Alternative Solution, Tute 4, Section 7.6, Question 49

$$(x-b)^2 + y^2 = a^2$$

$$2(x-b) + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x-b}{y}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{(x-b)^2}{y^2} = 1+\frac{(x-b)^2}{a^2-(x-b)^2} = \frac{a^2-(x-b)^2+(x-b)^2}{a^2-(x-b)^2} = \frac{a^2}{a^2-(x-b)^2}$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \frac{a}{\sqrt{a^2-(x-b)^2}}$$

$$A = \int_{x=b-a}^{x=b+a} 2\pi x \frac{a}{\sqrt{a^2-(x-b)^2}}\,dx$$

Now if we put $u = x-b$, then $du = dx$, $x = b-a \Rightarrow u = -a$, $x = b+a \Rightarrow u = a$.

$$A = 2\pi a\int_{u=-a}^{u=a} \frac{u+b}{\sqrt{a^2-u^2}}\,du$$

Put $u = a\sin(\theta), du = a\cos(\theta)d\theta$

$$u = -a \Rightarrow \theta = \frac{-\pi}{2}, u = a \Rightarrow \theta = \frac{\pi}{2}$$

$$A = 2\pi a\int_{\theta=\frac{-\pi}{2}}^{\theta=\frac{\pi}{2}} \frac{b+a\sin(\theta)}{a\cos(\theta)} a\cos(\theta)d\theta$$

$$= 2\pi a\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} b + a\sin(\theta)d\theta$$

$$= 2\pi a\left[b\theta - a\cos(\theta)\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$= 2\pi a\left(\frac{\pi}{2}b - a.0 - \left(\frac{-\pi}{2}b - a.0\right)\right)$$

$$= 2\pi^2 ab$$

And of course we double this for the surface area of the whole torus.