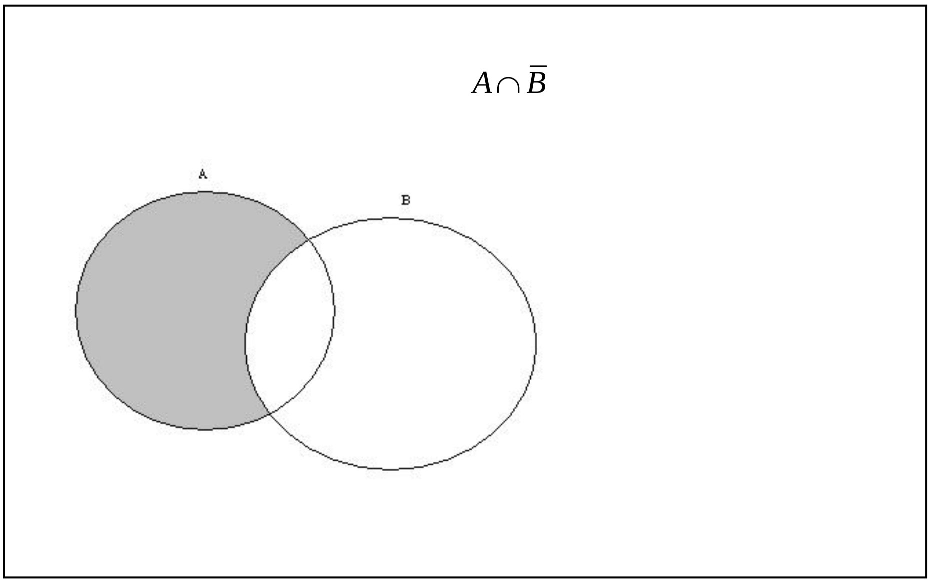
U={1, 2, 3, 4, 5, 6, 7, 8, 9 10}, *A*={1, 4, 7, 10}, *B*={1, 2, 3, 4, 5}, *C*={2, 4, 6, 8}.

4. *B*-*A*={2, 3, 5}

10. $A \cup U = U$

16. $(A \cup B) - (C - B) = \{1, 2, 3, 4, 5, 7, 10\} - \{6, 8\}$
$= \{1, 2, 3, 4, 5, 7, 10\}$

17.



25. If 10 take all 3, and 20 take French and music, then 20-10=10 must take French and music but not business.

32. $X \times Y = \{ (1,a), (1,b), (1,c), (2,a), (2,b), (2,c) \}$

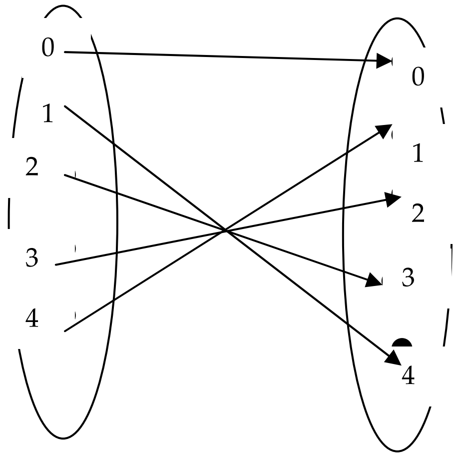
Section 2.2

38.

x	4*x* mod 5	Working
0	0	0 mod 5 = 0

1	4	4 mod 5 = 4
2	3	8 mod 5 = 3
3	2	12 mod 5 = 2
4	1	16 mod 5= 1

Ie $f = \{ (0,0), (1,4), (2,3), (3,2), (4,1) \}$



Yes, *f* is one to one, as every 2nd member has at most one 1st member relating to it. It is also onto, as every member of *X* appears as a 2nd member of an ordered pair.

81.

$$n = 2k + 1$$

$$\text{LHS} = \left\lfloor \frac{n^2}{4} \right\rfloor = \left\lfloor \frac{(2k+1)^2}{4} \right\rfloor$$

$$= \left\lfloor \frac{4k^2 + 4k + 1}{4} \right\rfloor$$

$$= \lfloor k^2 + k + 0.25 \rfloor$$

$$= k^2 + k$$

$$\text{RHS} = \frac{n-1}{2} * \frac{n+1}{2}$$

$$= \frac{2k}{2} * \frac{2k+2}{2}$$

$$= k(k+1)$$

$$= k^2 + k = \text{LHS}$$

Section 3.1

13. $R = \{ (a,b), (a,c), (b,a), (b,d), (c,c), (c,d) \}$

19. $R = \{ (1,1), (1,4), (2,2), (2,5), (3,3), (4,1), (4,4), (5,2), (5,5) \}$

(Note that it is important here to realise that negative numbers and zero can also be divisible by an integer, eg (1,4)∈*R* since -3 is divisible by 3.)

21. Domain of *R*={1, 2, 3, 4, 5}.

22. Range of *R*={1, 2, 3, 4, 5}.

32. Reflexivity: *R* is reflexive if (*x*,*x*)∈*R* for all *x*. So we need to ask is (*x*,*x*)∈*R*, ie is *x*=*x* true? Yes, it always is, so (*x*,*x*)∈*R* always, ie *R* is reflexive.
Symmetry: Its not possible for there to be any pairs (*x*,*y*) with *x*≠*y*. Hence the relation is anti-symmetric (see Example 3.1.15, 6th edition). This is an exception to the rule that you normally have to test both properties.
Anti-symmetry: as noted above, this relation is antisymmetric.
Transitivity: suppose (*x*,*y*) and (*y*,*z*)∈*R.* Then *x*=*y* and *y*=*z*, so *x*=*z*. Hence (*x*,*z*)∈*R* also. OR: as there are no pairs (*x*,*y*) and (*y*,*z*)∈*R* with *x*, *y* and *z* all different, the relation is transitive.

38. $R_1 \circ R_2 = \{ (1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,2) \}$
$R_2 \circ R_1 = \{ (1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,2) \}$

39. Eg $R = \{ (1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3)\ (4,4) \}$

Section 3.3

3.
$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

8. $R = \{(a,w), (a,y), (c,y), (d,w), (d,x), (d,y), (d,z)\}$

17.
A_1=

	2	*3*	*4*	*5*
2	1	0	1	0
3	0	1	0	0
4	0	0	1	0
5	0	0	0	1

A_2=

	1	*2*	*3*	*4*
1	0	0	0	0
2	1	0	0	0
3	1	1	0	0

4	1	1	1	0

A_1A_2=

	2	*3*	*4*	*5*
1	1	1	0	0
2	1	1	0	0
3	1	1	0	0
4	1	1	1	0

Hence $R_2oR_1 = \{(1,2), (1,3), (2,2), (2,3), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

Tutorial

2.1

52. No these sets are not equal, because one allows real numbers (ie decimal numbers) such as 0.83, 1.52, etc, while the other one only has the integers 1 and 2.

54.
$= \{ \varnothing, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\} \}$

60. Yes, this statement is true. Briefly, *X*-*Y* is everything that is in *X* but not *Y*, while *Y*-*X* is everything that is in *Y* but not *X*. Hence these two sets will have nothing in common, and so their intersection must be empty. Alternatively, use a Venn diagram.

63. This is rubbish. $\overline{X \cap Y}$ incudes things that are outside of *X*, assuming there are such things. So to construct a counter-example, we need only make sets *X* and *Y* proper subsets of the universe. For example, let $U = \{1,2,3,4,5\}$, $X = \{1,2,3\}$, $Y = \{3,4\}$. Then $\overline{X \cap Y} = \overline{\{3\}} = \{1,2,4,5\}$, which is not a subset of *X*.

75. $A \Delta B = \{1,4,5\}$

76. $A \Delta B$ is everything in *A* or *B* but not both.

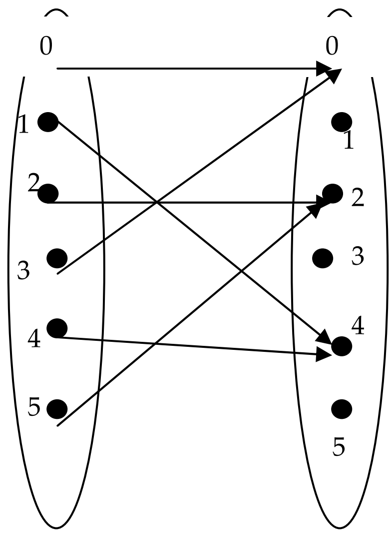
2.2

3. $A - B = \{7,10\}$.

39.

x	4x mod 6	Working
0	0	0 mod 6 = 0
1	4	4 mod 6 = 4
2	2	8 mod 6 = 2
3	0	12 mod 6 = 0
4	4	16 mod 6= 4
5	2	20 mod 6 = 2

$f = \{ (0,0), (1,4), (2,2), (3,0), (4,4), (5,2) \}$



f is not one to one, since, for example, 0 and 3 are both related to 0. It is not onto, since, eg, 1 does not appear anywhere as a 2^{nd} member.

3.1

11.

45. Suppose *R* and *S* are both transitive. Let (*x*,*y*) and (*y*,*z*) be in $R \cap S$. We need to ask is $(x,z) \in R \cap S$? Now if (*x*,*y*) and (*y*,*z*) are in *R* and *S* then $(x,z) \in R$ and *S*. Hence $(x,z) \in R \cap S$ also. So $R \cap S$ must be transitive.

48. Suppose *R* and *S* are both reflexive. Then for every $x \in X$, $(x,x) \in R$ (and *S* also, but one of *R* and *S* is enough for the union). Hence for every $x \in X$, $(x,x) \in R \cup S$. In other words, $R \cup S$ is reflexive.

3.3

6.

	1	*2*	*3*	*4*	*5*
1	0	1	1	1	1
2	0	0	1	1	1
3	0	0	0	1	1
4	0	0	0	0	1
5	0	0	0	0	0

10. $R = \{(w, w), (w, y), (y, w), (y, y), (z, z)\}$